Launch vehicle sizing

To design a vehicle which can give 9000m/s for 1 kg payload

$$m_{pay} = \frac{m_{prop}}{e^{\frac{\Delta v}{I_{sp} g_0}} - 1} - m_{inert}$$

1 kg payload mass is selected for normalisation for the Actual payload mass requirement

Table C.2. **Sizing Process for Staged Vehicles.** This process allows us to size individual stages and the entire vehicle.

Step	Comments
1. Choose the number of stages (n_{stage})	• Choose the minimum number of stages that is practical. • Choose different values for n_{stage} and compare the marginal differences.
2. Choose propellants for each stage	• These trades are discussed throughout the book.
3. Choose the inert-mass fraction for each stage	• Figs. 5.21, 5.22, and C.2 indicate reasonable choices. • There is a large dispersion in the numbers.
4. Allocate a fraction of Δv to each stage	• Let $f_1 \rightarrow f_{n_{stage}}$ be the fraction for each stage; 1 refers to the first stage, n_{stage} refers to the last stage. • $f_1 + f_2 + \ldots + f_{n_{stage}} = 1$ • $f_1 \Delta v_{tot} = \Delta v_1$ (Δv on first stage) $f_i \Delta v_{tot} = \Delta v_i$ (Δv on i-th stage) $f_{n_{stage}} \Delta v_{tot} = \Delta v_{n_{stage}}$ (Δv on last stage)
5. Size the stages and the vehicle	• We start at the uppermost stage and work back to the first stage. • The payload for each succeeding stage includes the previous stages and the actual payload for the mission.
6. Minimize the vehicle mass by optimizing the Δv fraction allotted to each stage	• We must vary f_1 through $f_{n_{stage}}$ to determine the combination that minimizes the vehicle's initial mass. • Usually requires a numerical iteration or optimizing algorithm which repeats steps 4 and 5 until we find a minimum initial mass of the vehicle.

Step 1 – choose the number of stages

- More the stage – less inert mass fraction
- Complexity and cost of development
- Choose the minimum number stages that is practical
- Choose different number of stages and optimum can be arrived at

Table C.1. Data on First Stages of Common Launch Vehicles. This is the basic data from Isakowitz [1991] used in Fig. C.1. Inert-mass fraction = (Gross Mass − Propellant Mass) / Gross Mass.

Stage	Propellant Mass (kg)	Gross Mass (kg)	Sea-Level I_{sp} (s)	f_{inert}
Atlas-E	112,900	121,000	233	0.067
Atlas-I	138,300	145,700	239.75	0.051
Atlas-II	155,900	165,700	240.75	0.059
Atlas-IIA	155,900	166,200	241.7	0.062
Atlas-IIAS	155,900	167,100	241.7	0.067
Delta	96,100	101,700	263.2	0.055
Titan-II	118,000	122,000	281	0.033
Titan-III	134,000	141,000	287	0.050
Titan-IV	155,000	163,000	287	0.049
Saturn S1-B	408,000	444,000	232	0.081
Saturn S1-C	2,080,000	2,210,000	264	0.059
Ariane-L33	233,000	251,000	248.5	0.072
Ariane-H150	155,000	170,000	409	0.088
Energia	820,000	905,000	354	0.094
Proton	410,200	455,600	285	0.100

Table 6.2. Mass Summary for Current Space-Propulsion SRMs. All masses are in kilograms. See text for explanation of miscellaneous masses.

