ENDURANCE

Endurance is the amount of time that an airplane can stay in the air on one load of fuel. The flight conditions for maximum endurance are different from those for maximum range.

$$\frac{dW_f}{dt} = -c_t T \longrightarrow dt = -\frac{dW_f}{c_t T}$$

Since T = *D* and *L* = W

$$dt = -\frac{dW_f}{c_t D} = -\frac{L}{D}\frac{1}{c_t}\frac{dW_f}{W}$$

$$E = -\int_{W_0}^{W_1} \frac{1}{c_t}\frac{L}{D}\frac{dW_f}{W} = \int_{W_1}^{W_0} \frac{1}{c_t}\frac{L}{D}\frac{dW_f}{W}$$

E is the time we can fly from W0 with full tank to W1 with empty tanks

If we assume flight at constant **Ct** and *L/D,*

$$\boxed{E = \frac{1}{c_t}\frac{L}{D}\ln\frac{W_0}{W_1}}$$

Endurance for Propeller-Driven Airplanes

The specific fuel consumption for propeller-driven airplanes is given in terms of power rather than thrust

$$c_t = \frac{cV_\infty}{\eta_{pr}}$$

therefore

$$E = \int_{W_1}^{W_0} \frac{\eta_{pr}}{cV_\infty} \frac{C_L}{C_D} \frac{dW_f}{W}$$

$$E = \int_{W_1}^{W_0} \frac{\eta_{pr}}{c} \sqrt{\frac{\rho_\infty S C_L}{2W}} \frac{C_L}{C_D} \frac{dW_f}{W}$$

$$E = \frac{\eta_{pr}}{c} \sqrt{2\rho_\infty S} \frac{C_L^{3/2}}{C_D} \left(W_1^{-1/2} - W_0^{-1/2} \right)$$

maximum endurance for a propeller-driven airplane corresponds to the following conditions:

1. Fly at maximum CL^3/2/CD.
2. Have the highest possible propeller efficiency.
3. Have the lowest possible specific fuel consumption.
4. Have the highest possible difference between *W0* and *W1* (carry a lot of fuel).
5. Fly at sea level, where ρ is the largest value.

Endurance for Jet-Propelled Airplanes

$$E = \frac{1}{c_t} \frac{L}{D} \ln \frac{W_0}{W_1}$$

is already expressed in terms of thrust specific fuel consumption,

maximum endurance for a jet-propelled airplane corresponds
to the following conditions:

1. Fly at maximum *LID.*
2. Have the lowest possible thrust specific fuel consumption.
3. Have the highest possible ratio of *W0* to ***W1*** (i.e., carry a lot of fuel).