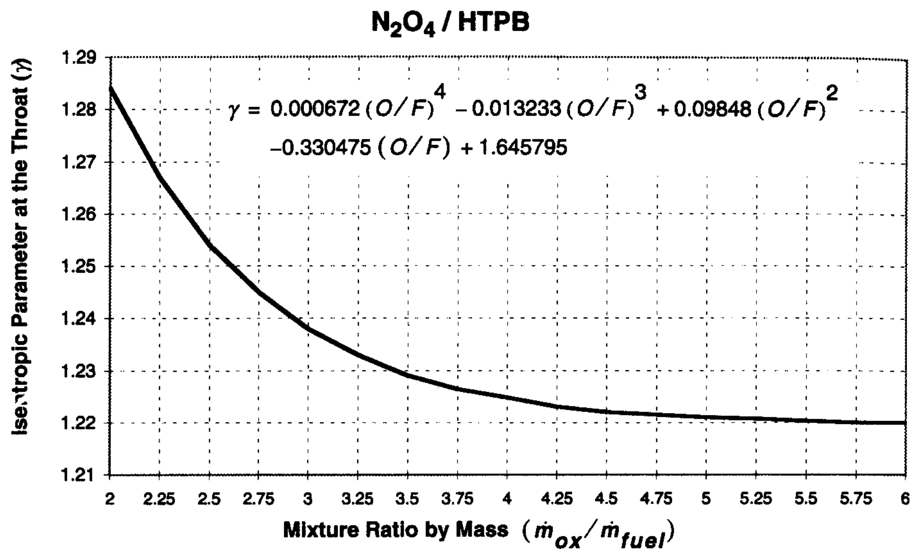

Fig. B.35. **Isentropic Parameter Versus Mixture Ratio.** The fuel is hydroxy terminated poly butadiene (HTPB), and the oxidizer is nitrogen tetroxide (N_2O_4). The equation gives a curve fit of the data.

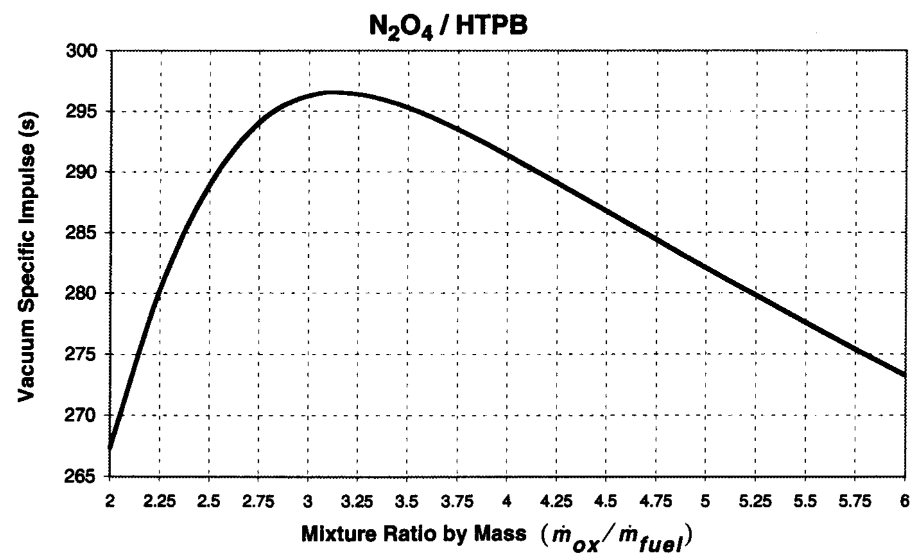

Fig. B.36. **Vacuum Specific Impulse Versus Mixture Ratio.** The fuel is hydroxy terminated poly butadiene (HTPB), and the oxidizer is nitrogen tetroxide (N_2O_4).