Motor Designation	Propellant	Insulation	Case	Nozzle	Igniter	Misc.	Total	f_{prop}*
RSRM	501,809	11,177	44,793	10,860	227	670	568,536	0.883
ASRM	548,670	8587	45,114	8469†	199	2251	613,290	0.895
Titan IV	268,168	20,478	27,401	4315	128	8660	329,150	0.815
SRMU	313,130	6443	15,684	6739	91	4892	346,979	0.902
Castor IVA	10,101	234	749	225	10	276	11,595	0.871
GEM	11,767	312	372	242	7.9	291	12,992	0.906
ORBUS 21	9707	145	354	143	16	7	10,374	0.936
ORBUS 6E	2721	64.1	90.9	105.2	9.5	5.3	2996	0.908
Star 48B	2010	27.1	58.3	43.8	0.0‡	2.2	2141	0.939
Star 37XFP	884	12.7	26.3	31.7	0.0‡	1.3	956	0.915
Star 63D	3250	71.4	106.3	60.8	1.0	11.6	3501	0.928
Orion 50SAL	12,160	265.2	547.9	235.4	9.1	21.0	13,239	0.918
Orion 50	3024	75.6	133.4	118.7	5.3	9.9	3367	0.898
Orion 38	770.7	21.9	39.4	52.8	1.3	10.6	896.7	0.859

$$^{*}f_{prop} = \frac{\text{mass of propellant}}{\text{total mass}}$$ (see Sec. 1.1.5)

† Excludes mass of actuation system which is included in miscellaneous mass.
‡ Igniter mass included in nozzle.

