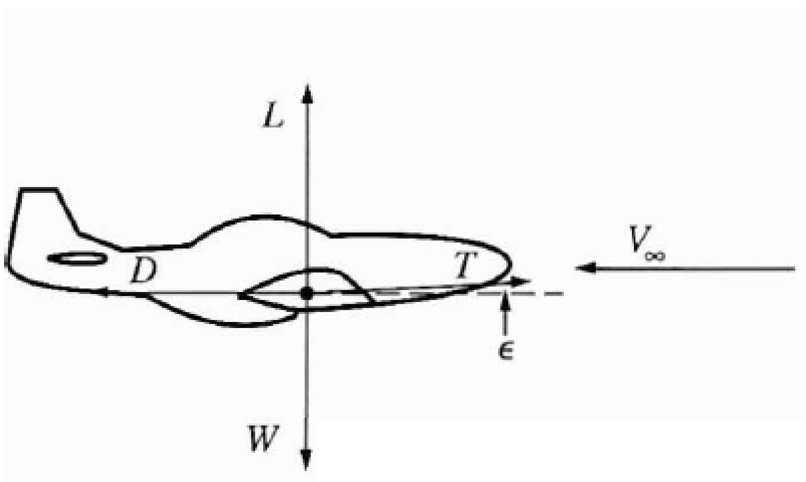
The Equations of Motion

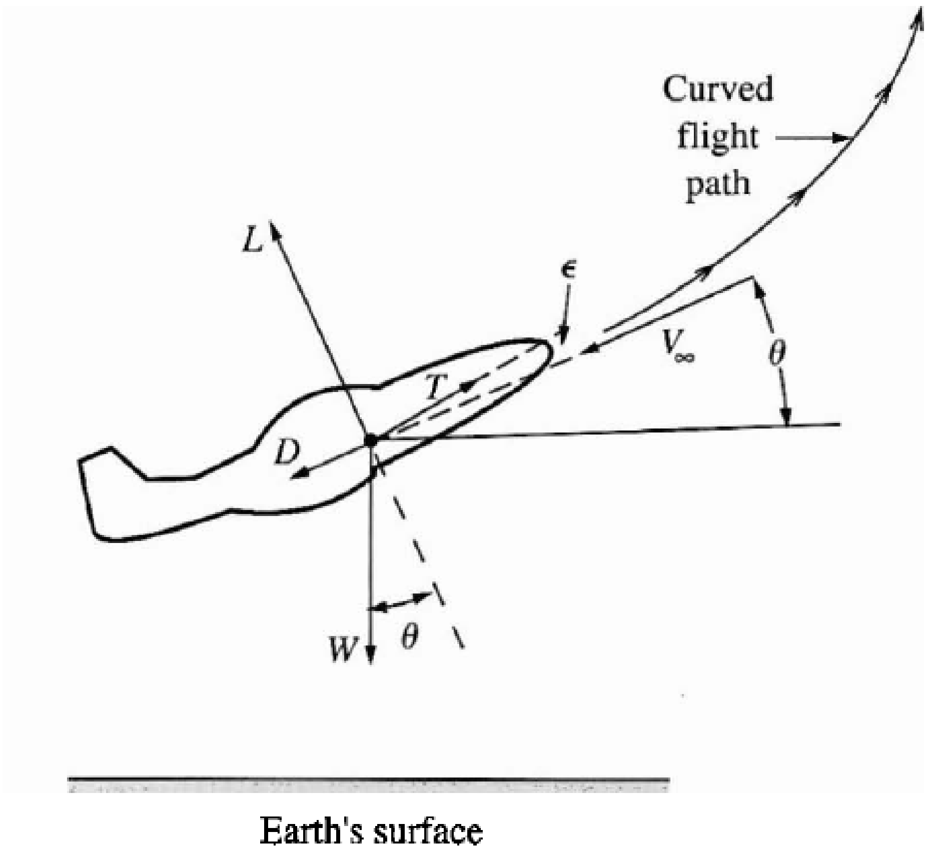
we make use of Aerodynamics mainly through the drag polar for a given airplane, and we consider the propulsive device simply in terms of thrust (or power) available and the specific fuel consumption. Our major concern is with the *movement* of a given airplane through the atmosphere, insofar as it is responding to the four forces of flight. This movement is governed by a set of equations called the *equations of motion*



