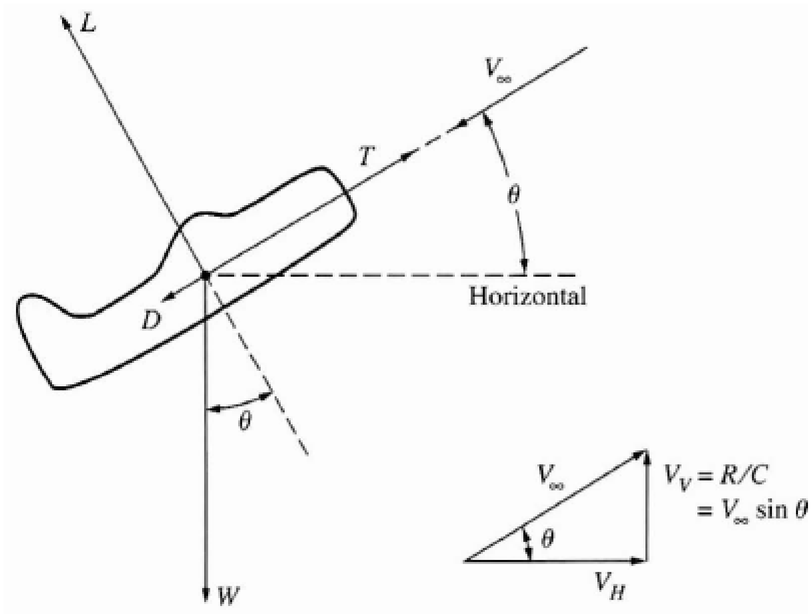
RATE OF CLIMB



$$
\boxed{\begin{aligned} T - D - W\sin\theta &= 0 \\ L - W\cos\theta &= 0 \end{aligned}}
$$

The vertical component of the velocity is, by definition, the ***rate*** of *climb* of the airplane;