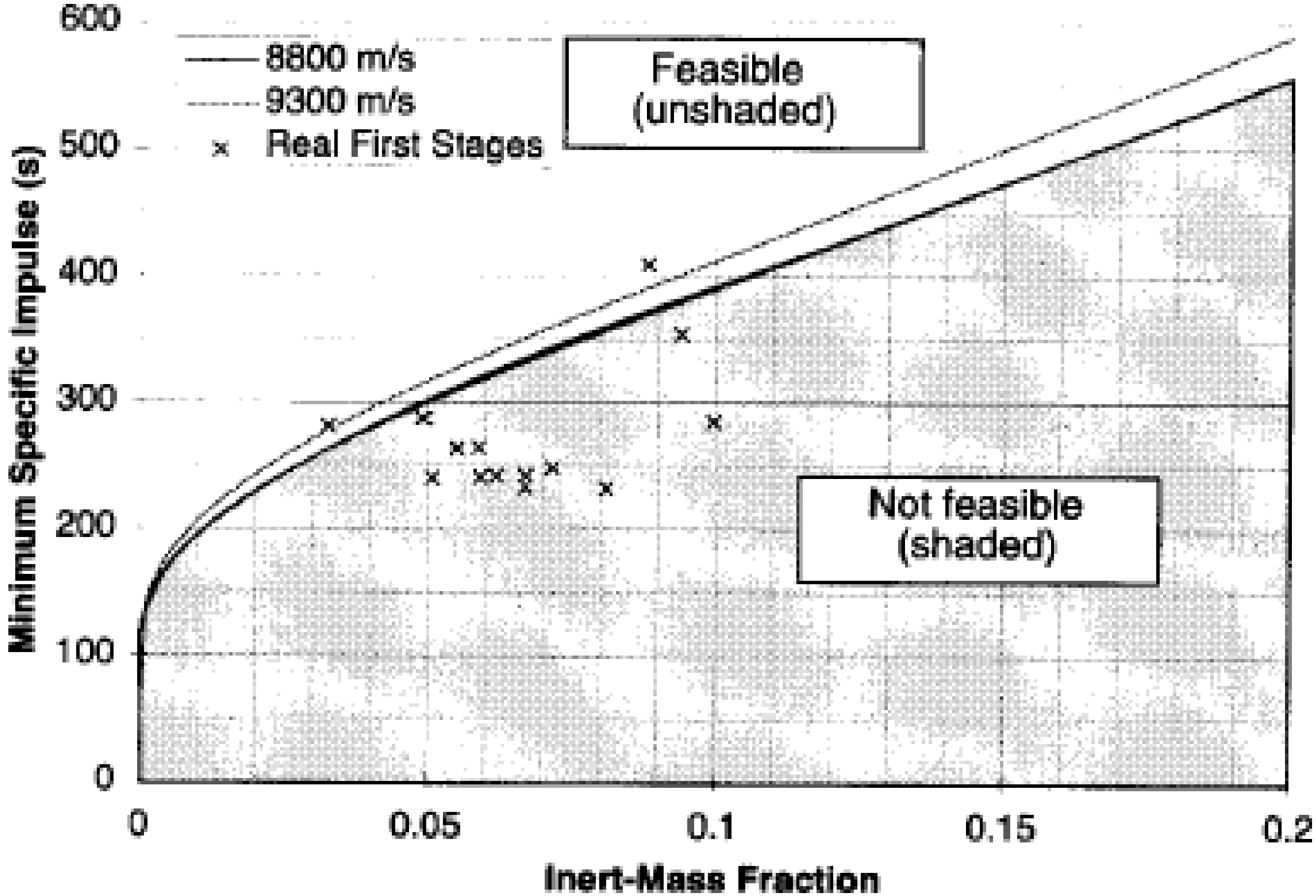

Fig. C.1. **Feasible Regions for Launch Systems**. The two curves shown here represent the minimum possible specific impulse, given a certain structural technology (f_{inert}), to perform a launch mission. Data for existing or historical (real) first stages is overlaid [Isakowitz,1991] and is listed in Table C.1. Several existing first-stage systems are feasible for a launch mission alone, based only on specific impulse and inert-mass fraction (other conditions may make these impractical or impossible).

Step 2 – choose the type of propellant

- Denser propellant for lower stages – lower ISP but lesser inert mass fraction
- But many other factors decide
- Determination of average propellant density – Add the volume – Divide total propellant with total volume

$$m_{fuel} = \frac{m_{prop}}{1 + O/F}$$

$$m_{ox} = m_{prop} \frac{O/F}{O/F + 1}$$

$$V_{fuel} = \frac{m_{fuel}}{\rho_{fuel}}$$

$$V_{ox} = \frac{m_{ox}}{\rho_{ox}}$$

Different possibilities of combinations

- The entire vehicle uses H_2/LOx, assuming 410 s I_{sp} for the first stages (slightly worse than the space value) and 435 s for all other stages (see Appendix B)
- The first stage uses RP-1, and the remaining stages use H_2/LOx, assuming a first stage I_{sp} of 290 s (slightly better than the sea-level value for the S-1C from Table C.1 or slightly worse than a space engine from Appendix B)
- The first stage uses hydrazine/N_2O_4, and the rest use H_2/LOx, assuming a first stage I_{sp} of 290 s (slightly worse than the vacuum value for Atlas (Table C.1) and worse than a space engine from Appendix B)
- All solid propellants, assuming 260 s for the first stage (slightly better than Scout at sea level—see Isakowitz [1991] or Chap. 6), and 290 s for all other stages (see Table 6.3)

Step 3 - Selection of inertial mass fraction for each stage

- Dispersion is large
- Depends on the complexity, type of propellant
- Design philosophy - conservative or aggressive

Single stage to orbit

• H_2/LOx	= 0.075	(Fig. C.2)
• RP-1/LOx	= 0.055	(Fig. C.2)
• Hydrazine/N_2O_4	= 0.035	(Fig. C.2)
• Solids	= 0.080	(Table 6.3)

