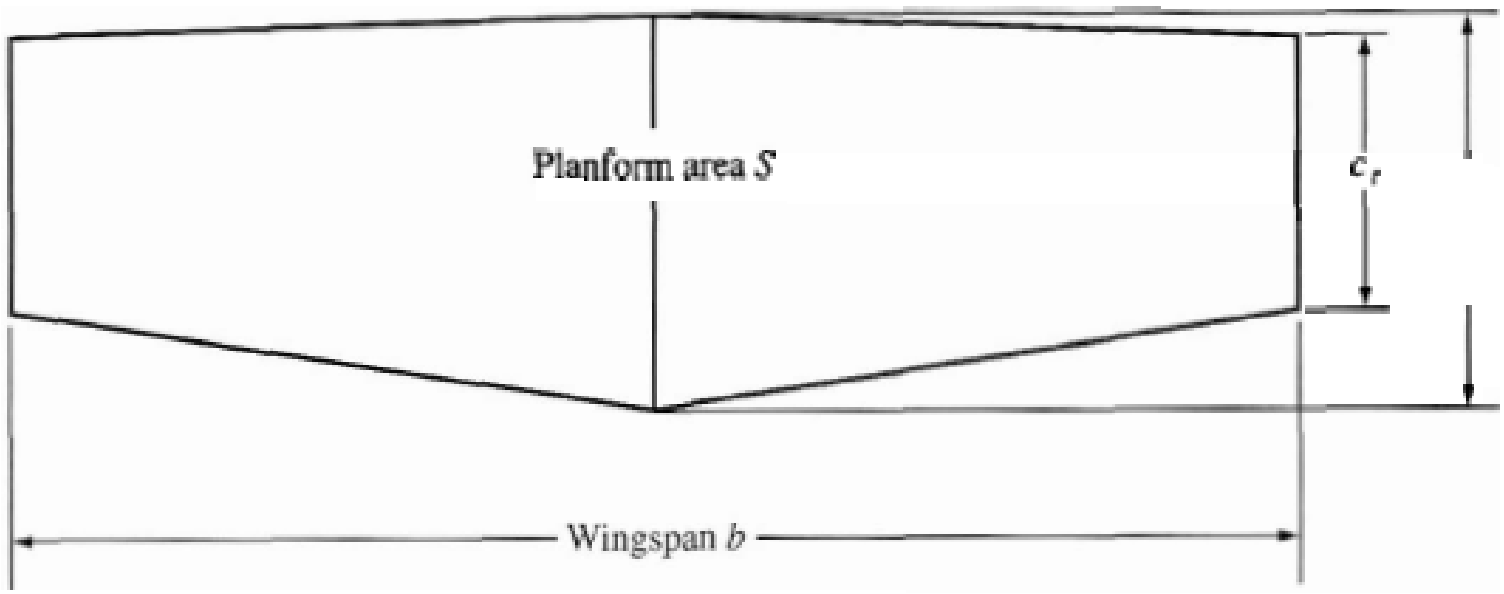
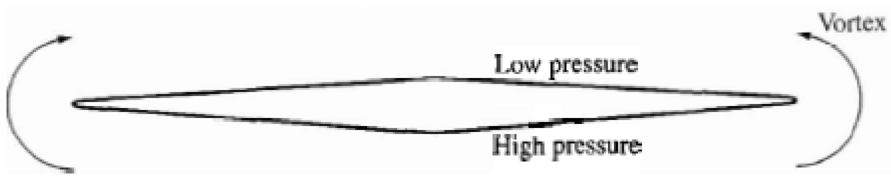
Lift for a Finite Wing

- all real wings are finite in span (airfoils are considered as infinite in the span)

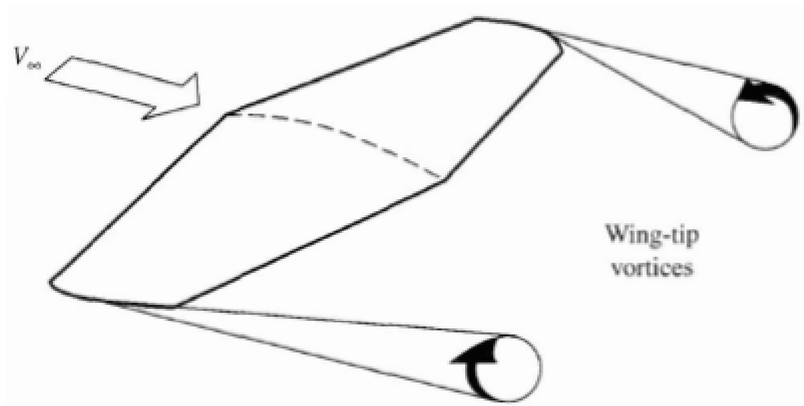


Aspect ratio $AR \equiv \frac{b^2}{S}$; Taper ratio $\equiv \frac{c_t}{c_r}$

The lift coefficient differs from that of an airfoil because there are strong vortices produced at the wing tips of the finite wing, which trail downstream. These vortices are analogous to mini-tornadoes, and like a tornado, they reach out in the flow field and induce changes in the velocity and pressure fields around the wing