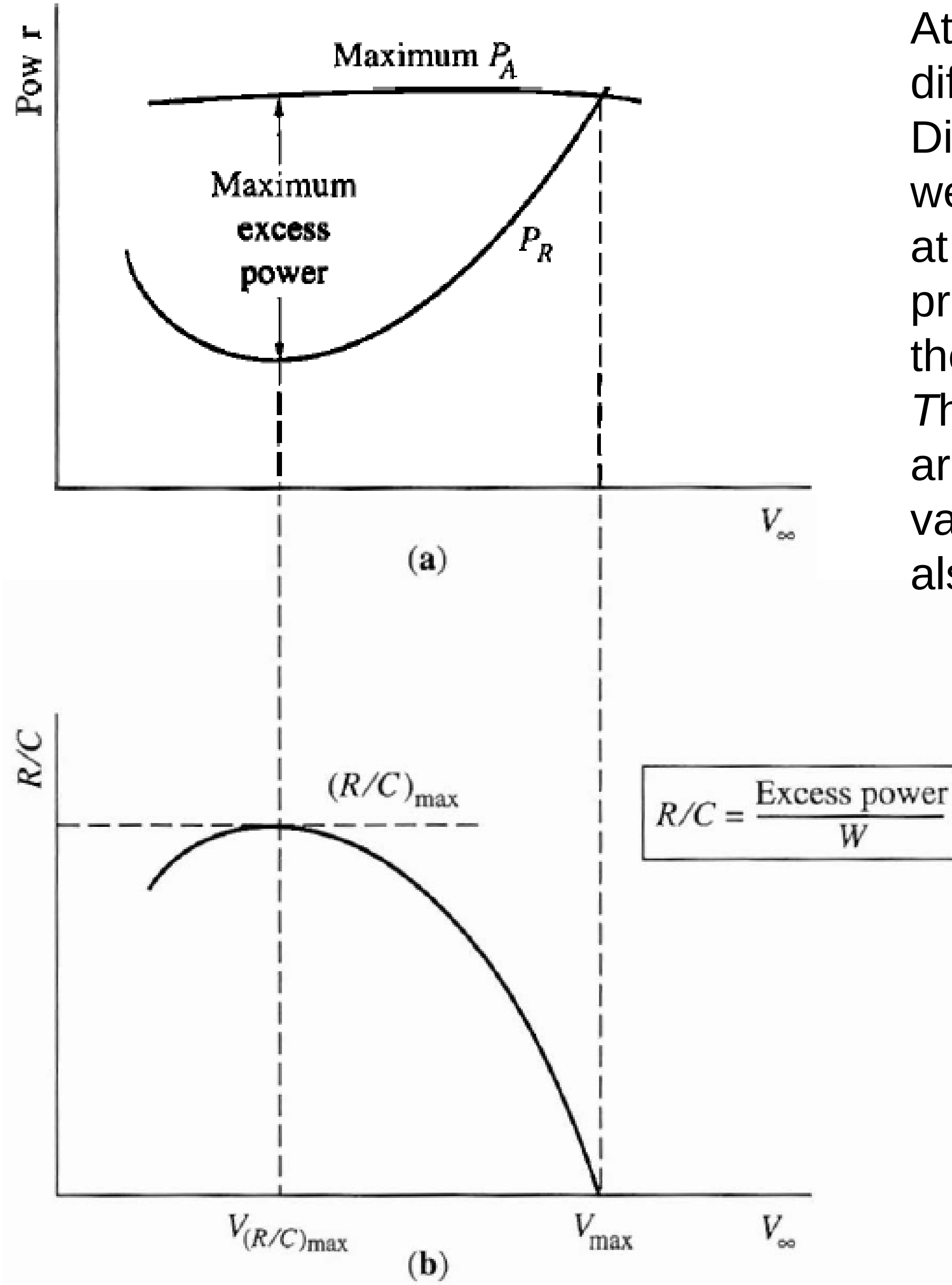
$$\boxed{R/C = V_\infty \sin\theta}$$

Excess of power

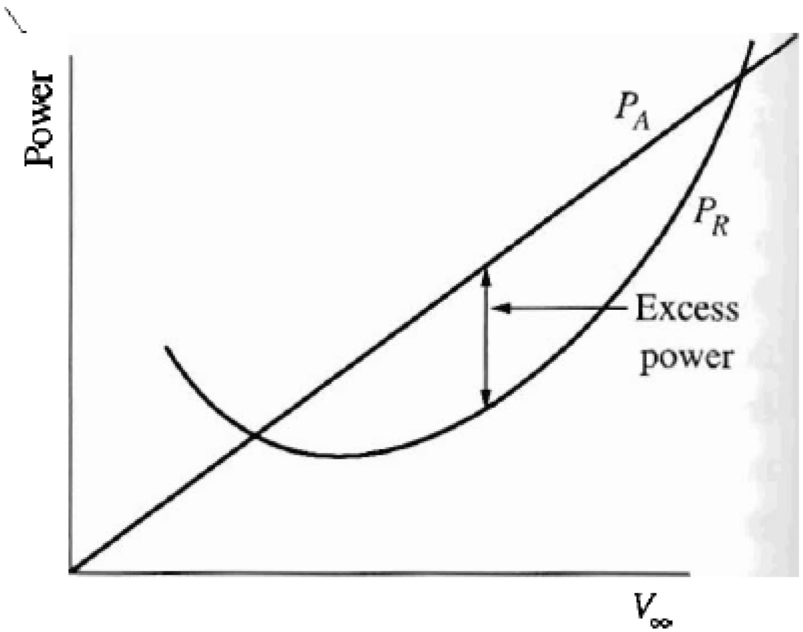
$$\boxed{V_\infty \sin\theta = R/C = \frac{TV_\infty - DV_\infty}{W}}$$

The higher the thrust, the lower the drag, and the lower the weight, the better the climb performance.

$L = W\cos\theta$ because, for climbing flight, part of the weight of the airplane is supported by the thrust, less lift is needed than for level flight and less lift means less drag due to lift. For a given velocity the drag in climbing flight is less than that for level flight.