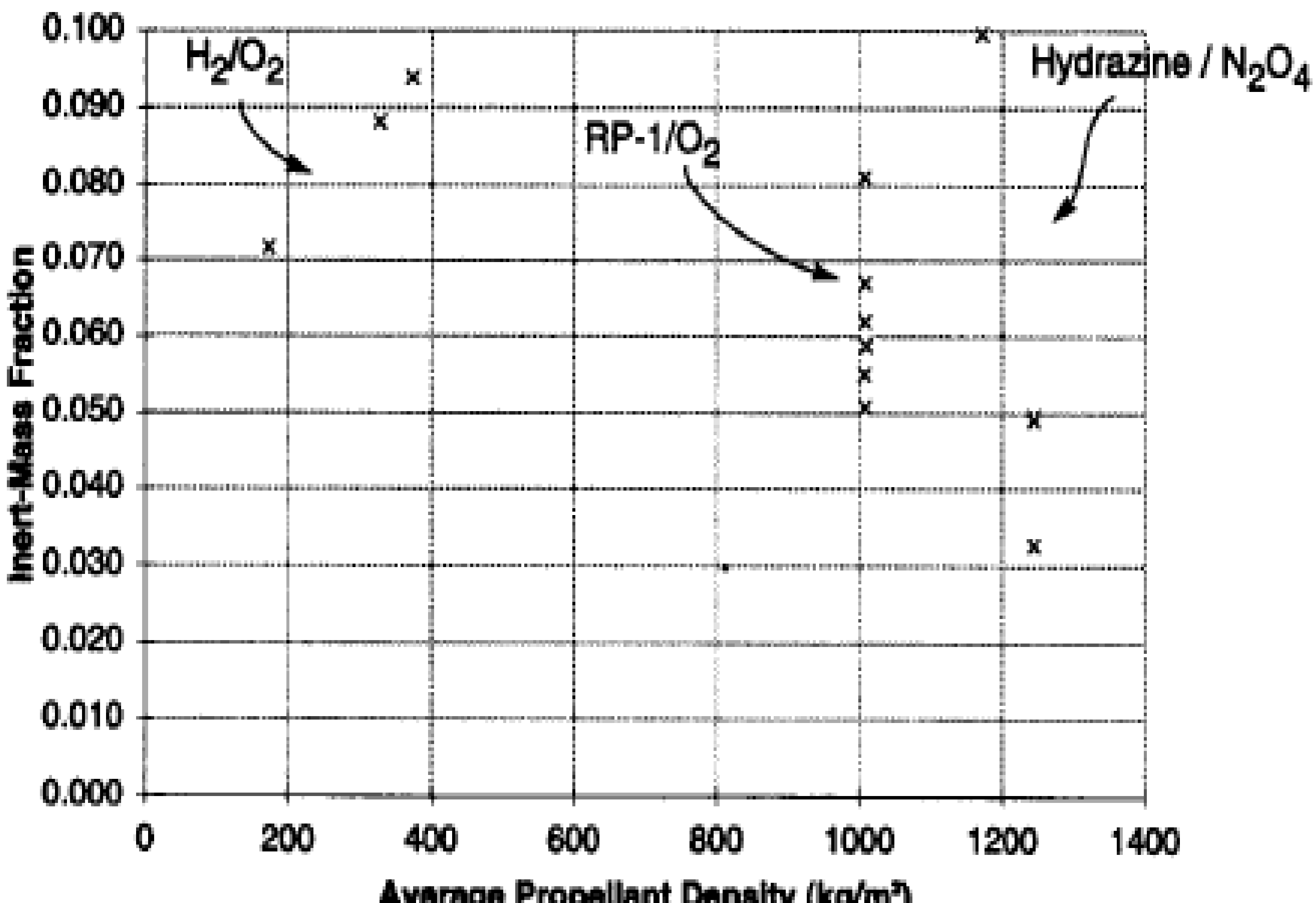

Fig. C.2. Inert-mass Fraction versus Average Propellant Density for the Vehicles Listed in Table C.1. As propellant density increases, inert-mass fraction decreases. But large dispersions indicate that other factors play a major role in these results. The density groupings indicated with text and arrow depend on the propellant combination used.

Multiple stages to orbit

- First stage, H_2/LOx = 0.095 (Fig. C-2 and Fig 5.29)
- First stage, RP-1/LOx = 0.070 (Fig. C-2 and Fig. 5.29)
- First stage, hydrazine/N_2O_4 = 0.050 (Fig. C-2 and Fig. 5.29)
- First stage, solid = 0.100 (Table 6.3)
- Others, H_2/LOx = 0.100 (Fig. 5.29)
- Others, RP-1/LOx = 0.085 (Fig. 5.29)
- Others, hydrazine/N_2O_4 = 0.075 (Fig. 5.29)
- Others, solid = 0.08 (Fig. 6.9)