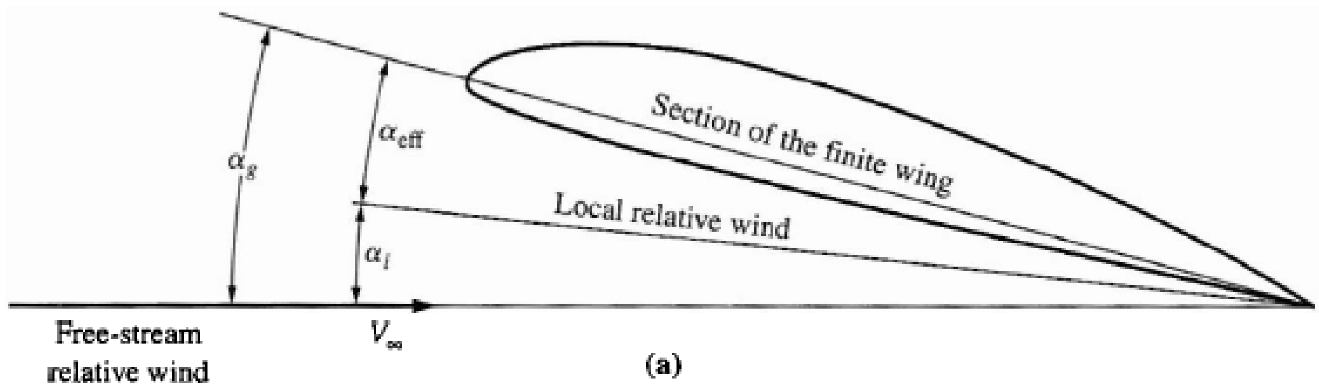


(a) Front view of wing



(b)

Imagine that you are standing on top of the wing you will feel a downward component of velocity over the span of the wing, induced by the vortices trailing downstream from both tips. This downward component of velocity is called ***downwash.***



The local downwash at your location combines with the free-stream relative wind to produce a ***local relative wind***. This local relative wind is inclined below the free-stream direction through the induced angle of attack α_i .
Hence, you are effectively feeling an angle of attack different from the actual geometric angle of attack of the wing relative to the free stream; you are sensing a ***smaller*** angle of attack*. For Example* if the wing is at a geometric angle of attack of ***5°,*** you are feeling an effective angle of attack which is smaller. Hence, the lift coefficient for the wing is going to be smaller than the lift coefficient for the airfoil. This explains the answer given to the question posed earlier.

High-Aspect-Ratio Straight Wing

The classic theory for such wings was worked out by Prandtl during World War I and is called ***Prandtl's lifting line theory.***

$$\boxed{a = \frac{a_0}{1 + a_0/(\pi e_1 \mathrm{AR})}}$$

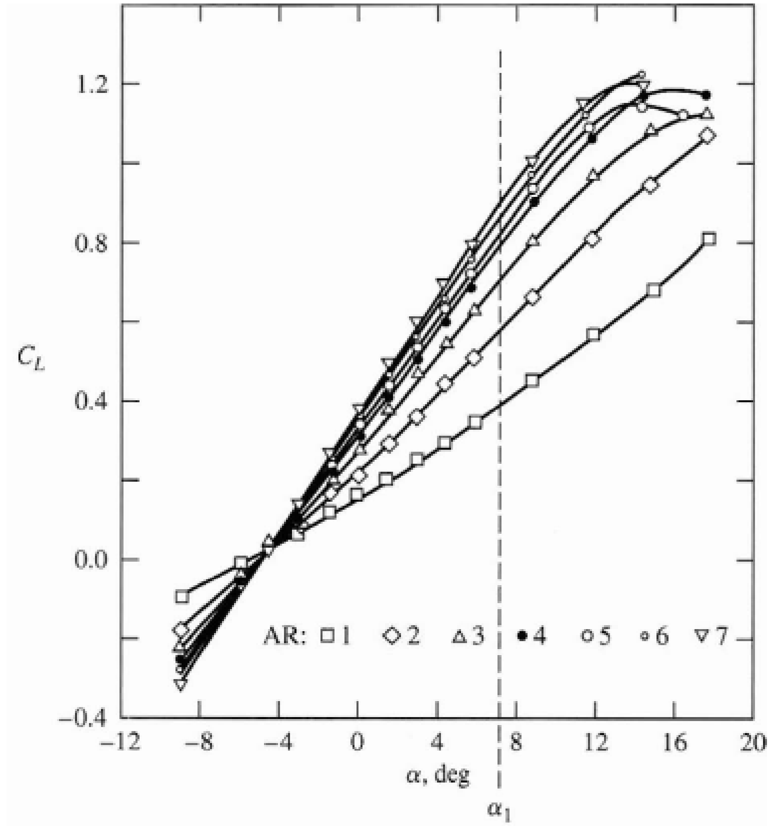
$$a_0 = \frac{dc_l}{d\alpha}$$ airfoil

$$a = \frac{dC_L}{d\alpha}$$ wing

lift slope per *radian* and **e1** is a factor that depends on the geometric shape of the wing, including the aspect ratio and taper ratio.

$$AR = \frac{b^2}{S}$$

Prandtl's lifting line theory does not apply to low-aspect-ratio wings. It holds for aspect ratios of about 4 or larger.



the lift slope for a finite wing decreases as the aspect ratio decreases.
The angle of attack for zero lift, denoted $\alpha_{CL=0}$
is the same for all the seven wings; at zero lift the induced effects theoretically disappear. At any given angle of attack larger than the value of CL becomes smaller as the aspect ratio is decreased.