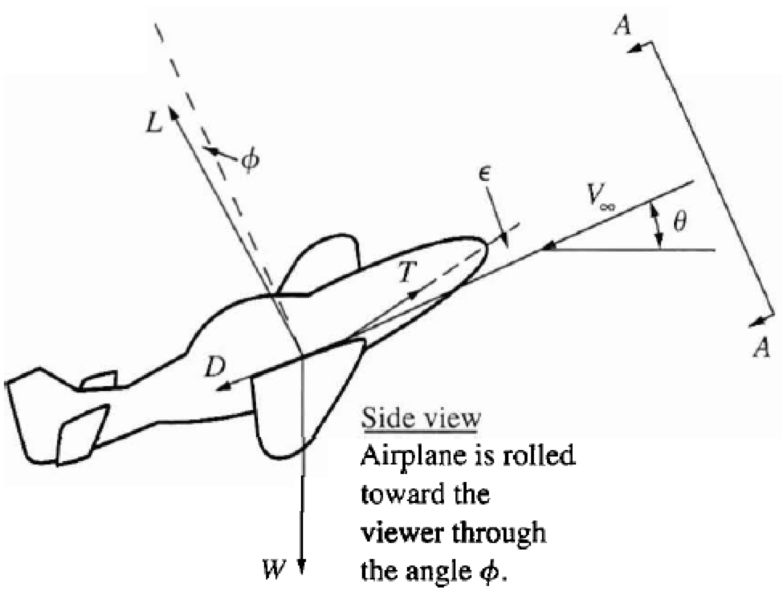
The four forces of flight-lift, drag, weight, and thrust are denoted by L, D, W, and T. The free-stream velocity *Vinf,* is always in the direction of the local flight of the airplane. By definition, the airplane lift and drag are perpendicular and parallel, respectively, to *Vinf*. Lift and drag are aerodynamic forces; L and D represent the lift and drag, respectively, of the *complete airplane,* including the wing, tail, fuselage, etc. The weight always acts toward the center of the earth.

The thrust is produced by whatever flight propulsion device is powering the airplane. In general, T is not necessarily in the free-stream direction can be inclined of angle eps relative to the flight path.