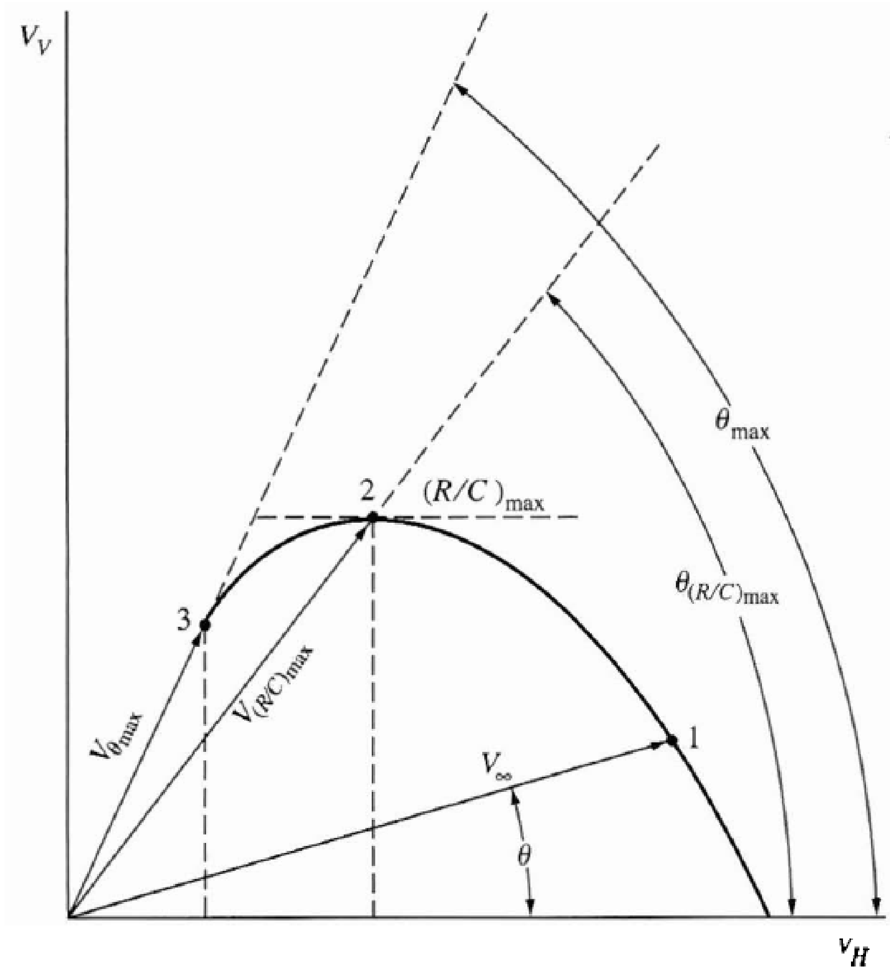


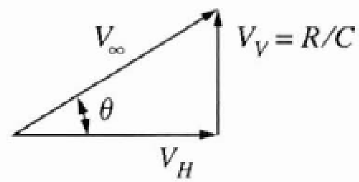
At any *V,* the excess power is the difference between the *PA* and ***PR*****.** Divide this excess power by the weight, obtaining the value of *RIC* at this velocity**.** *Carry* out this process for a range of V, obtaining the corresponding values of *RIC.* *T*he *PA* and *PR* curves sketched are for a given altitude, hence the variation of *RIC* versus velocity is also for a given altitude.



(b) Jet-propelled airplane

In case of a turbojet the thrust required is constant with the velocity therefore the power required has a linear behavior with V.





the hodograph diagram is a plot of the aircraft's vertical velocity *Vv* versus its horizontal velocity *VH***.** The hodograph diagram is slightly different the abscissa is the horizontal component of velocity *VH not* the total velocity V. Consider an arbitrary point on the hodograph curve, denoted by point 1**.** Draw a line from the origin to point 1. Geometrically, the length of the line is *V,* and the angle it makes with the horizontal axis is the corresponding climb angle at that velocity. Point 2 denotes the maximum *RIC;* the length of the line from the origin to point 2 is the airplane velocity at maximum *RIC* and the angle it makes with the horizontal axis is the climb angle for maximum *R/C. A* line drawn through the origin and tangent to the hodograph curve locates point **3.** The angle of this line relative to the horizontal defines the maximum possible climb angle. The maximum rate of climb does *not* correspond to the maximum climb angle. The maximum climb angle is important when you want to clear an obstacle.

