RANGE

- By definition, *range* is the total distance (measured with respect to the ground) traversed by an airplane on one load of fuel.
- W0 is the gross weight of the airplane including *everything;* full fuel load, payload, crew, structure
- Wf -weight of fuel; this is an instantaneous value, and it changes as fuel is consumed during flight.
- W1-weight of the airplane when the fuel tanks are empty.
- At any instant $W = W_1 + W_f$
- Since the fuel decreases during the flight $\frac{dW}{dt} = \frac{dW_f}{dt} = \dot{W}_f$
- because fuel is being consumed, both numbers are negative

Range is intimately connected with engine performance through the specific fuel consumption,

For a propeller-driven reciprocating engine combination

$$c \equiv -\frac{\dot{W}_f}{P}$$

P is the shaft power and the minus sign is necessary because *wf* is negative and *c* is always treated as a positive quantity.

For a jet-propelled airplane, the thrust specific fuel consumption is defined by (where T is the thrust available)

$$c_t \equiv -\frac{\dot{W}_f}{T}$$

c can be expressed in terms of ct

$$c_t = \frac{c V_\infty}{\eta_{\mathrm{pr}}}$$

$$c_t = -\frac{dW_f/dt}{T}$$

$$dt = -\frac{dW_f}{c_t T}$$

$$ds = V_\infty dt$$

$$ds = -\frac{V_\infty}{c_t T} dW_f$$

$$ds = -\frac{V_\infty}{c_t T} dW_f \longrightarrow ds = -\frac{V_\infty}{c_t T} dW = -\frac{V_\infty}{c_t} \frac{W}{T} \frac{dW}{W} = -\frac{V_\infty}{c_t} \frac{L}{D} \frac{dW}{W}$$

In steady level flight L=W and T=D

$$R = \int_0^R ds = \int_{W1}^{W0} -\frac{V_\infty}{c_t} \frac{L}{D} \frac{dW}{W} = \int_{W0}^{W1} \frac{V_\infty}{c_t} \frac{L}{D} \frac{dW}{W}$$

Usually L/D the velocity and the fuel consumption vary during the flight.
For a simplified analysis considering velocity, L/D and ct constant the Breguet range equation is obtained

$$R = \frac{V_\infty}{c_t} \frac{L}{D} \int_{W0}^{W1} \frac{dW}{W} = \frac{V_\infty}{c_t} \frac{L}{D} \ln \frac{W_0}{W_1}$$

At first glance, it would appear that to obtain the largest possible range, we would want to fly simultaneously at the highest possible velocity and at the largest possible value of L/D. However, V and L/D are not independent. Keep in mind that for a given airplane L/D varies with angle of attack, which in turn changes as V, changes in level flight. To obtain maximum range, we need to fly at a condition where the product V(L/D) is maximized. This condition is different for propeller-driven and jet-propelled airplanes.

Range for Propeller-Driven Airplanes

the specific fuel consumption for propeller/reciprocating engine power plants is fundamentally expressed in terms of power

$$R = \frac{V_\infty}{c_t}\frac{L}{D}\ln\frac{W_0}{W_1} = \frac{\eta_{pr}}{cV_\infty}V_\infty\frac{L}{D}\ln\frac{W_0}{W_1} \longrightarrow \boxed{R = \frac{\eta_{pr}}{c}\frac{L}{D}\ln\frac{W_0}{W_1}}$$

1. Fly at maximum L/D.
2. Have the highest possible propeller efficiency.
3. Have the lowest possible specific fuel consumption.
4. Have the highest possible ratio of gross weight to empty weight (carry a lot of fuel).

To fly at the maximum efficiency, the velocity has to be $V_{(L/D)_{max}} = \left(\frac{2}{\rho_\infty}\sqrt{\frac{K}{C_{D,0}}}\frac{W}{S}\right)^{1/2}$

Range for Jet-Propelled Airplanes

$$R = \int_0^R ds = \int_{W1}^{W0} -\frac{V_\infty}{c_t}\frac{L}{D}\frac{dW}{W} = \int_{W0}^{W1} \frac{V_\infty}{c_t}\frac{L}{D}\frac{dW}{W}$$

$$L = W = \tfrac{1}{2}\rho_\infty V_\infty^2 S C_L$$

$$V_\infty = \sqrt{\frac{2W}{\rho_\infty S C_L}}$$

$$V_\infty \frac{L}{D} = \sqrt{\frac{2W}{\rho_\infty S C_L}}\frac{C_L}{C_D} = \sqrt{\frac{2W}{\rho_\infty S}}\frac{C_L^{1/2}}{C_D}$$

Clearly, maximum range for a jet is *not* dictated by maximum *LID,* but rather the maximum value of the product *V(L/D).* The product *V(L/D)* is maximum when the airplane is flying at a maximum value of *CL^1/2/Cd.*

Substituting we obtain

$$R = \int_{W_1}^{W_0} \frac{1}{c_t} \sqrt{\frac{2W}{\rho_\infty S}} \frac{C_L^{1/2}}{C_D} \frac{dW}{W}$$

Assuming ***ct*** , ***density,*** S, and *CL^1/2/ CD* constant,

$$R = \frac{1}{c_t} \sqrt{\frac{2}{\rho_\infty S}} \frac{C_L^{1/2}}{C_D} \int_{W_1}^{W_0} \frac{dW}{W^{1/2}} \longrightarrow \boxed{R = \frac{2}{c_t} \sqrt{\frac{2}{\rho_\infty S}} \frac{C_L^{1/2}}{C_D} \left(W_0^{1/2} - W_1^{1/2}\right)}$$

the flight conditions for maximum range for a jet-propelled airplane are

1. Fly at maximum *CL^1/2/Cd.*
2. Have the lowest possible thrust specific fuel consumption.
3. Fly at high altitude
4. Carry a lot of fuel.

the range of a propeller-driven airplane does not explicitly depend on ***the density,*** and hence the influence of altitude appears only implicitly via the altitude effects on *the propeller efficiency* and ***c.*** However, ***p,*** appears directly for the range of a jet-propelled airplane, and hence the altitude has a first-order effect on range. This explains why, in part, when you fly in your jumbo jet across the Atlantic Ocean to London, you cruise at altitudes above 30,000 ft. Of course, when taken in the limit of ***p,*** going to zero, shows the range going to infinity. This is nonsense, the highest altitude that a given airplane can reach is limited by its absolute ceiling.

The flight conditions associated with *(CL^1/2/Cd)max,*

$$V_{(C_L^{1/2}/C_D)_{\max}} = \left(\frac{2}{\rho_\infty}\sqrt{\frac{3K}{C_{D,0}}}\frac{W}{S}\right)^{1/2}$$

$$\left(\frac{C_L^{1/2}}{C_D}\right)_{\max} = \frac{3}{4}\left(\frac{1}{3KC_{D,0}^3}\right)^{1/4}$$