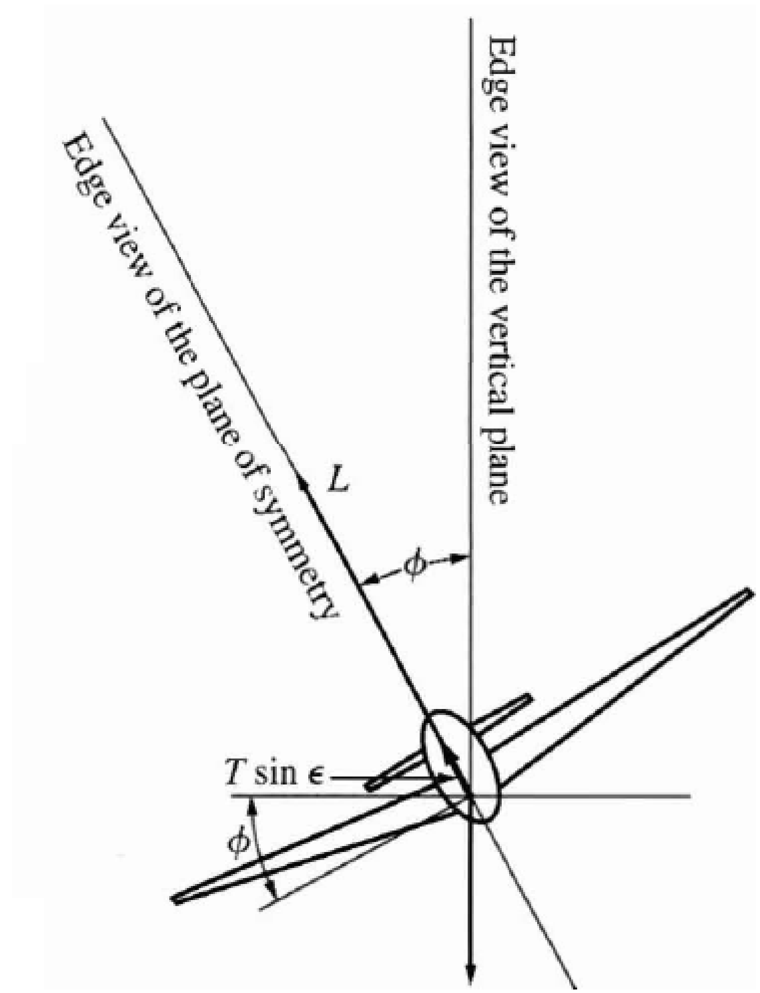
Consider an airplane climbing (or descending) along a flight path that is angled to the horizontal. In general, the flight path is curved. instantaneous angle of the flight path, relative to the horizontal, is theta. Hence Vinf is inclined at angle *theta,* which is called the local ***climb*** angle of the airplane.



L and D are perpendicular and parallel to Vinf. Weight W, acts toward the center of the earth, and hence is perpendicular to the earth's surface. For the airplane in climbing flight, the direction of W is inclined at the angle *theta* relative to the lift.



If the airplane rotates about the longitudinal axis (the axis along the fuselage from the nose to the tail) the page is no longer the symmetry plane of the aircraft. Instead, the plane of symmetry is as shown in the head-on front view.



This front view is a projection of the airplane and the forces on plane AA taken perpendicular to the local free-stream velocity Vinf. In this head-on front view, the plane of symmetry of the airplane is inclined to the local vertical through the roll angle **phi.**