Analytical Approach

$$T - D - W\sin\theta = 0$$
$$L - W\cos\theta = 0$$

$$\boxed{V_\infty \sin\theta = R/C = \frac{TV_\infty - DV_\infty}{W}}$$

$$C_L = \frac{L}{q_\infty S} = \frac{W\cos\theta}{q_\infty S}$$

$$D = q_\infty S C_D = q_\infty S\left(C_{D,0} + KC_L^2\right)$$

$$D = q_\infty S\left[C_{D,0} + K\left(\frac{W\cos\theta}{q_\infty S}\right)^2\right]$$

If we approximate cos (theta)=1

$$V_\infty \sin\theta = R/C = V_\infty\left[\frac{T}{W} - \frac{1}{2}\rho_\infty V_\infty^2 \left(\frac{W}{S}\right)^{-1} C_{D,0} - \frac{W}{S}\frac{2K}{\rho_\infty V_\infty^2}\right]$$

The corresponding climb angle is $$\sin\theta = \frac{R/C}{V_\infty}$$

$$\sin\theta = \frac{T}{W} - \frac{1}{2}\rho_\infty V_\infty^2 \left(\frac{W}{S}\right)^{-1} C_{D,0} - \frac{W}{S}\frac{2K}{\rho_\infty V_\infty^2}$$

increasing the thrust-to-weight ratio increases *R/C.* A decrease in *CD0* or K, or in both, increases *R/C.* The effect of increasing altitude usually is to decrease *RIC.* All three terms in the equation are sensitive to altitude through rho. The effect of altitude on T depends on the type of power plant used. However, for turbojets, turbofans, and supercharged piston engines with propellers, thrust at a given V, decreases with altitude. For an airplane with any reasonable climb capacity, the dominant term is *T/* W; hence when *T/* W decreases with increasing altitude, *R/C* also decreases.

Wing loading also affects *R/C.* At a given arbitrary V, this effect is a mixed bag. Note from the drag terms that increasing *W/S* decreases the zero-lift drag and increases the drag due to lift. Hence, in the low-velocity range where drag due to lift is dominant, an increase in the design *W/S* results in a decrease in *RIC* at the same *V.* However, in the high-velocity range where zero-lift drag is dominant, an increase in *W/S* results in an increase in *RIC* at the same *V.*

Maximum Climb Angle

$$\sin\theta = \frac{T}{W} - \frac{D}{W} \qquad W = \frac{L}{\cos\theta}$$

$$\sin\theta = \frac{T}{W} - \frac{\cos\theta}{L/D}$$

Assuming that cos (theta)=1

$$\sin\theta = \frac{T}{W} - \frac{1}{L/D}$$

Maximum Climb Angle for a Jet

For a jet-propelled airplane where the thrust is essentially constant with velocity

$$\boxed{\sin\theta_{\max} = \frac{T}{W} - \frac{1}{(L/D)_{\max}}}$$

$$\boxed{\sin\theta_{\max} = \frac{T}{W} - \sqrt{4C_{D,0}K}}$$

The corresponding velocity can be found as

$$L = W\cos\theta = \tfrac{1}{2}\rho_\infty V_\infty^2 S C_L$$

$$C_L = \sqrt{\frac{C_{D,0}}{K}}$$

$$W\cos\theta_{\max} = \tfrac{1}{2}\rho_\infty V_{\theta_{\max}}^2 S\sqrt{\frac{C_{D,0}}{K}}$$

$$\boxed{V_{\theta_{\max}} = \sqrt{\frac{2}{\rho_\infty}\left(\frac{K}{C_{D,0}}\right)^{1/2}\frac{W}{S}\cos\theta_{\max}}}$$

the rate of climb that corresponds to the maximum climb angle is

$$(R/C)_{\theta_{\max}} = V_{\theta_{\max}}\sin\theta_{\max}$$

Theta max does *not* depend on wing loading, but from *V_theta_max* varies directly as *(W/S)****^2.*** Hence, everything else being equal, the rate of climb is higher for higher wing loadings. Since *(LID)max* does not depend on altitude, then *Theta_max* decreases with altitude because T decreases with altitude. However *Vtheta_max* increases with altitude. These are competing effects in determining *(R/C)_theta_max.* However, the altitude effect on T usually dominates, and *(R/C)theta_max,* usually decreases with increasing altitude.

