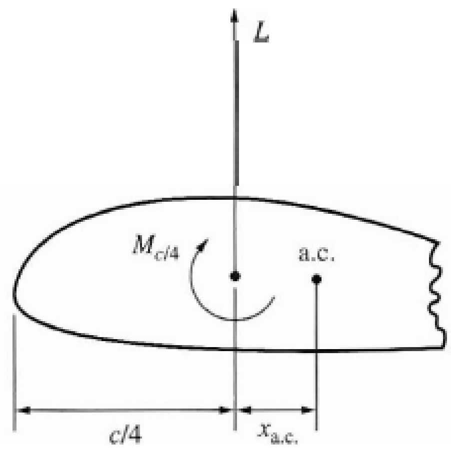
THE AERODYNAMIC CENTER

- The aerodynamic center is the point on a body about which the moments are independent of the angle of attack.



$$M_{a.c.} = Lx_{a.c.} + M_{c/4}$$

$$\frac{M_{a.c.}}{q_\infty Sc} = \frac{L}{q_\infty S}\left(\frac{x_{a.c.}}{c}\right) + \frac{M_{c/4}}{q_\infty Sc}$$

$$c_{m_{a.c.}} = c_l\left(\frac{x_{a.c.}}{c}\right) + c_{m_{c/4}}$$

Differentiating with respect to angle of attack a gives

$$\frac{dc_{m_{a.c.}}}{d\alpha} = \frac{dc_l}{d\alpha}\left(\frac{x_{a.c.}}{c}\right) + \frac{dc_{m_{c/4}}}{d\alpha}$$

If the aerodynamic center is the point about which moments are independent of the angle of attack.

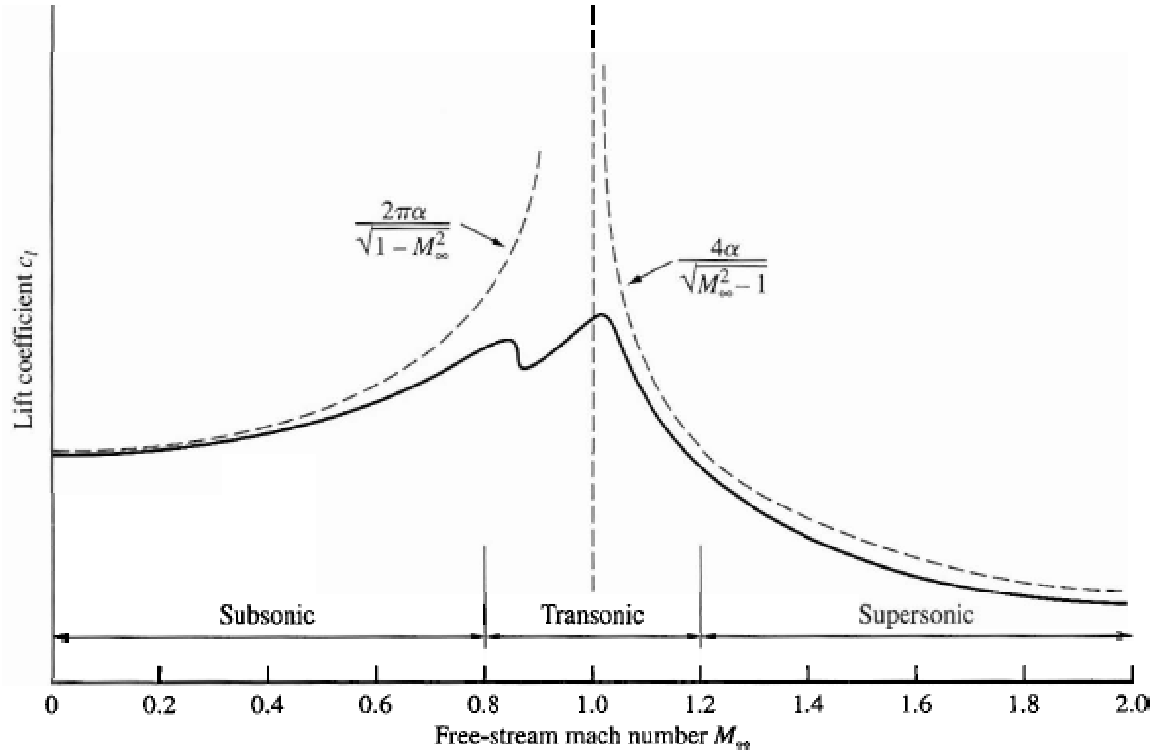
$$\frac{dc_{m_{a.c}}}{d\alpha} = 0$$

$$\boxed{0 = \frac{dc_l}{d\alpha}\left(\frac{x_{\text{a.c.}}}{c}\right) + \frac{dc_{m_{c/4}}}{d\alpha}}$$

$$\frac{x_{\text{a.c.}}}{c} = -\frac{dc_{m_{c/4}}/d\alpha}{dc_l/d\alpha} = -\frac{m_0}{a_0}$$

for a body with linear lift and moment curves, where m0 and a0 are the values, the aerodynamic center does exist as a fixed point on the airfoil.