Appendix C
Launch Vehicles and Staging

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Whenever we encounter missions requiring a large Δv, we run the risk of not being able to perform the missions with certain technologies. For example, from Table 2.10 we find that a typical launch Δv ranges from about 8.8 km/s to 9.3 km/s. In Sec. 1.1.5, we find there is a "not feasible" condition that gives us a relationship [Eq. (1.29)] between the mission Δv, average specific impulse (I_{sp}), and the inert-mass fraction (f_{inert}). Figure C.1 shows the regions that are and are not feasible for a launch mission using Eq. (1.29). The lower curve corresponds to the relation between inert-mass fraction and specific impulse for Δv = 8800 m/s. The upper curve corresponds to Δv = 9300 m/s. Specific impulses below the curve values for a particular Δv are not feasible. We have also overlaid discrete values for first stages of existing launch vehicles [Isakowitz, 1991]. Table C.1 lists the data for these stages. The systems above the line to the left of f_{inert} = 0.5 are the core first stages for the Titan vehicles, and the system above the line at f_{inert} = 0.088 is the Ariane 5 core stage. Of these possibilities, only the Titan-II and the Ariane 5 have enough Δv to get themselves to orbit. To be conservative, we have assumed the sea-level value for specific impulse for all of the "real" data. In some cases, Isakowitz does not specify the sea-level I_{sp}, so we simply reduce the vacuum I_{sp} by 5%.

There are at least two other considerations. First, the Titan-II and Ariane-5 can get themselves to orbit but without much payload. For example, using Eq. (1.20), we find the allowable payload is given by

$$m_{pay} = \frac{m_{prop}}{e^{\frac{\Delta v}{I_{sp} g_0}} - 1} - m_{inert} \qquad \text{(C.1)}$$

We assume a conservative Δv = 8800 m/s and the inert mass is the difference between the gross mass and propellant mass in Table C.1 (this difference does not include a payload mounting structure or a fairing). If so, we can find the payload masses for Titan-II and Ariane-5 (payload masses for the other systems are negative):

- Titan-II = 858 kg
- Ariane-5 = 2,224 kg

The second consideration occurs when we are close to the feasible limit. As we approach this limit, our design space becomes very sensitive to small changes in key parameters. For example, Fig. 1.6 shows us that, for a given inert-mass fraction, as our specific impulse decreases, the slope of the mass curve gets steeper.

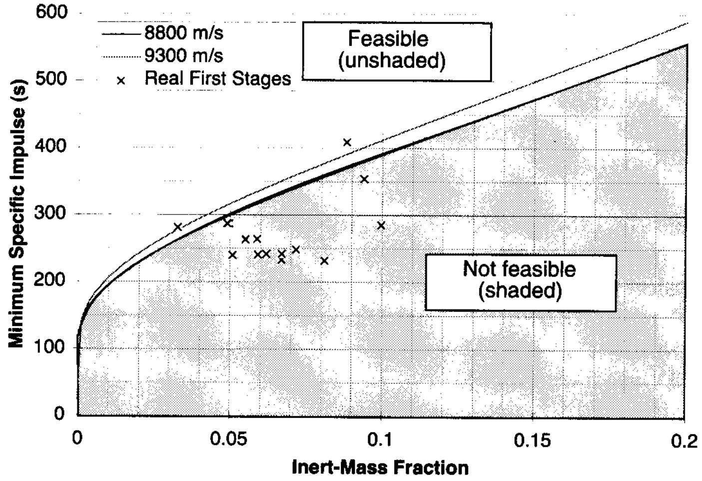

Fig. C.1. **Feasible Regions for Launch Systems.** The two curves shown here represent the minimum possible specific impulse, given a certain structural technology (f_{inert}), to perform a launch mission. Data for existing or historical (real) first stages is overlaid [Isakowitz,1991] and is listed in Table C.1. Several existing first-stage systems are feasible for a launch mission alone, based only on specific impulse and inert-mass fraction (other conditions may make these impractical or impossible).