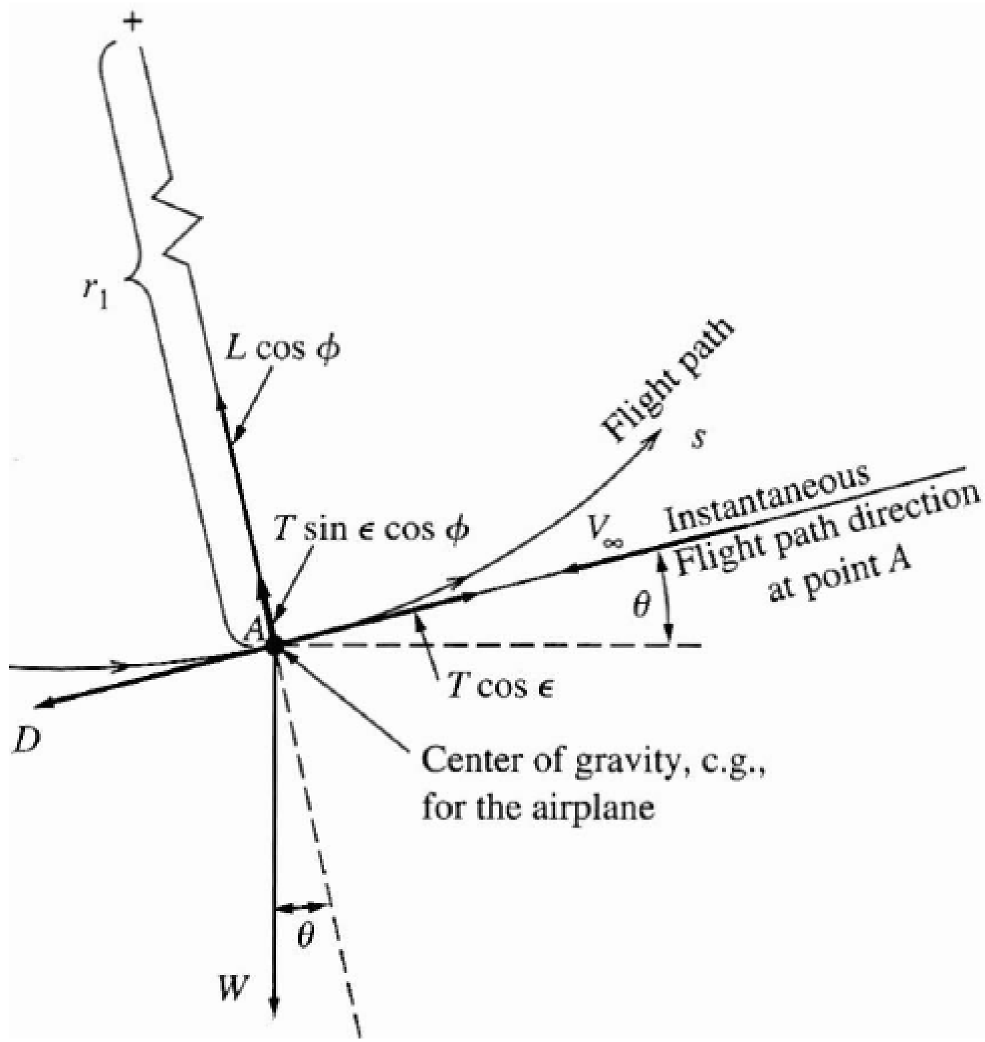
THE EQUATIONS OF MOTION

The equations of motion for an airplane are simply statements of Newton's second law, $\mathbf{F} = m\mathbf{a}$

Since we are interested in the *translational* motion of the airplane only, let us replace the airplane with a point mass at its center of gravity, with the four forces of flight acting through this point. Both D and W are in the plane of the page. The component of lift in this plane is L cos(theta). The component of force parallel to the flight path is,

$$F_{\parallel} = T\cos\epsilon - D - W\sin\theta$$

$$\boxed{m\frac{dV_\infty}{dt} = T\cos\epsilon - D - W\sin\theta}$$



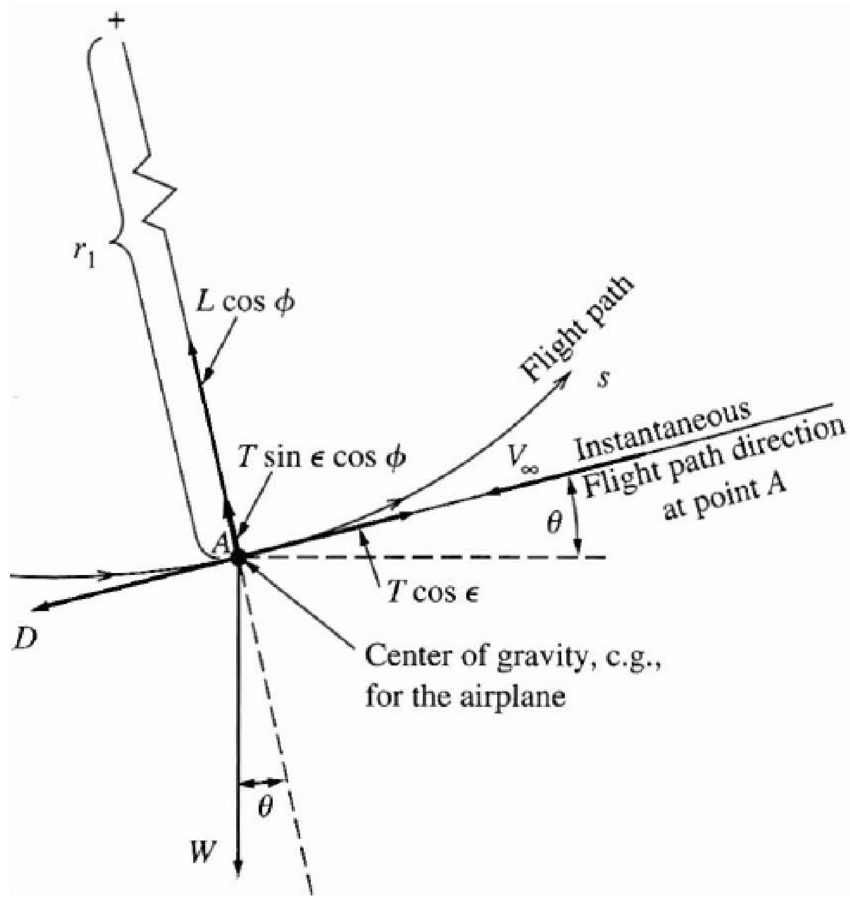
in the direction perpendicular to the flight path, the component of force is