Prandtl's lifting line theory, also holds for subsonic compressible flow,

$$a_{\text{comp}} = \frac{a_{0,\text{comp}}}{1 + a_{0,\text{comp}}/(\pi e_1 \text{AR})}$$

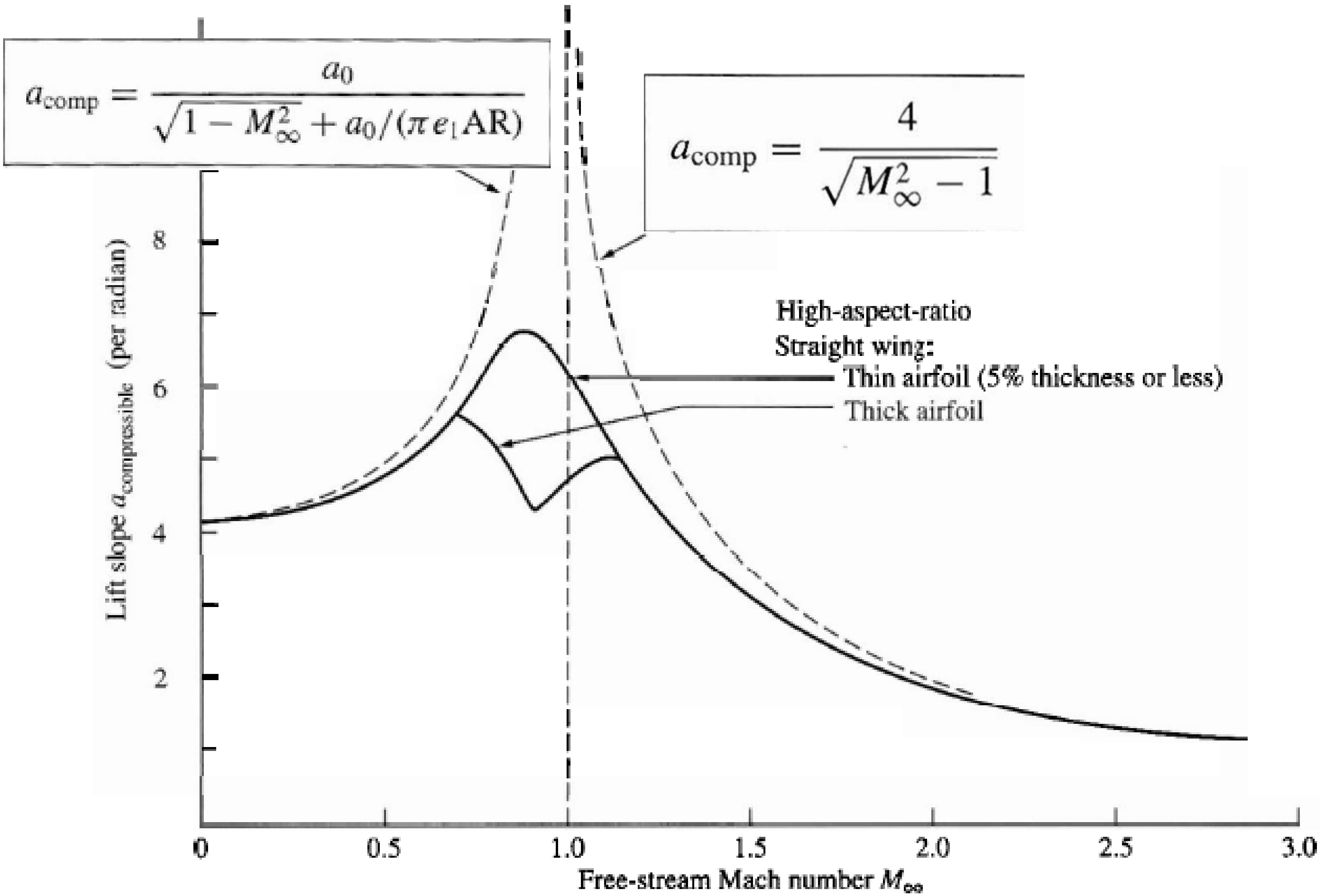
where $$a_{0,\text{comp}} = \frac{a_0}{\sqrt{1 - M_\infty^2}}$$

Substituting we have $$\boxed{a_{\text{comp}} = \frac{a_0}{\sqrt{1 - M_\infty^2} + a_0/(\pi e_1 \text{AR})}}$$

It gives a quick, but approximate correction to the lift slope; because it is derived from linear subsonic flow theory it is not recommended for use for Ma greater than 0.7.

For supersonic flow over a high-aspect-ratio straight wing, the lift slope can be approximated from supersonic linear theory

$$\boxed{a_{\text{comp}} = \frac{4}{\sqrt{M_\infty^2 - 1}}}$$

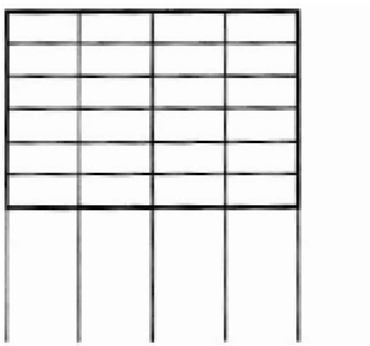


$a_{comp} = \dfrac{a_0}{\sqrt{1 - M_\infty^2} + a_0/(\pi e_1 \mathrm{AR})}$
$a_{comp} = \dfrac{4}{\sqrt{M_\infty^2 - 1}}$
High-aspect-ratio
Straight wing:
Thin airfoil (5% thickness or less)
Thick airfoil
Lift slope $a_{compressible}$ (per radian)
8
6
4
2
0
0.5
1.0
1.5
2.0
2.5
3.0
Free-stream Mach number M_∞

Low-Aspect-Ratio Straight Wings

- When applied to straight wings at AR < 4, the equations for high AR do not apply because are derived from a theoretical model which represents the finite wing with a single lifting line across the span of the wing. However, when the aspect ratio is small, the same intuition leads to some misgivings-how can a short, stubby wing be properly modeled by a single lifting line? The fact is-it cannot.
- Instead of a single spanwise lifting line, the low-aspect-ratio wing must be modeled by a large number of spanwise vortices, each located at a different chordwise station

Modern panel methods can quickly and accurately calculate the inviscid flow properties of low-aspect-ratio straight wings,