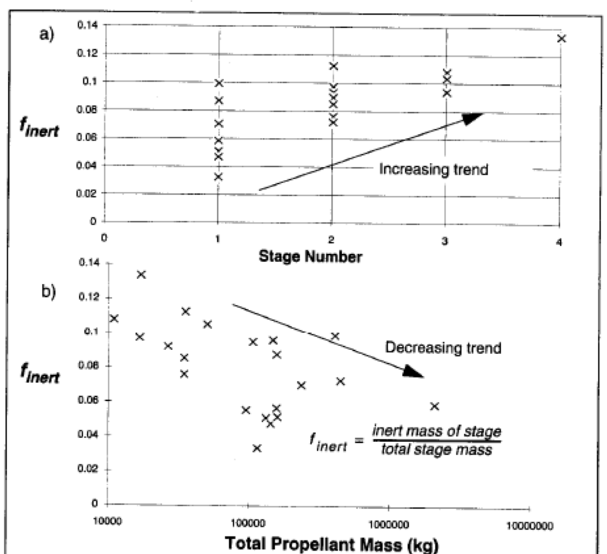

Fig. 5.21. Trends in Structural Mass Fraction for Launch Vehicles. Graph (a) is a plot of mass fractions versus stage number. Graph (b) is a plot of the same data as in (a) but is a function of propellant mass. These plots show that as propellant mass increases, the structural mass fraction decreases. As the stage number goes up, the mass fraction also tends to increase [Isakowitz, 1991].

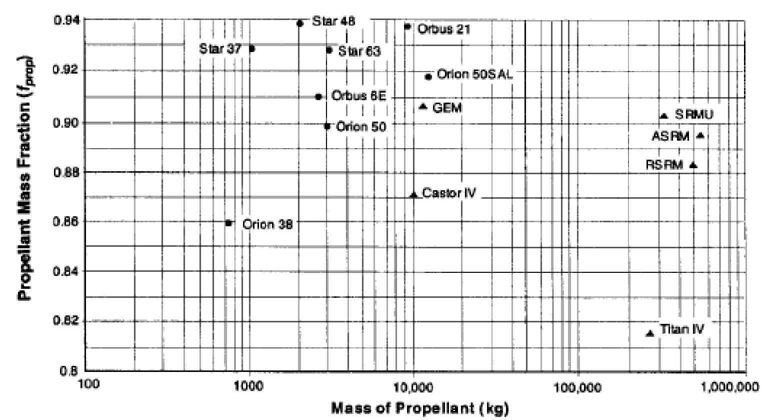

Fig. 6.9. Trends in Propellant Mass Fraction for Solid Rocket Motors. Booster motors (shown as triangles) usually have a lower propellant mass fraction than space motors (shown as solid circles).

Step 4 - Allocate ΔV to each stage

- Let $f_1 \rightarrow f_{n_{stage}}$ be the fraction for each stage; 1 refers to the first stage, n_{stage} refers to the last stage.
- $f_1 + f_2 + \ldots + f_{n_{stage}} = 1$
- $f_1 \Delta v_{tot} = \Delta v_1$ (Δ*v* on first stage)

  $f_i \Delta v_{tot} = \Delta v_i$ (Δ*v* on *i*-th stage)

  $f_{n_{stage}} \Delta v_{tot} = \Delta v_{n_{stage}}$ (Δ*v* on last stage)

Allocate ΔV fraction to each stage

In general form $\sum \Delta V_i = f_i * V_{total} = 1$

Size the stages and the vehicle

Sizing starts with the upper most stage and working down stage by stage downwards

Given the payload mass, inert mass fraction, ΔV, ISP
--- propellant Mass, inert mass and the total mass at the start of the stage