Maximum Climb Angle for a propeller-driven airplane

$$T_A = \frac{\eta_{pr} P}{V_\infty}$$

Eta_pr is the propeller efficiency and P is the shaft power from the reciprocating piston engine. The product *etaP* is the power available ***Pa,*** which we assume to be constant with velocity.

$$\sin\theta = \frac{\eta_{pr} P}{V_\infty W} - \frac{1}{2}\rho_\infty V_\infty^2 \left(\frac{W}{S}\right)^{-1} C_{D,0} - \frac{W}{S}\frac{2K}{\rho_\infty V_\infty^2}$$

To find the velocity for the maximum climb angle, we have to differentiate with respect to V

$$\frac{d(\sin\theta)}{dV_\infty} = -\frac{\eta_{pr} P}{W V_\infty^2} - \rho_\infty V_\infty \left(\frac{W}{S}\right)^{-1} C_{D,0} + 2\frac{W}{S}\frac{K}{\frac{1}{2}\rho_\infty V_\infty^3}$$

$$\cancel{V_{\theta_{max}}^4} + \frac{\eta_{pr}(P/W)(W/S)}{\rho_\infty C_{D,0}} V_{\theta_{max}} - \frac{4(W/S)^2 K}{\rho_\infty^2 C_{D,0}} = 0$$

Can be neglected

$$V_{\theta_{max}} \approx \frac{4(W/S)K}{\rho_\infty \eta_{pr}(P/W)}$$

Maximum Rate of Climb for a jet propelled

For a jet-propelled airplane where T is relatively constant with *V* conditions associated with maximum rate of climb can be found by differentiating

$$V_\infty \sin\theta = R/C = V_\infty \left[\frac{T}{W} - \frac{1}{2}\rho_\infty V_\infty^2 \left(\frac{W}{S}\right)^{-1} C_{D,0} - \frac{W}{S}\frac{2K}{\rho_\infty V_\infty^2} \right]$$

$$\frac{d(R/C)}{dV_\infty} = \frac{T}{W} - \frac{3}{2}\rho_\infty V_\infty^2 \left(\frac{W}{S}\right)^{-1} C_{D,0} + \frac{W}{S}\frac{2K}{\rho_\infty V_\infty^2}$$

And setting the derivative equal to zero.

$$[L/D]_{\max} = 1/\sqrt{4KC_{D,0}}$$

$$V_\infty^2 - \frac{2(T/W)(W/S)}{3\rho_\infty C_{D,0}} - \frac{4K(W/S)^2}{3\rho_\infty^2 C_{D,0} V_\infty^2} = 0$$

$$V_\infty^4 - \frac{2(T/W)(W/S)}{3\rho_\infty C_{D,0}} V_\infty^2 - \frac{(W/S)^2}{3\rho_\infty^2 C_{D,0}^2 (L/D)_{\max}^2} = 0$$

$$V_{(R/C)_{\max}} = \left\{ \frac{(T/W)(W/S)}{3\rho_\infty C_{D,0}} \left[1 + \sqrt{1 + \frac{3}{(L/D)^2_{\max}(T/W)^2}} \right] \right\}^{1/2}$$

An equation for the maximum rate of climb is obtained by substituting V(R/C)max

$$V_\infty \sin\theta = R/C = V_\infty \left[\frac{T}{W} - \frac{1}{2}\rho_\infty V_\infty^2 \left(\frac{W}{S}\right)^{-1} C_{D,0} - \frac{W}{S}\frac{2K}{\rho_\infty V_\infty^2} \right]$$