An approximate relation for the lift slope for low-aspect-ratio straight wings was obtained by H. B. Helmbold in Gemany in 1942

$$a = \frac{a_0}{\sqrt{1 + [a_0/(\pi AR)]^2} + a_0/(\pi AR)}$$

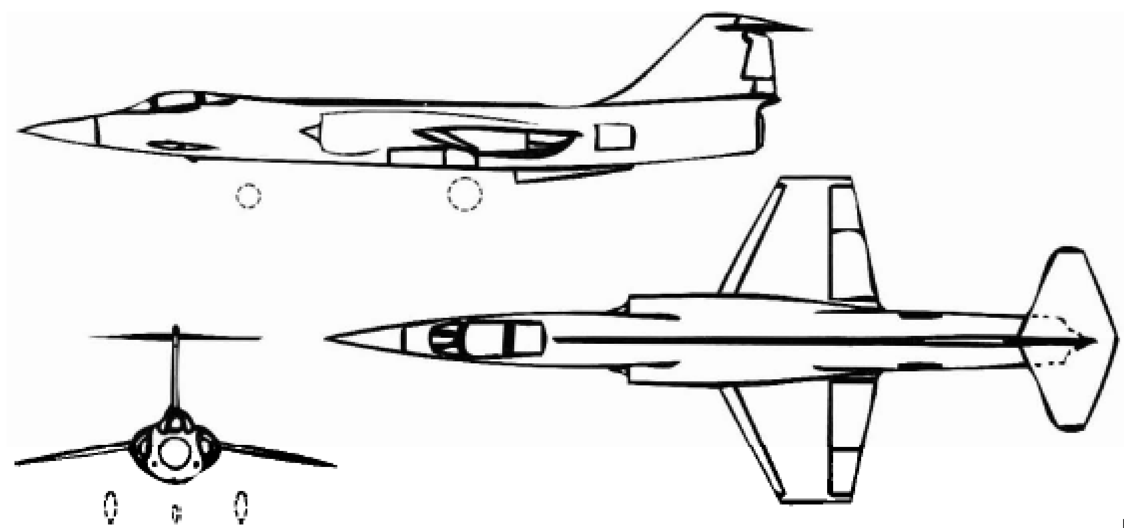
For subsonic compressible flow, is modified as follows

$$a_{comp} = \frac{a_0}{\sqrt{1 - M_\infty^2 + [a_0/(\pi AR)]^2} + a_0/(\pi AR)}$$

In the case of supersonic flow over a low-aspect-ratio straight wing,

$$a_{comp} = \frac{4}{\sqrt{M_\infty^2 - 1}}\left(1 - \frac{1}{2AR\sqrt{M_\infty^2 - 1}}\right)$$

At subsonic speeds, a low-aspect-ratio wing is plagued by large induced drag, and hence subsonic aircraft (since World War I) do not have low-aspect-ratio wings. On the other hand, a low-aspect-ratio straight wing has low supersonic wave drag, and this is why such a wing was used on the F-104-the first military fighter designed for sustained Mach 2 flight. At subsonic speeds, and especially for takeoff and landing, the low-aspect-ratio wings were a major liability to the F-104.

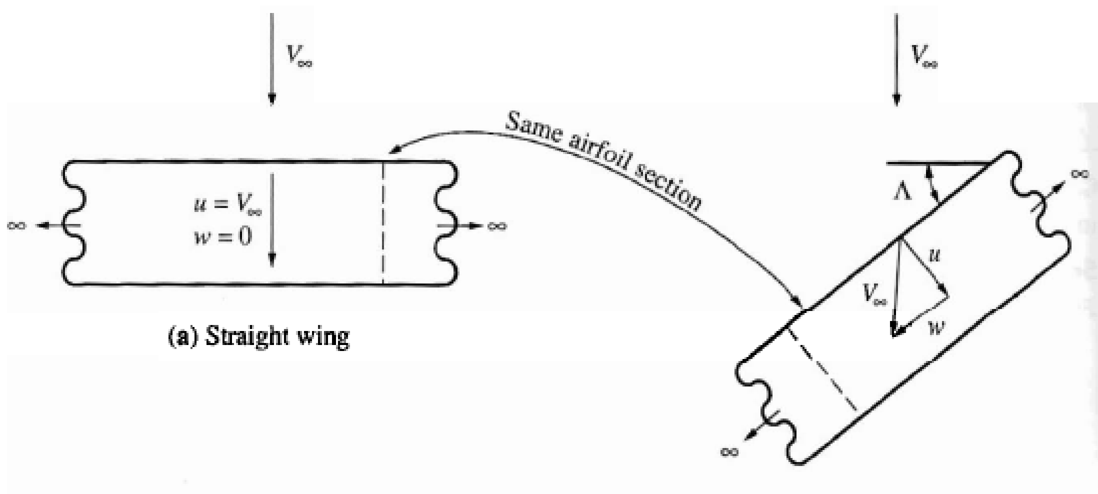


F104

Fortunately, there are two other wing platforms that reduce wave drag without suffering nearly as large a penalty at subsonic speeds, namely, the swept wing and the delta wing.