This mass becomes the payload mass of the succeeding lower stage

This process continues

Example – two stage LV using LH2 – LOX for 1kg P/L

First stage ΔV fraction – 46% Second stage ΔV fraction – 54%
ISP of 435 seconds

$$f_1 = 0.46 \rightarrow \Delta v_1 = 0.46\,(9000) = 4140 \text{ m/s}$$

$$f_2 = 0.54 \rightarrow \Delta v_2 = 0.54\,(9000) = 4860 \text{ m/s}$$

Stage 2 - Mass of propellant – 2.779 kg
Mass of inert – 0.309 k
Payload mass - 1 kg
Total weight (including P/L) - 4.008 kg

$$m_{pay} = \frac{m_{prop}}{e^{\frac{\Delta v}{I_{sp} g_0}} - 1} - m_{inert}$$

Stage 1 - Mass of propellant – 9.066 kg
Mass of inert – 0.952 kg

Total weight at lift off - 14.106 kg

Step – 5 Optimisation of ΔV fraction – Iteration method

Consider a two stage vehicle

Select the range of ΔV fraction f 1 for the first stage – say $f_{1\text{-}}$start to f_1–end with step Δf_1 - like 0.45 to 0.55 with 0.1 increment

Select f_1 as f_1 start (lowest range)

