POWER REQUIRED

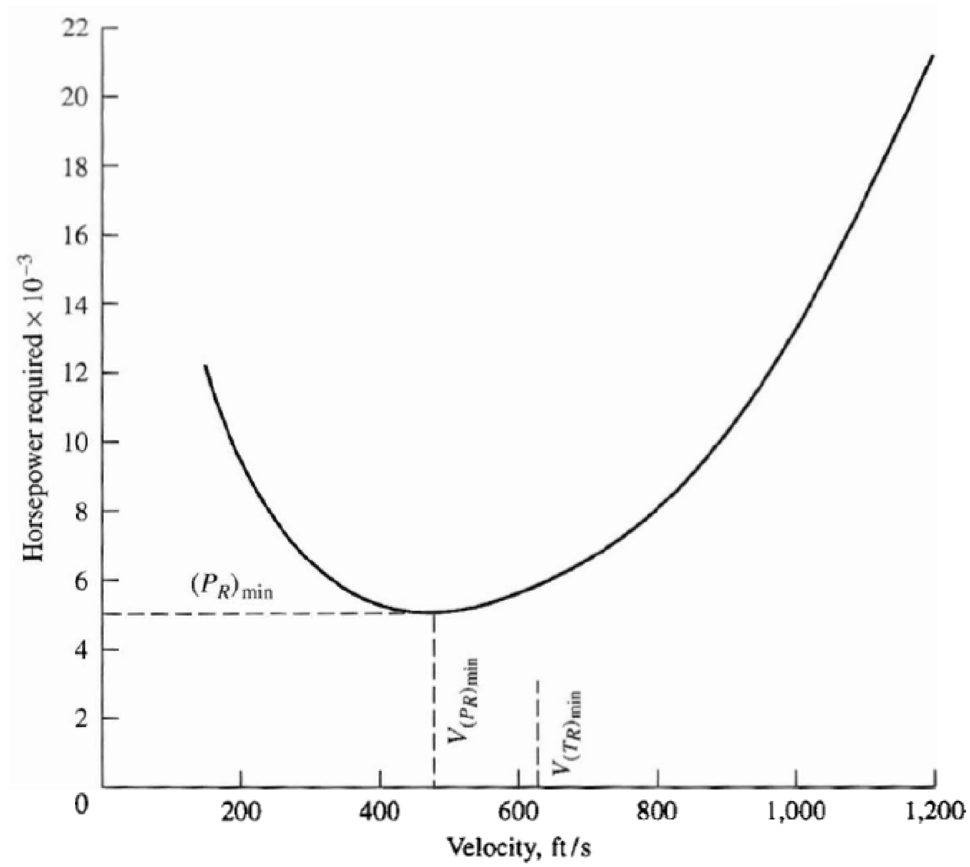
$$P_R = T_R V_\infty = \frac{W}{C_L/C_D} V_\infty$$

Since L = W for steady, level flight,

$$L = W = \tfrac{1}{2}\rho_\infty V_\infty^2 S C_L$$

$$V_\infty = \sqrt{\frac{2W}{\rho_\infty S C_L}}$$

$$P_R = \sqrt{\frac{2W^3 C_D^2}{\rho_\infty S C_L^3}}$$



The minimum power required occurs when the airplane is flying such that $\frac{C_L^{3/2}}{C_D}$ is maximum. By replacing *CD* with the drag polar, this expression can be written as

$$\frac{C_L^{3/2}}{C_D} = \frac{C_L^{3/2}}{C_{D,0} + KC_L^2}$$

To find the conditions that hold for a maximum value we have to differentiate with respect to *CL,* and set the result equal to zero.

$$\frac{d\left(C_L^{3/2}/C_D\right)}{dC_L} = \frac{\left(C_{D,0} + KC_L^2\right)\left(\frac{3}{2}C_L^{1/2}\right) - C_L^{3/2}(2KC_L)}{C_{D,0} + KC_L^2} = 0$$

$$\frac{3}{2}C_{D,0}C_L^{1/2} + \frac{3}{2}KC_L^{5/2} - 2KC_L^{5/2} = 0$$

$$\boxed{C_{D,0} = \tfrac{1}{3}KC_L^2} \longrightarrow C_L = \sqrt{3C_{D,0}/K}$$

$$\frac{C_L^{3/2}}{C_D}$$

$$\boxed{\left(\frac{C_L^{3/2}}{C_D}\right)_{\max} = \frac{1}{4}\left(\frac{3}{KC_{D,0}^{1/3}}\right)^{3/4}}$$

$\frac{C_L^{3/2}}{C_D}$ *is a maximum value, the zero-lift drag equals one third the drag due to* ***lift***

When $C_L = \sqrt{3C_{D,0}/K}$ $\longrightarrow$ $\dfrac{C_L^{3/2}}{C_D}$ Is max

$$W = \frac{1}{2}\rho_\infty V_{(C_L^{3/2}/C_D)_{\max}} S \sqrt{\frac{3C_{D,0}}{K}}$$

$$\boxed{V_{(C_L^{3/2}/C_D)_{\max}} = \left(\frac{2}{\rho_\infty}\sqrt{\frac{K}{3C_{D,0}}}\frac{W}{S}\right)^{1/2}}$$

$$V_{(C_L^{3/2}/C_D)_{\max}} = \left(\frac{1}{3}\right)^{1/4} V_{(L/D)_{\max}}$$

the power available, denoted by ***PA,*** is the power provided by the power plant of the airplane.

$$P_A = T_A V_\infty$$

The maximum power available compared with the power required allows the calculation of the maximum velocity of the airplane.

