

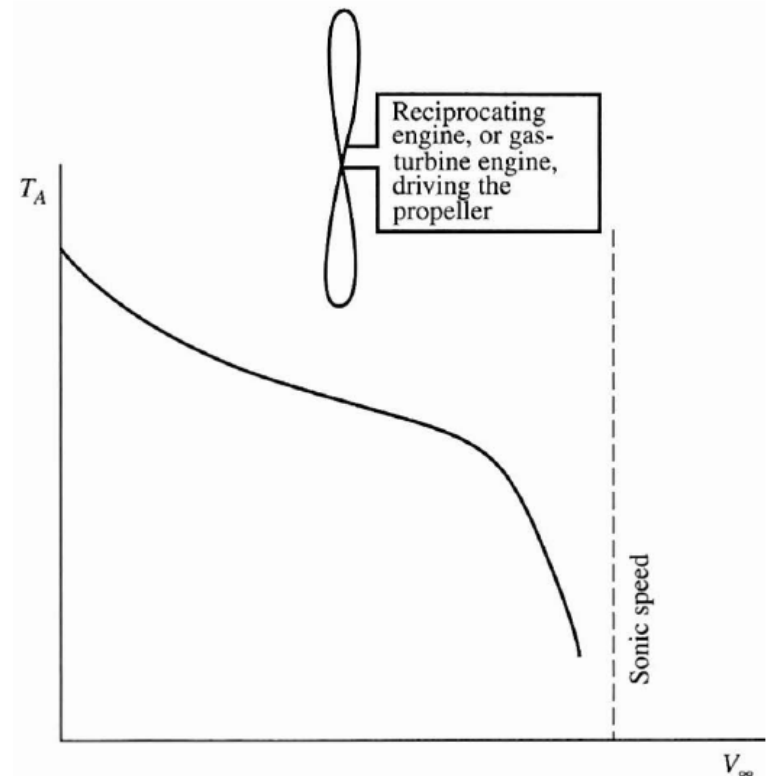
# THRUST AVAILABLE AND THE MAXIMUM VELOCITY OF THE AIRPLANE

Thrust available, denoted by  $TA$ , is the thrust provided by the power plant of the airplane. Unlike the thrust required  $TR$  which has almost everything to do with the airframe (including the weight) of the airplane and virtually nothing to do with the power plant, the thrust available  $TA$  has almost everything to do with the power plant and virtually nothing to do with the airframe. This statement is not completely true; there is always some aerodynamic interaction between the airframe and the power plant. For conventional, low-speed airplanes, this interaction is usually small. However, for modern transonic and supersonic airplanes, it becomes more of a consideration.

# Propeller-Driven Aircraft

An aerodynamic force is generated on a propeller that is translating and rotating through the air. The component of this force in the forward direction is the thrust of the propeller. For a propeller/reciprocating engine combination, this propeller thrust is the thrust available. For a turboprop engine, the propeller thrust is augmented by the jet exhaust, albeit by only a small amount (typically almost **5%**). The combined propeller thrust and jet thrust is the thrust available  **$T_A$**  for the turboprop.

The qualitative variation of  **$T_A$**  with  $V$ , for propeller-driven aircraft is sketched in Figure. The thrust is highest at zero velocity (called the static thrust) and decreases with an increase in  $V$ . The thrust rapidly decreases as  $V$ , approaches sonic speed; this is because the propeller tips encounter compressibility problems at high speeds, including the formation of shock waves. It is for this reason (at least to the present) that propeller-driven aircraft have been limited to low to moderate subsonic speeds.



The propeller is attached to a rotating shaft which delivers **power** from a reciprocating piston engine or a gas turbine (as in the case of the turboprop). For this reason, **power** is the more appropriate characteristic of such power plants rather than thrust. The values of **TA** for a propeller-driven airplane can be readily obtained from the power ratings as follows. The power available from a propeller/reciprocating engine combination is

$$P_A = \eta_{pr} P$$

where  $\eta_{pr}$  is the propeller efficiency and P is the shaft power from the piston engine.