f_2 is $(1 – f_1)$

$\Delta V1 = f_1 * \Delta V$ total $\Delta V2 = f_2 * \Delta V$ total

Calculate the initial mass – lift off mass

Increment f_1 with Δf_1 and repeat

Plot f_1 versus initial mass

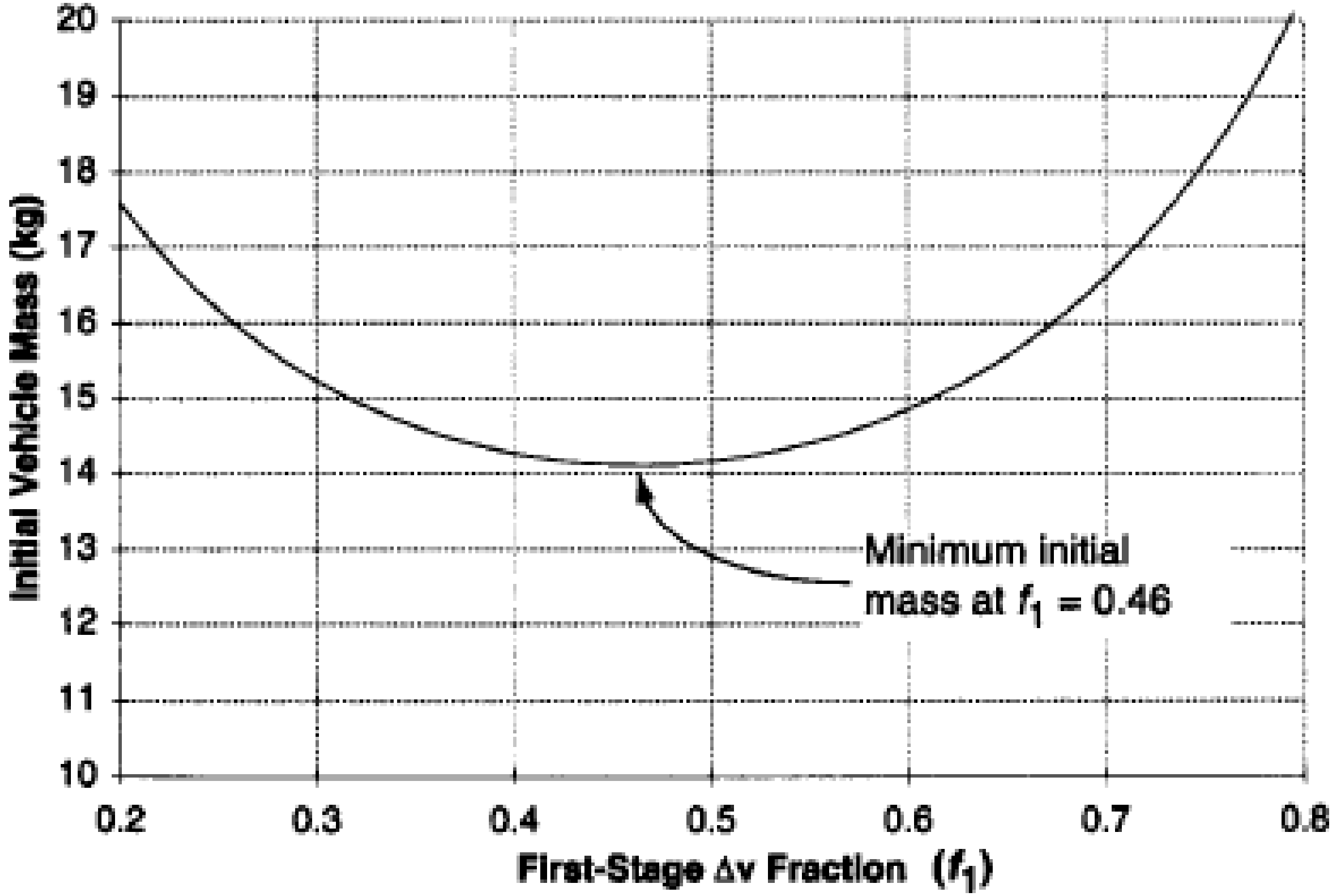

Fig. C.3. **Two-Stage H_2/O_2 Vehicle Initial Mass versus First-Stage Δ*v* Fraction.** As we vary f_1 between 0.2 and 0.8, we see a minimum at $f_1 = 0.46$.

For three stage vehicle – three fractions for are
f_1, f_2 and f_3

$$f_3 = 1 - f_2 - f_1$$

By iterative method, we can find minimise these values

Table C.3. Results of the Single-Stage-to-Orbit Example. Based on the assumed parameters, RP-1/O_2 and solids are not feasible. The H_2/O_2 system is lighter than the hydrazine/N_2O_4 system. Remember, we have normalized our vehicle masses by assuming a 1-kg payload. For other payloads, multiply these numbers by the payload mass to get actual mass.

	H_2 / O_2	Hydrazine / N_2O_4
Specific impulse (s)	410	290
Inert-mass fraction	0.075	0.035
Propellant mass (kg)	26.06	127.04
Inert mass (kg)	2.11	4.61
Final mass (kg)	3.11	5.61
Initial mass (kg)	**29.17**	**132.64**
Mass of payload / initial mass	3.43 %	0.75 %
Minimum feasible I_{sp} [Eq. (1.29)]	354.2 s	273.66 s

Table C.4. **Results of Analysis for Two-Stage Vehicles.** The vehicle made up completely of propellants with high specific impulse outperforms all others. A two-stage, all-solid vehicle seems impractical. Remember, we have normalized our vehicle masses by assuming a 1-kg payload. For other payloads, multiply these numbers by the payload mass to get actual mass.

	All H_2O_2	**RP-1 and H_2**	**N_2H_4 and H_2**	**All Solids**
Stage 1 - I_{sp} (s)	410	290	290	260
Stage 2 - I_{sp} (s)	435	435	435	290
Stage 1 - Inert-mass fraction	0.095	0.070	0.050	0.100
Stage 2 - Inert-mass fraction	0.100	0.100	0.100	0.080
Stage 1 - Δv (m/s)	4140	2610	2880	3780
Stage 2 - Δv (m/s)	4860	6390	6120	5220
Stage 1 - Propellant mass (kg)	9.066	12.328	12.558	63.179
Stage 1 - Inert mass (kg)	0.952	0.928	0.661	7.020
Stage 2 - Propellant mass (kg)	2.668	5.648	4.956	9.708
Stage 2 - Inert mass (kg)	0.296	0.628	0.551	0.844
Initial vehicle mass (kg)	**14.106**	**20.531**	**19.726**	**81.752**
Payload mass/Initial mass	7.1 %	4.9 %	5.1%	1.2 %

Three stage analysis 12.3kg 47.4 kg

PSLV has 5 stages due to many solid stages – not considered optimum