Propeller-Driven Aircraft

Propellers are driven by reciprocating piston engines or by gas turbines (turboprop). The engines in both these cases are rated in terms of ***power*** (not thrust , as in the case of jet engines). Hence, for propeller-driven airplanes, ***power available*** is much more germane than thrust available

$$P_A = \eta_{pr} P$$

Power *P* is reasonably constant with V

For an unsupercharged engine,

$$\frac{P}{P_0} = \frac{\rho}{\rho_0}$$

For a supercharged engine, *P* is essentially constant with altitude up to the critical design altitude of the supercharger. Above this critical altitude, *P* decreases the velocity and altitude variations of *PA* for a turboprop are as follows:

Power available *PA* is reasonably constant with V, (or M).
The altitude effect is approximated by

$$\frac{P_A}{P_{A,0}} = \left(\frac{\rho}{\rho_0}\right)^n \qquad n = 0.7$$

Turbojet Engines

Turbojet engines are rated in terms of thrust. Hence,

$$\boxed{P_A = T_A V_\infty}$$

At subsonic speeds, ***TA*** is essentially constant.

For supersonic speeds,

$$\frac{T_A}{(T_A)_{\text{Mach 1}}} = 1 + 1.18(M_\infty - 1)$$

The effect of altitude on ***TA*** is given by

$$\boxed{\frac{P_A}{(P_A)_0} = \frac{\rho}{\rho_0}}$$

Turbofan Engines

Turbojet engines are rated in terms of thrust. Hence,

$$P_A = T_A V_\infty$$

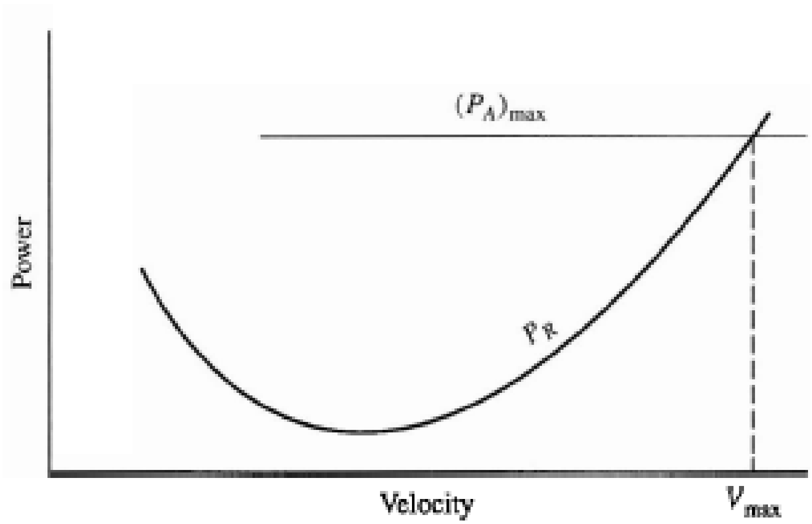
At subsonic speeds, ***TA*** is essentially constant.

$$T_A/(T_A)_{V=0} = AM_\infty^{-n}$$

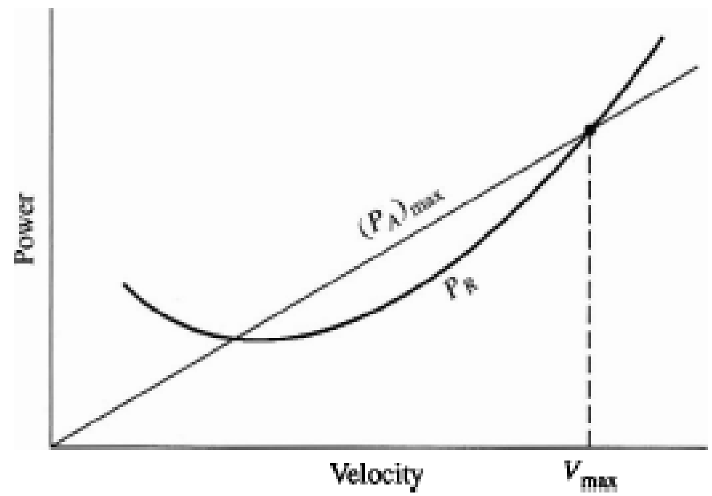
The effect of altitude on ***TA*** is given by

$$\frac{P_A}{(P_A)_0} = \left[\frac{\rho}{\rho_0}\right]^m$$

Maximum Velocity



For a propeller driven airplane, the power available ***PA*** is essentially constant with velocity. The intersection of the maximum power available curve and the power required curve defines the maximum velocity for straight and level flight



For a turbojet-powered airplane, assuming ***TA*** is constant with velocity, the power available at subsonic speeds varies linearly with V. The high-speed intersection of the maximum power available curve and the power required curve defines the maximum velocity for straight and level flight.