Variation of Cl with Ma

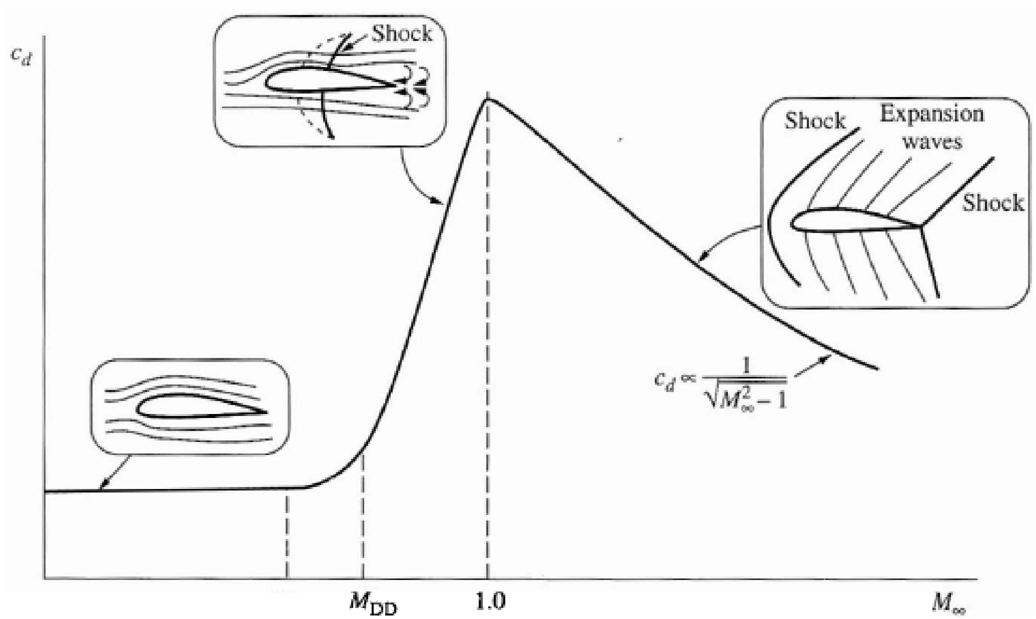


At subsonic speeds, the "compressibility effects" associated with increasing *Ma,* result in a progressive increase in *Cl.* The reason for this is that the lift is mainly due to the pressure distribution on the surface. As *Ma,* increases, the differences in pressure from one point to another on the surface become more pronounced.

Hence, *Cl* increases as *Ma,* increases. The Prandtl-Glauert rule, the first and simplest (and also the least accurate) of the several formulas for subsonic "compressibility corrections," predicts that *Cl* will rise inversely proportional to (1-Ma^2)^0.5.

In the supersonic region, the dashed curve shows the theoretical supersonic variation for a thin airfoil, where *Cl* = 4a/J-. The oscillatory variation of ***Cl*** near Mach=1 is typical of the transonic regime, and is due to the shock wave-boundary layer interaction that is prominant for transonic Mach numbers.

Dependence of Cd with Ma



C***d*** stays relatively constant with ***Ma,*** up to, and slightly beyond the critical Mach number (that free-stream Mach number at which sonic flow is first encountered at some location on the airfoil). The drag in the subsonic region is mainly due to friction, and the "compressibility effect" on friction in the subsonic regime is small. The flow over the airfoil in this regime is smooth and attached, with no shock waves present.

As M, increases above ***Ma critical,*** a large pocket of locally supersonic flow forms above, and sometimes also below, the airfoil. These pockets of supersonic flow are terminated at the downstream end by shock waves. The presence of these Shocks will affect the pressure distribution in such a fashion as to cause an increase in pressure drag (this drag increase is related to the loss of total pressure across the shock waves). However, the dominant effect is that the shock wave interacts with the boundary layer on the surface, causing the boundary layer to separate. Finally, in the supersonic regime, ***Cd*** gradually decreases,