This means that any small change or error in our design causes very large changes in the system. This situation is undesirable!

So, what can we do to resolve the dilemma of having technology—such as specific impulse or inert-mass fraction—that cannot do a large Δv mission? The obvious answer is to find or develop a solution that allows us to increase specific impulse or to decrease the inert-mass fraction. Looking again at Fig. 1.6, we find that, if we can increase the specific impulse of our propulsion system above 700 s, the mass curves become very flat and almost independent of structural technology (f_{inert}). Two technologies that can achieve this level of specific impulse at high thrust-to-weight ratios are nuclear fission (Chap. 8) and, perhaps, beamed-laser propulsion (Sec. 11.3.2).

Finding technology that can lower the inert-mass fraction can relieve us from a requirement for high specific impulse. This fact is also illustrated in Fig. 1.6, where we see that lower f_{inert}s shift our specific-impulse requirement, for a given initial mass, to a lower number. But existing systems are pretty good, and it is difficult to drastically improve structural technology. Having said this, we *can* drastically improve the "integrated inert-mass fraction" (the average mass fraction, integrated over a mission) by discarding inert mass as it becomes unnecessary. This approach is called *staging*. The basic philosophy behind staging is presented in Sec. 2.6.1.

Table C.1. **Data on First Stages of Common Launch Vehicles.** This is the basic data from Isakowitz [1991] used in Fig. C.1. Inert-mass fraction = (Gross Mass − Propellant Mass) / Gross Mass.

Stage	Propellant Mass (kg)	Gross Mass (kg)	Sea-Level I_{sp} (s)	f_{inert}
Atlas-E	112,900	121,000	233	0.067
Atlas-I	138,300	145,700	239.75	0.051
Atlas-II	155,900	165,700	240.75	0.059
Atlas-IIA	155,900	166,200	241.7	0.062
Atlas-IIAS	155,900	167,100	241.7	0.067
Delta	96,100	101,700	263.2	0.055
Titan-II	118,000	122,000	281	0.033
Titan-III	134,000	141,000	287	0.050
Titan-IV	155,000	163,000	287	0.049
Saturn S1-B	408,000	444,000	232	0.081
Saturn S1-C	2,080,000	2,210,000	264	0.059
Ariane-L33	233,000	251,000	248.5	0.072
Ariane-H150	155,000	170,000	409	0.088
Energia	820,000	905,000	354	0.094
Proton	410,200	455,600	285	0.100

Evaluating Staging

Having discussed the rationale for staging, how do we choose the number of stages, and how do we size the individual stages? In Sec. 2.6.1, we see that increasing the number of stages decreases the initial mass of our vehicle (Fig. 2.11). However, increasing the number of stages usually increases the cost of our system, if we have to design all of the stages from scratch. In fact, if we choose *n* stages, the cost for this system can be greater than *n* times the cost of a single stage. This claim assumes, of course, that doing a mission with a single stage is practical. The process outlined in Table C.2 allows us to size a vehicle with a number of stages.

To illustrate how we can evaluate staging, we look at several launch-vehicle systems as an example. We assume an average ascent $\Delta v = 9000$ m/s and a payload of 1 kg. The choice of payload mass allows us to normalize all of the other masses. This means we simply multiply all of the normalized masses by the payload mass to get the actual design mass. In summary:

- $\Delta v = 9000$ m/s
- payload mass (m_{pay}) = 1 kg

Table C.2. Sizing Process for Staged Vehicles. This process allows us to size individual stages and the entire vehicle.