Swept Wings

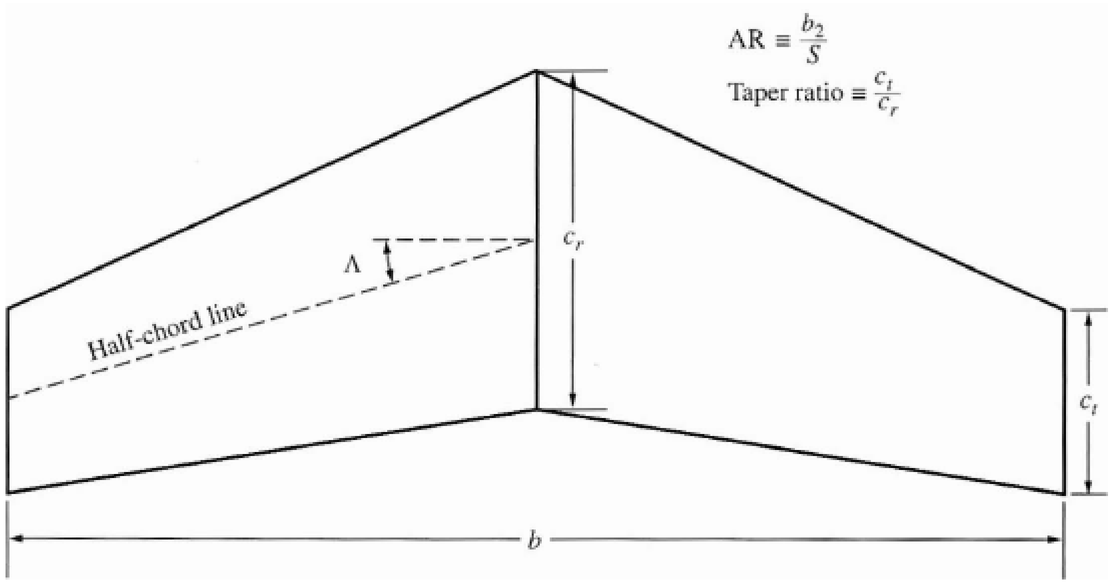
The main function of a swept wing is to reduce wave drag at transonic and supersonic speeds. Consider a straight wing and a swept wing in a flow with a free-stream velocity V. Assume that the aspect ratio is high for both wings, so that we can ignore tip effects. Let ***u*** and ***w*** be the components of V, perpendicular and parallel to the leading edge, respectively. The pressure distribution over the airfoil section oriented perpendicular to the leading edge is mainly governed by the chordwise component of velocity ***u;*** the spanwise component of velocity ***w*** has little effect on the pressure distribution. For the straight wing the chordwise velocity component ***u*** is the full V, for the swept wing the chordwise component of the velocity *u* is smaller than V: $u = V \cos \Lambda$



(a) Straight wing

(b) Swept wing

Since u for the swept wing is smaller than *u* for the straight wing, the difference in pressure between the top and bottom surfaces of the swept wing will be less than the difference in pressure between the top and bottom surfaces of the straight wing. Sincelift is generated by these differences in pressure, the lift on the swept wing will **be** less than that on the straight wing.



The wingspan b is the straight-line distance between the wing tips, the wing planform area is *S,* and the aspect ratio and the taper ratio are defined AR = b^2/S and taper ratio ***c,/c,.*** For the tapered wing, the sweep angle AR is referenced to the half-chord

an approximate calculation of the lift slope for a swept finite wing, Kuchemann suggests the following approach. The lift slope for an infinite swept wing should be $a_0 \cos\Lambda$

therefore

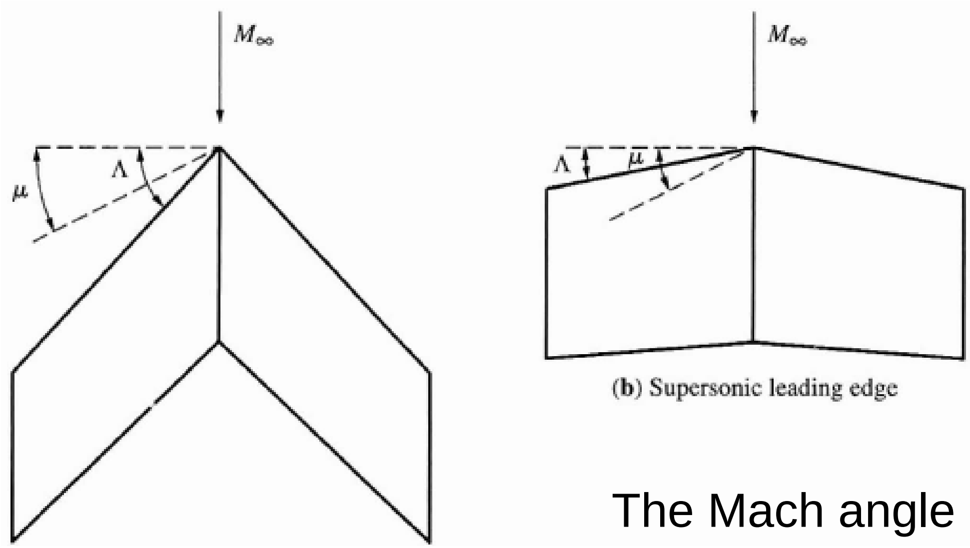
$$a = \frac{a_0 \cos\Lambda}{\sqrt{1 + [(a_0 \cos\Lambda)/(\pi AR)]^2} + (a_0 \cos\Lambda)/(\pi AR)}$$

The subsonic compressibility effect is added by replacing

a_0 with $a_0\sqrt{1-Ma}$

$$a_{\text{comp}} = \frac{a_0 \cos \Lambda}{\sqrt{1 - M_\infty^2 \cos^2 \Lambda + [(a_0 \cos \Lambda)/(\pi \text{AR})]^2} + (a_0 \cos \Lambda)/(\pi \text{AR})}$$

Supersonic Delta wings



(b) Supersonic leading edge

(a) Subsonic leading edge

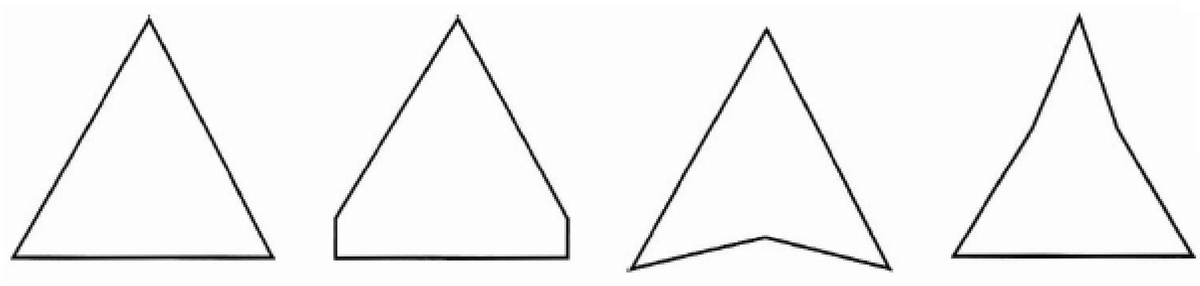
For a swept wing moving at supersonic speeds, the aerodynamic properties depend on the location of the leading edge relative to a Mach wave emanating from the apex of the wing.