We obtain

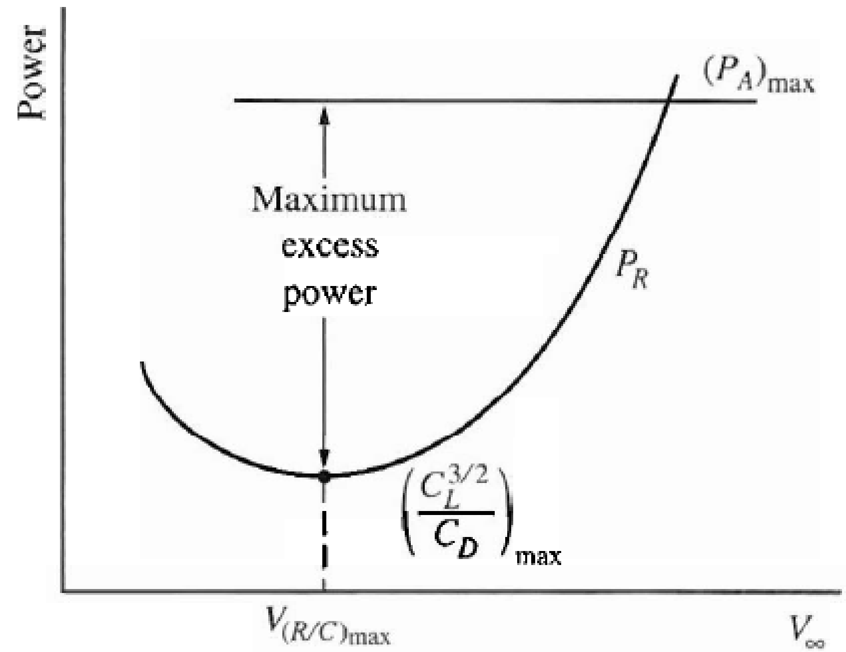
$$(R/C)_{\max} = \left[\frac{(W/S)Z}{3\rho_\infty C_{D,0}} \right]^{1/2} \left(\frac{T}{W}\right)^{3/2} \left[1 - \frac{Z}{6} - \frac{3}{2(T/W)^2(L/D)^2_{\max} Z} \right]$$

Where $$Z \equiv 1 + \sqrt{1 + \frac{3}{(L/D)^2_{\max}(T/W)^2}}$$

Maximum Rate of Climb for a propeller-driven airplane

$$(R/C)_{\max} = \frac{\text{maximum excess power}}{W}$$

Since the available power is constar with W, the maximum excess power *RIC* occurs at the flight velocity for minimum power required therefore:



$$\boxed{V_{(R/C)_{\max}} = \left(\frac{2}{\rho_\infty}\sqrt{\frac{K}{3C_{D,0}}}\frac{W}{S}\right)^{1/2}}$$

An equation for the maximum rate of climb is obtained by substituting V(R/C)max

$$R/C = \frac{\eta_{\mathrm{pr}} P}{W} - V_\infty \left[\frac{1}{2}\rho_\infty V_\infty^2 \left(\frac{W}{S}\right)^{-1} C_{D,0} + \frac{W}{S}\frac{2K}{\rho_\infty V_\infty^2}\right]$$

As a result

$$(R/C)_{\max} = \frac{\eta_{\mathrm{pr}} P}{W} - \left[\frac{2}{\rho_\infty}\sqrt{\frac{K}{3C_{D,0}}}\left(\frac{W}{S}\right)\right]^{1/2} \frac{1.155}{(L/D)_{\max}}$$

the dominant influence on *(RIC)max* is the power to-weight ratio. More power means a higher rate of climb. The effect of wing loading is secondary, but interesting. *(RIC)max decreases* with an increase in *W/S.* This is in contrast to the case of a jet-propelled airplane.

Propeller-driven airplanes are penalized in terms of *(RIC)max* if they have a high wing loading. Finally, the effect of increasing altitude is to increase *V(RIC)mx* and decrease *(R/* C)max. *(RIC)max* decreases with increasing altitude.