Step	Comments
1. Choose the number of stages (n_{stage})	• Choose the minimum number of stages that is practical. • Choose different values for n_{stage} and compare the marginal differences.
2. Choose propellants for each stage	• These trades are discussed throughout the book.
3. Choose the inert-mass fraction for each stage	• Figs. 5.21, 5.22, and C.2 indicate reasonable choices. • There is a large dispersion in the numbers.
4. Allocate a fraction of Δv to each stage	• Let $f_1 \rightarrow f_{n_{stage}}$ be the fraction for each stage; 1 refers to the first stage, n_{stage} refers to the last stage. • $f_1 + f_2 + \ldots + f_{n_{stage}} = 1$ • $f_1 \Delta v_{tot} = \Delta v_1$ (Δv on first stage) $f_i \Delta v_{tot} = \Delta v_i$ (Δv on i-th stage) $f_{n_{stage}} \Delta v_{tot} = \Delta v_{n_{stage}}$ (Δv on last stage)
5. Size the stages and the vehicle	• We start at the uppermost stage and work back to the first stage. • The payload for each succeeding stage includes the previous stages and the actual payload for the mission.
6. Minimize the vehicle mass by optimizing the Δv fraction allotted to each stage	• We must vary f_1 through $f_{n_{stage}}$ to determine the combination that minimizes the vehicle's initial mass. • Usually requires a numerical iteration or optimizing algorithm which repeats steps 4 and 5 until we find a minimum initial mass of the vehicle.

Choose Propellants for Each Stage

The process for choosing the propellants for a particular stage is discussed throughout the rest of the book, where we have already discussed the usual considerations of specific impulse, handling, toxicity, and others.

But one point needs to be stressed. There is a common perception that the choice of propellants can be based on the density of the propellants. Further, this perception drives us to choose denser, and usually lower-specific-impulse, propellants (such as RP-1/LOx) for lower stages and less-dense, higher-specific-impulse propellants (such as H_2/LOx) for upper stages. We reason that higher-density propellants allow us a better (lower) inert-mass fraction, which leads us to a lighter first stage. Although this reasoning may be correct (depending on the mission and requirements), the overall vehicle mass usually increases above what is achievable with higher-performing propellants.

The Saturn family of launch vehicles used RP-1/LOx on the first stage and H_2/LOx on second and third stages. This approach is now universally accepted. However, keep in mind that these vehicles were huge because they were intended for the very large Δv mission of going from the Earth's surface to the Moon and back. If designers had made the first stage with H_2/LOx, it would have been too big to transport to the launch site and would have made vertical assembly and operation of the vehicle even more difficult than it was. Although the mix of propellants was appropriate for Saturn-V, it may not be appropriate for other missions.

If we look at the vehicles listed in Table C.1 and plot the inert-mass fraction versus the average propellant density, we get the result shown in Fig. C.2. We determine the average propellant density as follows:

- From the oxidizer-to-fuel ratio (*O/F*) for the individual systems (see Isakowitz [1991]), determine the mass of fuel and oxidizer based on the propellant mass [use Eqs. (5.29) and (5.30)]
- Determine the fuel and oxidizer volumes using Eqs. (5.31) and (5.32) and the density data given in Appendix B
- Add the volumes together to get the total volume
- Divide the total propellant mass by the total volume to determine the average propellant density

In Fig. C.2, the propellants on the left are LH_2/LOx (*O/F* range 5–6), the middle band contains RP-1/LOx systems (*O/F*s about 2.25) and the right-hand band reflects values for hydrazine/N_2O_4 (*O/F*s about 1.9). Clearly, inert-mass fraction decreases as propellant density increases, but large dispersions indicate other important factors are at play.