Since power is given by force times velocity

$$P_A = T_A V_\infty \longrightarrow$$

$$T_A = \frac{\eta_{pr} P}{V_\infty}$$

Similarly, for a turboprop,

$$P_A = \eta_{pr} P_{es} \longrightarrow$$

$$T_A = \frac{\eta_{pr} P_{es}}{V_\infty}$$

both P and  $P_{es}$ , are relatively constant with **Vinf**. By assuming a variable-pitch propeller such that the variation of *the efficiency*, with **V**, is minimized, TA decreases as V, increases.

# Jet-Propelled Aircraft

For subsonic speeds  $T_A$  is about constant with the velocity

for **supersonic speeds** 
$$\frac{T_A}{(T_A)_{\text{Mach } 1}} = 1 + 1.18(M_\infty - 1)$$

The effect of the altitude is 
$$\frac{T_A}{(T_A)_0} = \frac{\rho}{\rho_0}$$

For the high-bypass-ratio turbofans commonly used for civil transports, thrust decreases with increasing velocity.

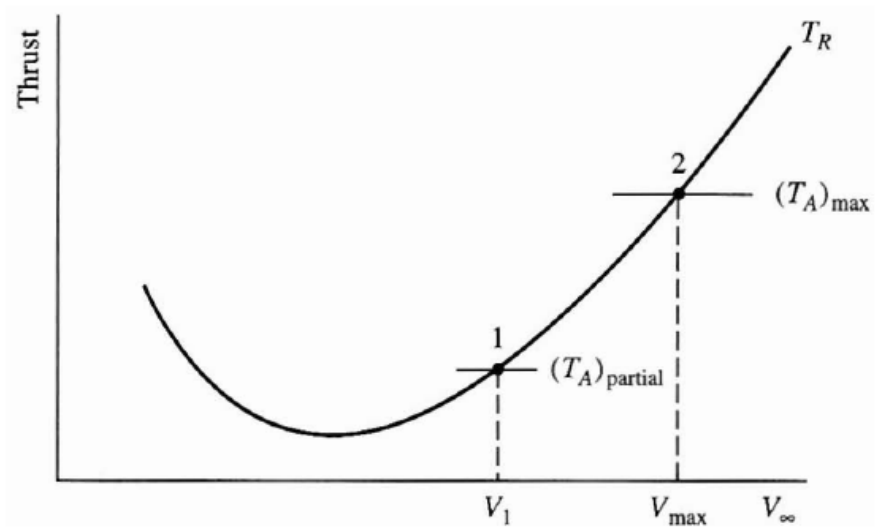
$$\frac{T_A}{(T_A)_{V=0}} = AM_\infty^{-n}$$

where  $(T_A)_{V=0}$  is the static thrust available (thrust at zero velocity) at standard sea level, and  $A$  and  $n$  are functions of altitude, obtained by correlating the data for a given engine. On the other hand, for a low-bypass-ratio turbofan, the thrust variation with velocity is much closer to that of a turbojet, essentially constant at subsonic speeds and increasing with velocity at supersonic speeds.

The altitude variation of thrust for a high-bypass-ratio civil turbofan is correlated where  $(T_A)_0$  is the thrust available at sea level and  $\rho_0$  is standard sea-level density. 
$$\frac{T_A}{(T_A)_0} = \left[ \frac{\rho}{\rho_0} \right]^m$$