$$\boxed{F_\perp = L\cos\phi + T\sin\epsilon\cos\phi - W\cos\theta}$$

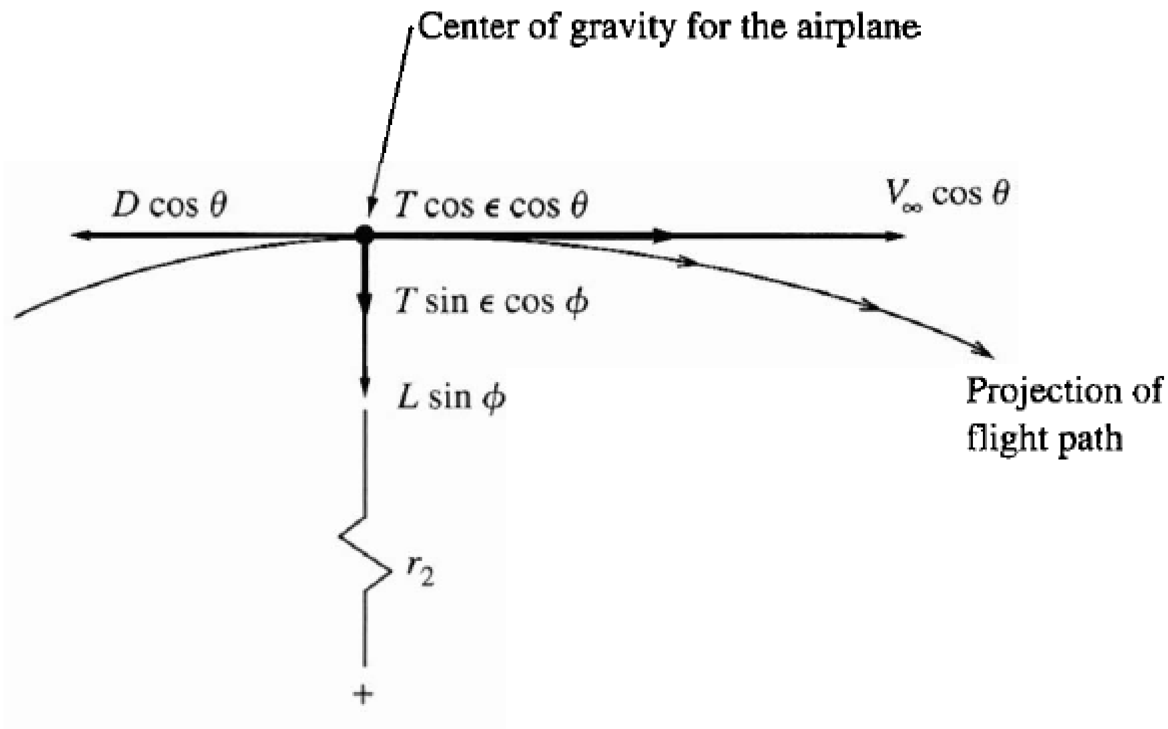
The radial acceleration of the curvilinear motion, perpendicular to the flight path, is