For our example problem, we look at several possibilities:

- The entire vehicle uses H_2/LOx, assuming 410 s I_{sp} for the first stages (slightly worse than the space value) and 435 s for all other stages (see Appendix B)
- The first stage uses RP-1, and the remaining stages use H_2/LOx, assuming a first stage I_{sp} of 290 s (slightly better than the sea-level value for the S-1C from Table C.1 or slightly worse than a space engine from Appendix B)
- The first stage uses hydrazine/N_2O_4, and the rest use H_2/LOx, assuming a first stage I_{sp} of 290 s (slightly worse than the vacuum value for Atlas (Table C.1) and worse than a space engine from Appendix B)
- All solid propellants, assuming 260 s for the first stage (slightly better than Scout at sea level—see Isakowitz [1991] or Chap. 6), and 290 s for all other stages (see Table 6.3)

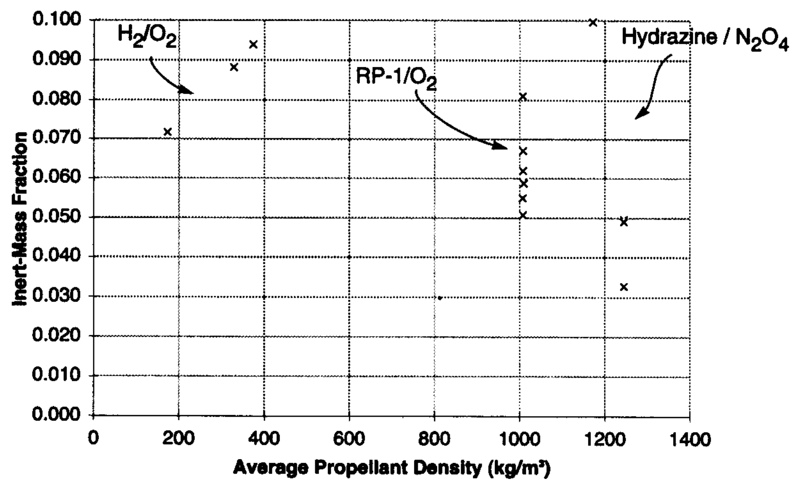

Fig. C.2. **Inert-mass Fraction versus Average Propellant Density for the Vehicles Listed in Table C.1.** As propellant density increases, inert-mass fraction decreases. But large dispersions indicate that other factors play a major role in these results. The density groupings indicated with text and arrow depend on the propellant combination used.

Choose the Inert-mass Fraction for Each Stage

Figures 5.29, 5.30, and C.2 show the trends in inert-mass fraction for liquid rockets. Table 6.2 and Figs. 6.9 and 6.10 show trends for solids. But the large dispersions in these figures are frustrating. For example, the inert-mass fraction for the Atlas family of vehicles ranges from 0.051 to 0.067 (see Table C.1). How can fractions vary by 30% for similar technology and propellants?

The dispersion in mass fractions depends on all of the design requirements and constraints that are part of any design. The Atlas-E is a simple vehicle that has no parallel stages and does not have much mass stacked on top. By contrast, the Atlas-IIAS first stage has solid rockets strapped to its side and has a large upper stage (Centaur) and payload on top. It makes sense that this more complex vehicle should have a larger mass fraction. When choosing an inert-mass fraction, we must consider complexity, plus propellant type and mass, and then decide how aggressive or conservative we want to be.

For our example, we choose the following inert-mass fractions:

Single stage to orbit

- H_2/LOx = 0.075 (Fig. C.2)
- RP-1/LOx = 0.055 (Fig. C.2)
- Hydrazine/N_2O_4 = 0.035 (Fig. C.2)
- Solids = 0.080 (Table 6.3)

Multiple stages to orbit

- First stage, H_2/LOx = 0.095 (Fig. C-2 and Fig 5.29)
- First stage, RP-1/LOx = 0.070 (Fig. C-2 and Fig. 5.29)
- First stage, hydrazine/N_2O_4 = 0.050 (Fig. C-2 and Fig. 5.29)
- First stage, solid = 0.100 (Table 6.3)
- Others, H_2/LOx = 0.100 (Fig. 5.29)
- Others, RP-1/LOx = 0.085 (Fig. 5.29)
- Others, hydrazine/N_2O_4 = 0.075 (Fig. 5.29)
- Others, solid = 0.08 (Fig. 6.9)