# Maximum Velocity

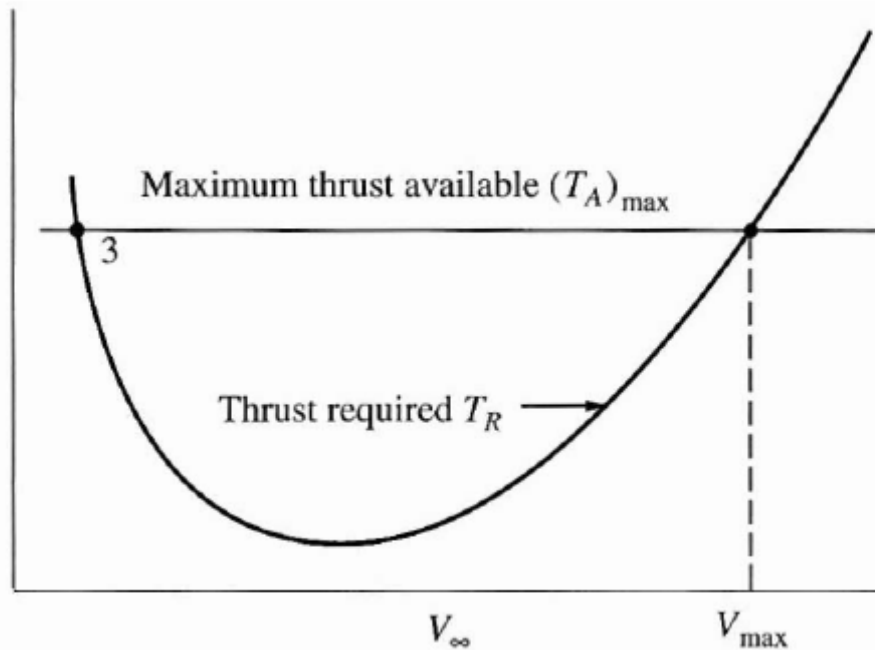
Consider a given airplane flying at a given altitude, with a  $TR$  curve as sketched in Figure. For steady, level flight at a given velocity  $V_1$ , the value of  $TA$  is adjusted such that  $TA = TR$  at that velocity. This is denoted by point 1. The pilot of the airplane can adjust  $TA$  by adjusting the engine throttle in the cockpit. For point 1 the engine is operating at partial throttle, and the resulting value of  $TA$  is denoted by  $(TA)_{\text{partial}}$ .



Then the throttle is pushed all the way forward, maximum thrust available is produced, denoted by  $(TA)_{\text{max}}$ . The airplane will accelerate to higher velocities, and  $TR$  will increase, until  $TR = (TA)_{\text{max}}$ , denoted by point 2. When the airplane is at point 2 in, any further increase in velocity requires more thrust than is available from the power plant. Hence, for steady, level flight, point 2 defines the maximum velocity  $V$ , at which the given airplane can fly at the given altitude.

the thrust available curve is the variation of  $TA$  with velocity at a given throttle setting and altitude. For the throttle full forward,  $(TA)_{max}$  is obtained. The maximum thrust available curve is the variation of  $(TA)$  with velocity at a given altitude. For turbojet and low-bypass-ratio turbofans, we have seen that at subsonic speeds, the thrust is essentially constant with velocity. Hence, the thrust available curve is a horizontal line.

***In steady, level flight, the maximum velocity, of the airplane is determined by the high speed intersection of the thrust required and thrust available curves.***



Note that there is a low-speed intersection of the  $(TA)$  and  $TR$  curves, point **3**. At first glance, this would appear to define the minimum velocity of the airplane in steady, level flight. However, what is more usual is that the minimum velocity of the airplane is determined by its stalling speed, which depends strongly on  $CL_{max}$ , and wing loading.

The generic solution of the velocity as a function of  $T_R$  is

$$V_{\infty} = \left[ \frac{(T_R/W)(W/S) \pm (W/S)\sqrt{(T_R/W)^2 - 4C_{D,0}K}}{\rho_{\infty}C_{D,0}} \right]^{1/2}$$

replacing  $V_{inf}$ , with  $V_{max}$ , and  $T_R$  with  $(T_A)_{max}$ , and taking the plus sign in the quadratic expression because we are interested in the highest velocity, we have the direct calculation of the maximum velocity:

$$V_{\max} = \left\{ \frac{[(T_A)_{\max}/W](W/S) + (W/S)\sqrt{[(T_A)_{\max}/W]^2 - 4C_{D,0}K}}{\rho_{\infty}C_{D,0}} \right\}^{1/2}$$

$V_{max}$ , depends on the maximum thrust-to-weight ratio  $(T_A)_{max}/W$ , (2) wing loading  $W/S$ , (3) the drag polar via  $C_{D0}$  and  $K$ , and (4) altitude via  $\rho$ .

1.  $V_{max}$ , increases as  $(T_A)_{max}/W$  increases.
2.  $V_{max}$ , increases as  $W/S$  increases.
3.  $V_{max}$ , decreases as  $C_{D0}$  and/or  $K$  increases.