The Mach angle is given by $\mu = \cos^{-1}(1/Ma)$

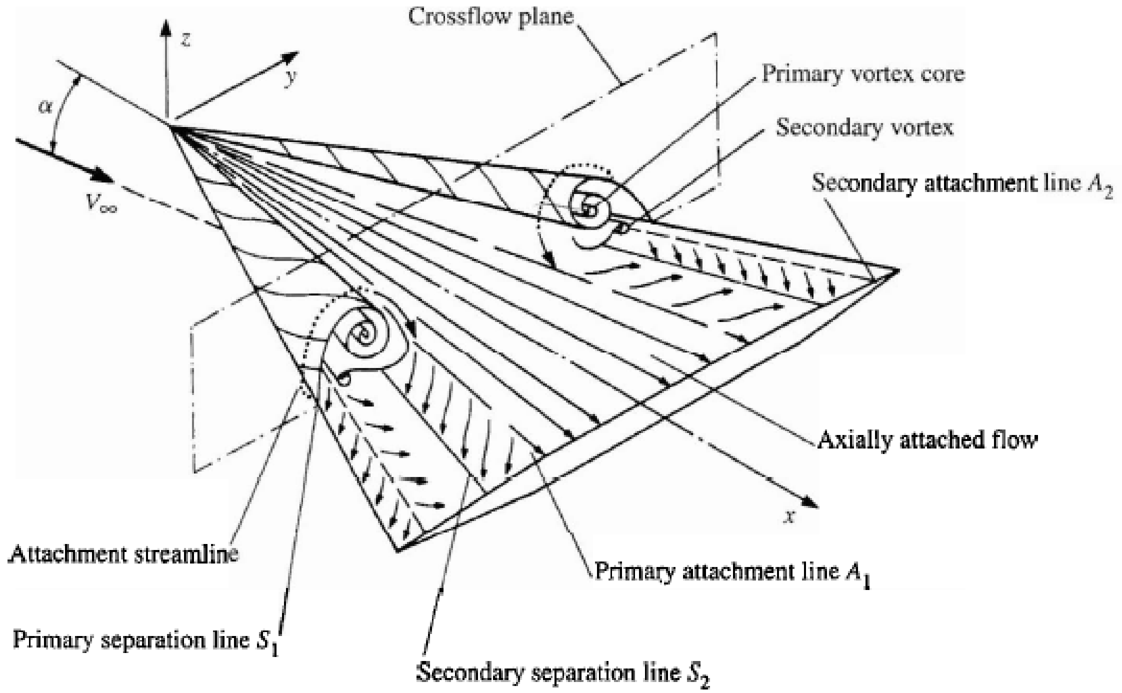
If the wing leading edge is swept inside the Mach cone the component of *Ma* perpendicular to the leading edge is subsonic; hence, the swept wing is said to have a *subsonic leading edge.* For the wing in supersonic flight, there is a weak shock that emanates from the apex, but there is *no* shock attached elsewhere along the wing leading edge. In contrast, if the wing leading edge is swept outside the Mach cone the component of *Ma,* perpendicular to the leading edge is supersonic; hence the swept wing is said to have a *supersonic leading edge.* For this wing in supersonic flight, there wiil be a shock wave attached along the entire leading edge. **A** swept wing with a subsonic leading edge behaves somewhat as a wing at subsonic speeds, although the actual free-stream Mach number is supersonic.

Delta Wings

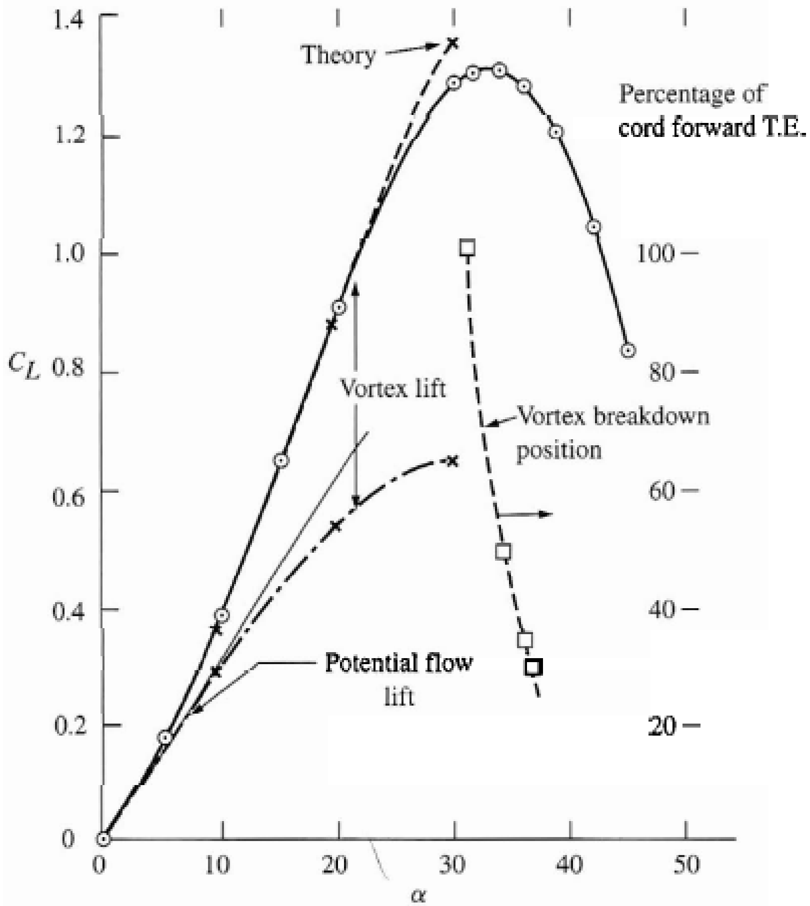
Swept wings that have planforms such as shown in Fig are called delta wings***.***