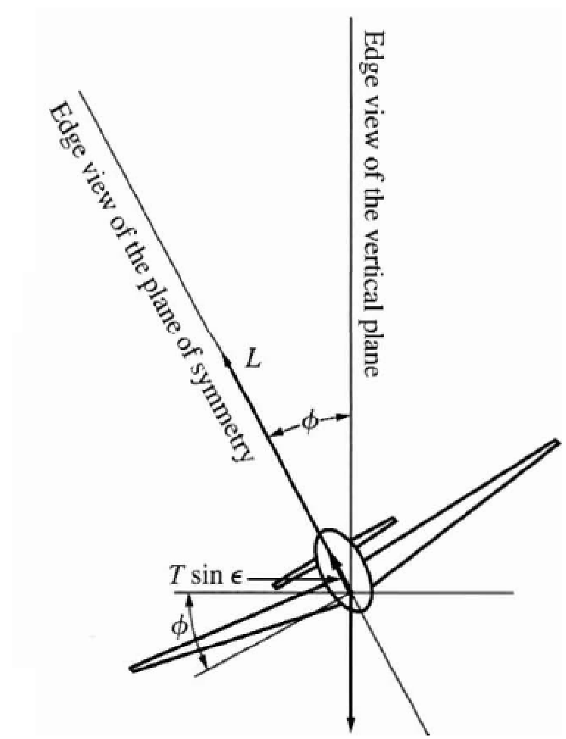
$$a_\perp = \frac{V_\infty^2}{r_1}$$

$$\boxed{m\frac{V_\infty^2}{r_1} = L\cos\phi + T\sin\epsilon\cos\phi - W\cos\theta}$$



The projection of the curved flight path on this horizontal plane is sketched





$$F_2 = L \sin\phi + T \sin\epsilon \sin\phi \qquad a_2 = \frac{(V_\infty \cos\theta)^2}{r_2}$$

$$\boxed{m\frac{(V_\infty \cos\theta)^2}{r_2} = L \sin\phi + T \sin\epsilon \sin\phi}$$

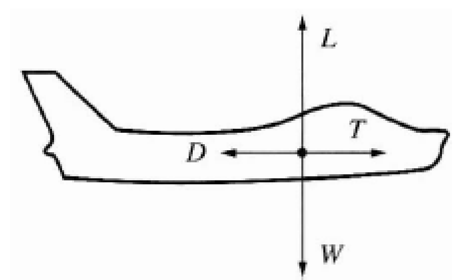
THRUST REQUIRED (DRAG) IN LEVEL FLIGHT

- the climb angle theta and roll angle phi are zero. The accelerations are also zero (The normal acceleration V^2/r is zero by definition of steady flight since the flight path is a straight line, where the radius of curvature ***r*** is infinitely large.) The resulting equations are:

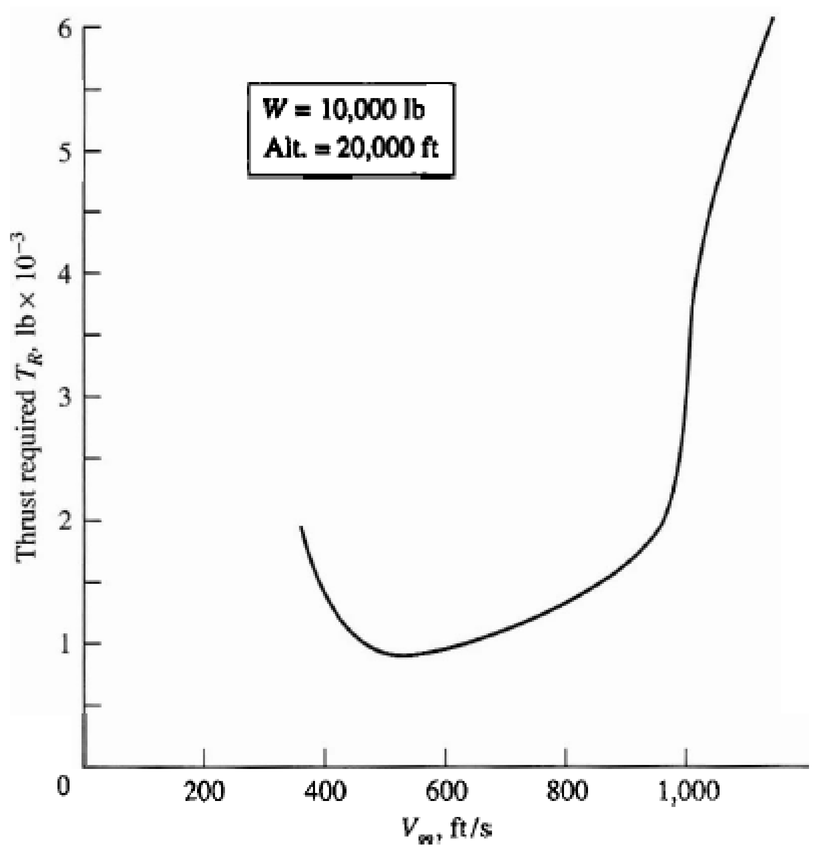
$$0 = T\cos\epsilon - D \qquad\qquad 0 = L + T\sin\epsilon - W$$

If we neglect the angle of the thrust respect to the horizontal (eps=0) we have:

$$\boxed{T = D} \qquad\qquad \boxed{L = W}$$



To maintain speed and altitude, enough thrust must be generated to exactly overcome the drag and to keep the airplane going this is the ***thrust required***. The thrust required $\boldsymbol{T_R}$ depends on the velocity, the altitude, and the aerodynamic shape, size, and weight of the airplane-it is an airframe associated feature rather than anything having to do with the engines themselves. Indeed, the thrust required is simply equal to the ***drag*** of the airplane-it is the thrust required to overcome the aerodynamic drag. The thrust required is simply the drag of the airplane, hence the thrust required curve is nothing other than a plot of drag versus velocity for a given airplane at a given altitude.