Allocate a Fraction of Δv to Each Stage

How do we vary the proportions between stages? We want to divide up the Δv so the vehicle's total mass is minimized! We define f_i as the fraction of Δv allocated to the i-th stage. The constraint on f_i is that the sum of all of the fractions equals one. The Δv for each stage becomes

$$\Delta v_i = f_i \Delta v_{tot} \tag{C.2}$$

The best combination of these numbers minimizes the vehicle mass. In the special case of inert-mass fractions and specific impulses being equal for all stages, the fraction is

$$f_i = \frac{1}{n_{stage}} \tag{C.3}$$

For the more special case of a single-stage-to-orbit, the only fraction is $f_1 = 1$. For all other situations, we must rely on results of numerical analysis. We discuss this approach below.

Size the Stages and Vehicle

To size the vehicle, we start with the uppermost stage and work down the vehicle stack, stage by stage. Given the payload mass, Δv, specific impulse, and inert-mass fraction for each stage, we can determine the propellant mass, inert mass, and initial mass of that stage using Eqs. (1.27), (1.24), and (1.26) respectively. This initial mass then becomes the payload mass for the succeeding stage, and we repeat the analysis. As an example, consider a two-stage launch vehicle using all H_2/LOx propulsion. We assume the specific impulse and inert mass decisions as listed above. We also assume the Δv is divided, with 46% on the first stage and 54% on the second stage. (This assumption is justified by the analysis discussed in the next section.) If our payload mass is 1 kg, the numbers for the second (upper) stage are:

$$f_1 = 0.46 \rightarrow \Delta v_1 = 0.46\,(9000) = 4140 \text{ m/s}$$

$$f_2 = 0.54 \rightarrow \Delta v_2 = 0.54\,(9000) = 4860 \text{ m/s}$$

$$m_{prop_2} = \frac{m_{pay}\left[e^{\left(\frac{\Delta v_2}{I_{sp_2} g_0}\right)} - 1\right]\left(1 - f_{inert_2}\right)}{1 - f_{inert_2} e^{\left(\frac{\Delta v_2}{I_{sp_2} g_0}\right)}}$$

$$= \frac{(1)\left[e^{\frac{4860}{435\,(9.81)}} - 1\right](1 - 0.1)}{1 - 0.1 e^{\frac{4860}{435\,(9.81)}}} = 2.779 \text{ kg}$$

$$m_{inert_2} = \frac{f_{inert_2}}{1 - f_{inert_2}} m_{prop_2} = \frac{0.1}{1 - 0.1}(2.779) = 0.309 \text{ kg}$$

$$m_{i_2} = m_{pay} + m_{prop_2} + m_{inert_2} = 1 + 2.779 + 0.309 = 4.088 \text{ kg}$$

Now, for the first stage:

$$m_{prop_1} = \frac{(4.088)\left[e^{\frac{4140}{410\,(9.81)}} - 1\right](1 - 0.095)}{1 - 0.095 e^{\frac{4140}{410\,(9.81)}}} = 9.066 \text{ kg}$$

$$m_{inert_1} = \frac{0.095}{1 - 0.095}(9.066) = 0.952 \text{ kg}$$

$$m_i = 4.088 + 9.066 + 0.952 = 14.106 \text{ kg}$$

Optimize the Δv Fraction

So, how do we allocate Δv between stages? For a two-stage vehicle, we can vary one of the Δv fractions over the range from 0 to 1. If we choose to vary f_1, then f_2 is determined from the requirement that both numbers add up to 1. As we vary f_1, we can calculate the initial mass of the vehicle. If we plot the result of f_1 versus the initial mass, we can see the minimum value of initial mass, giving us our optimum distribution of Δv. The algorithm is as follows:

1. Choose a range of f_1 and divide this range into several increments that are Δf_1 apart
2. Let f_1 be the lowest value in the range of f_1
3. Let $f_2 = 1 - f_1$
4. Let $\Delta v_1 = f_1 \times \Delta v_{tot}$ and $\Delta v_2 = f_2 \times \Delta v_{tot}$
5. Calculate the initial mass of the vehicle with these Δv fractions
6. Let $f_1 = f_1 + \Delta f_1$, if we have not reached the end of our range
7. Go back to step 3

To illustrate this algorithm, we look at the example from above for the two-stage H_2/O_2 system. Figure C.3 shows how the initial mass varies as a function of f_1. The minimum initial vehicle mass is at $f_1 = 0.46$, as we used in our example above.