(a) Simple delta (b) Cropped delta (c) Notched delta (d) Double delta dominant aspect of this flow is the two vortices that are formed along the highly swept leading edges, and that trail downstream over the top of the wing. This vortex pattern is created by the following mechanism. The pressure on the bottom surface of the wing is higher than the pressure on the top surface.



Thus, the flow on the bottom surface in the vicinity of the leading edge tries to curl around the leading edge from the bottom to the top. If the leading edge is relatively sharp, the flow will separate along its entire length. This separated flow curls into a primary vortex above the wing just inboard of each leading edge. The stream surface which has separated at the leading edge loops above the wing and then reattaches along the primary attachment line. The primary vortex is contained within this loop. **A** secondary vortex is formed underneath the primary vortex, with its own separation line, and its own reattachment line***.*** Unlike many separated flows in aerodynamics, the vortex pattern over a delta wing is a friendly flow in regard to the production of lift. The vortices are strong and generally stable. They are a source of high energy, relatively high vorticity flow, and the local static pressure in the vicinity of the vortices is small. Hence, the vortices create a lower pressure on the top surface than would exist if the vortices were not there. This increases the lift compared to what it would be without the vortices.



The difference between the experimental data and the potential flow lift is the ***vortex lift.*** The vortex lift is a major contributor to the overall lift; The lift slope is small, on the order of 0.05 per degree. The lift, however, continues to increase over a large range of angle of attack (the stailing angle of attack is about **35°).**

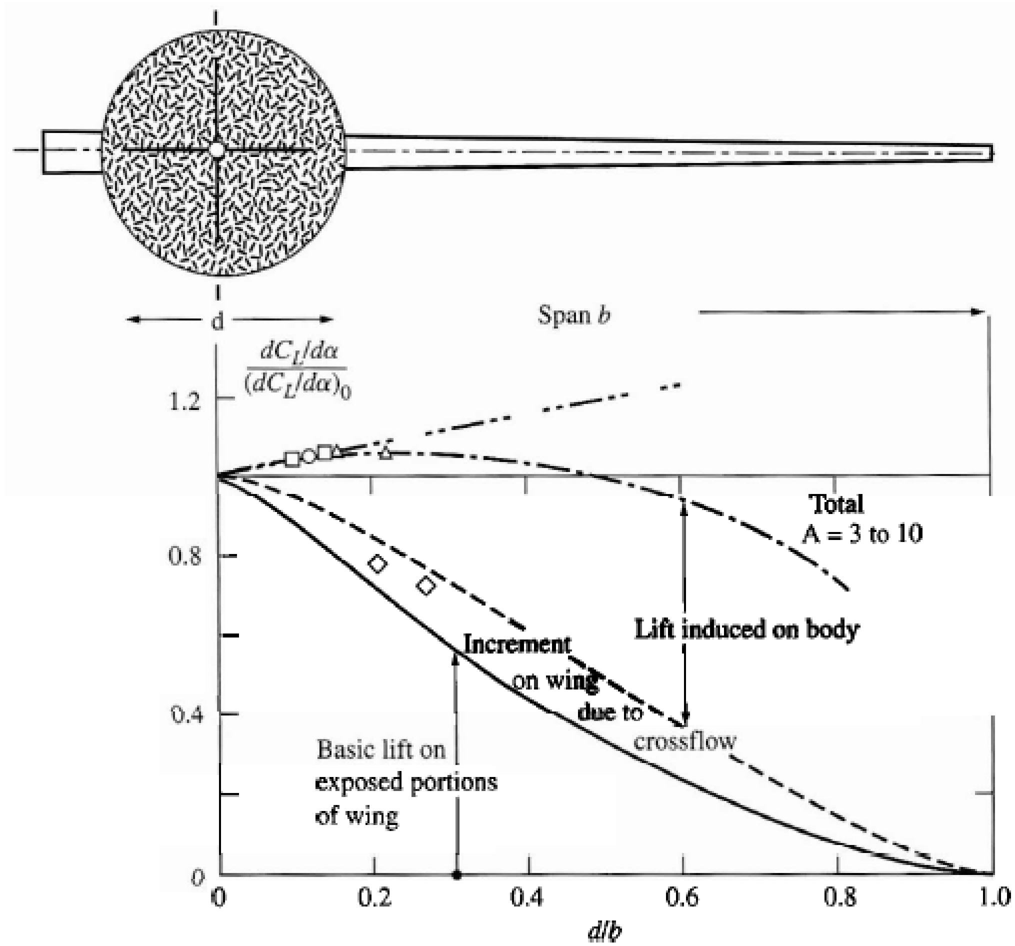
The net result is a reasonable value of **CL,max=**1.35. The lift curve is ***nonlinear,*** in contrast to the linear variation exhibited by conventional wings for subsonic aircraft. The vortex lift is mainly responsible for this nonlinearity.