Graphical approach

The drag polar is $C_D = C_{D,0} + KC_L^2$

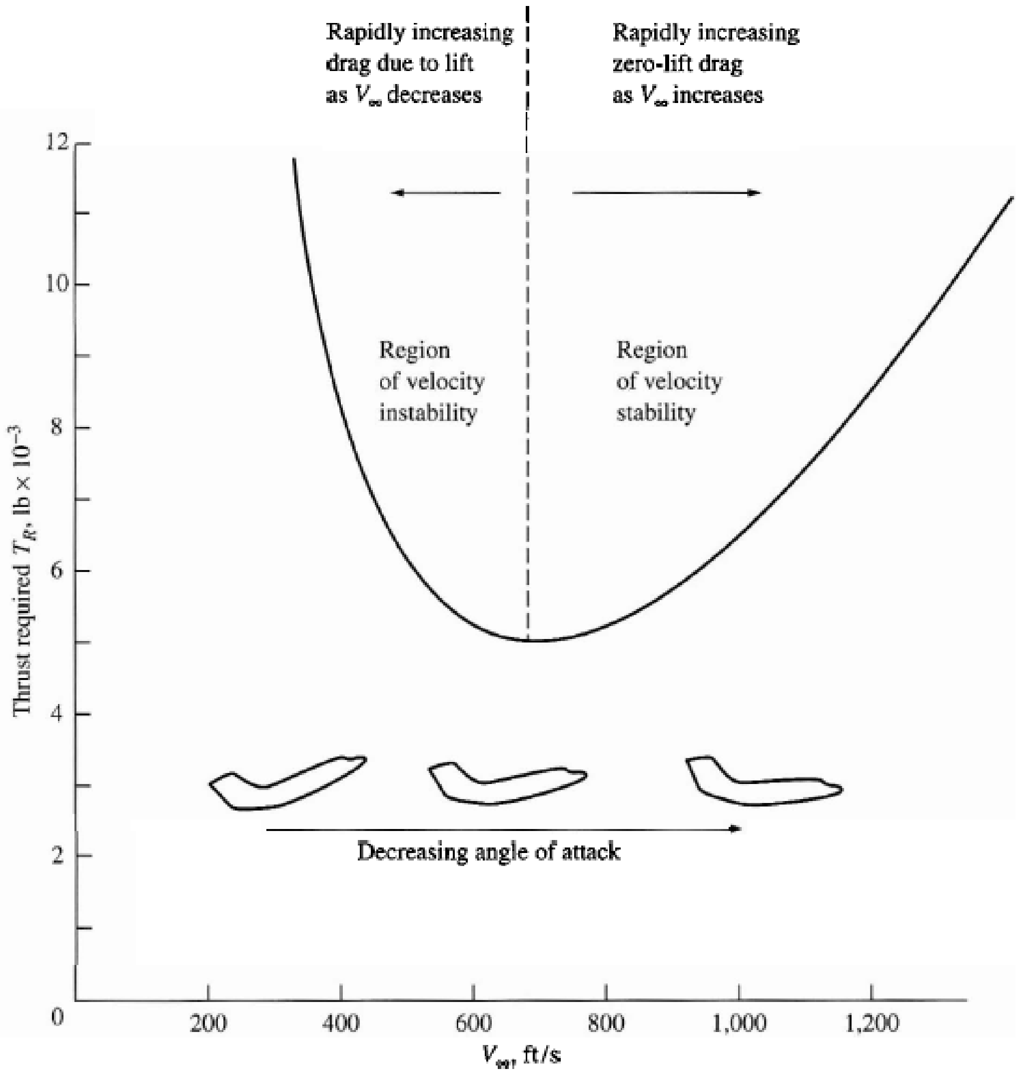
Choose a value of V, for this value to have W=L we need: $C_L = \frac{2W}{\rho_\infty V_\infty^2 S}$

With the value of Cl we can calculate $C_D = C_{D,0} + KC_L^2$

We can calculate the required Thrust $T_R = D = \frac{1}{2}\rho_\infty V_\infty^2 S C_D$

Repeat for several values of Velocity

At the lowest values of *V, CL* is very large; but as V increases, *CL* decreases. At very low velocity, the necessary lift is generated by flying at a high lift coefficient, hence at a high angle of attack. However, as *V,* increases, a progressively lower **CL** is required to sustain the weight of the airplane because the necessary lift is generated progressively more by the increasing dynamic pressure (the angle of attack of the airplane progressively decreases).



At low velocity, where ***CL*** is high, the total drag is dominated by the drag due to lift. Since the drag due to lift is proportional to the ***square*** of ***CL,*** since ***CL*** decreases rapidly as ***V,*** increases, the drag due to lift rapidly decreases, in spite of the fact that the dynamic pressure is increasing. This is why the ***TR*** curve first ***decreases*** as ***V increases.***

In contrast, the zero-lift drag increases as the square of V. At high velocity, the total drag is dominated by the zero-lift drag. Hence, as the velocity of the airplane increases, there is some velocity at which the increasing zero-lift drag exactly compensates for the decreasing drag due to lift; this is the velocity at which ***TR*** is a minimum. At higher velocities, the rapidly increasing zero-lift drag causes ***TR*** to increase with increasing velocity