The next time you have an opportunity to watch a delta-wing aircraft take off or land, for example, the televised landing of the space shuttle, note the large angle of attack of the vehicle. Also, this is why the Concorde supersonic transport, with its low-aspect-ratio deltalike wing, lands at a high angle of attack. In fact, the angle of attack is so high that the front part of the fuselage must be mechanically drooped upon landing in order for the pilots to see the runway.

Wing-Body Combinations

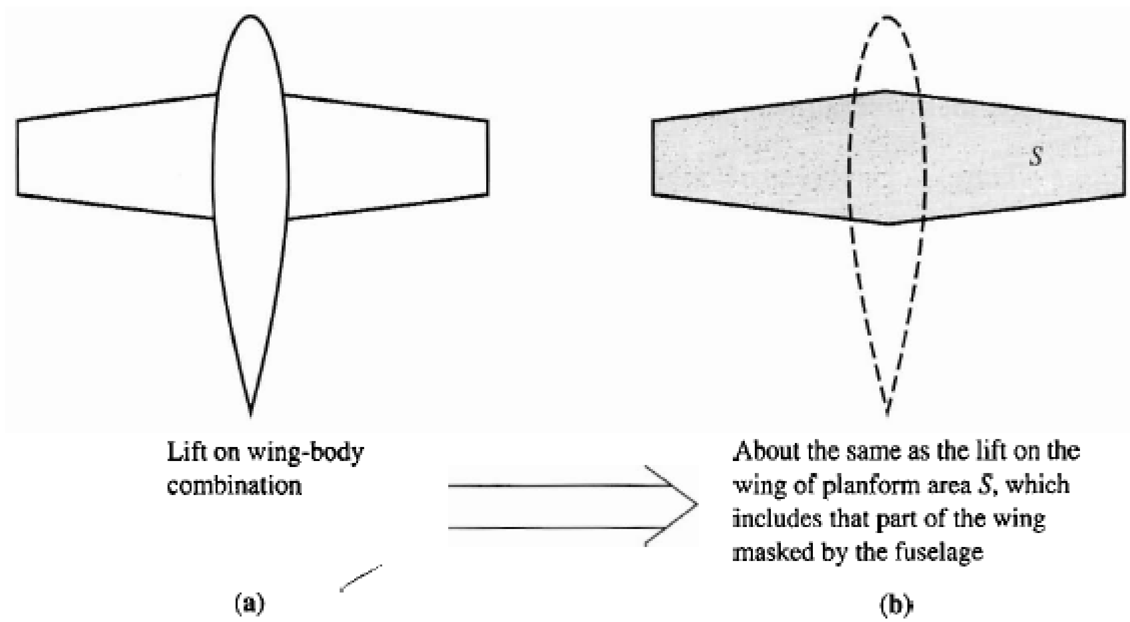
- even a pencil at an angle of attack will generate lift, albeit small.
- Hence, lift is produced by the fuselage of an airplane as well as the wing.
- The mating of a wing with a fuselage is called a *wing-body combination.*
- The lift of a wing-body combination is *not* obtained by simply adding the lift of the wing alone to the lift of the body alone. Rather, as soon as the wing and body are mated, the flow field over the body modifies the flow field over the wing, and vice versa-this is called the *wing-body interaction.*

The lift slope of the wing-body combination, divided by the lift slope of the wing alone is shown as a function of d/b (d is the fuselage diameter). The magnitudes of the three contributions to the lift are identified in Fig. as (1) the basic lift due to exposed portions of the wing, (2) the increase in lift on the wing due to crossflow from the fuselage acting favorably on the pressure distribution on the wing, and (3) the lift on the fuselage.



For a range of d/b from 0 (wing only) to 6 (which would be an inordinately fat fuselage with a short, stubby wing), the *total* lift for the wing-body combination is essentially constant (within about 5%). Hence, the lift of the wing-body combination can be treated as simply the lift on the complete wing by itself, including that portion of the wing that is masked by the fuselage.

For subsonic speeds, this is a reasonable approximation
for preliminary airplane performance and design considerations. Hence, in al1 our future references to the planform area of a wing of an airplane, it will be construed



DRAG

minimizing drag has been one of the strongest drivers in the historical development of applied aerodynamics. In airplane performance and design, drag is perhaps the most important aerodynamic quantity.

There are only two sources of aerodynamic force on a body moving through a fluid: the pressure distribution and the shear stress distribution acting over the body surface. Therefore, there are only two general types of drag:
1) Pressure drag-due to a net imbalance of surface pressure acting in the drag direction
2) Friction drag-due to the net effect of shear stress acting in the drag direction

For a purely laminar flow, $$c_f = \frac{1.328}{\sqrt{\mathrm{Re}}}$$

One of the used formulas for turbulent flows is $$(c_f)^{-1/2} = 4.13 \log(\mathrm{Re}\, c_f)$$

The location at which transition actually occurs on the surface is a function of a number of variables; suffice it to say that the transition Reynolds number is

$$\mathrm{Re}_{\mathrm{trans}} = \frac{\rho_\infty V_\infty x_{\mathrm{tr}}}{\mu_\infty} \approx 350{,}000 \text{ to } 1{,}000{,}000$$

No simple formulas exist to estimate the pressure drag.

The induced flow effects due to the wing-tip vortices result in an extra component of drag on a three-dimensional lifting body. This extra drag is called induced drag. Induced drag is purely a pressure drag. It is caused by the wing tip vortices which generate an induced, perturbing flow field over the wing, which in turn perturbs the pressure distribution over the wing surface in such a way that the integrated pressure distribution yields an increase in drag-the induced drag ***Di.*** For a high-aspect-ratio straight wing,

$$\frac{C_L^2}{e\mathrm{AR}}$$

$$e = \frac{1}{1+\delta}$$