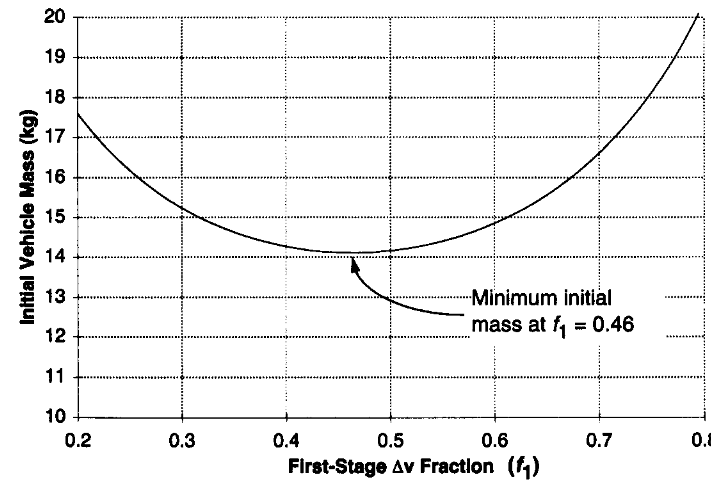

Fig. C.3. Two-Stage H_2/O_2 Vehicle Initial Mass versus First-Stage Δv Fraction. As we vary f_1 between 0.2 and 0.8, we see a minimum at $f_1 = 0.46$.

Doing this analysis for more than two stages is more difficult. We need to vary two Δvs over some range to find a minimum. For a three-stage system, we repeat the above algorithm for a range of f_2 values, choosing the minimum f_1 value for each f_2 (remember $f_3 = 1 - f_1 - f_2$). We can then plot the initial vehicle mass (each point being minimized for f_1) and choose the f_2 with the minimum initial-mass value.

Summary of Example Results

Let us now apply this analytical approach to our example problem. We start by looking at the single-stage-to-orbit problem. No optimizing is required because all of the Δv goes onto the only stage. Only the H_2/O_2 system and the hydrazine system turn out to be feasible for this mission, given our assumed numbers. The results are shown in Table C.3. The hydrogen-fueled vehicle is definitely lighter than the hydrazine-fueled vehicle.

Table C.3. Results of the Single-Stage-to-Orbit Example. Based on the assumed parameters, RP-1/O_2 and solids are not feasible. The H_2/O_2 system is lighter than the hydrazine/N_2O_4 system. Remember, we have normalized our vehicle masses by assuming a 1-kg payload. For other payloads, multiply these numbers by the payload mass to get actual mass.

	H_2 / O_2	Hydrazine / N_2O_4
Specific impulse (s)	410	290
Inert-mass fraction	0.075	0.035
Propellant mass (kg)	26.06	127.04
Inert mass (kg)	2.11	4.61
Final mass (kg)	3.11	5.61
Initial mass (kg)	**29.17**	**132.64**
Mass of payload / initial mass	3.43 %	0.75 %
Minimum feasible I_{sp} [Eq. (1.29)]	354.2 s	273.66 s

Now, let us look at the four possibilities described above for two-stage vehicles. Table C.4 shows the results of the analysis. Notice that the vehicle using pure H_2/O_2 is substantially lighter than the vehicle with RP-1 fuel on the first stage and H_2 on the second stage. If we add up the mass for the first stage, we find that the all-H_2/O_2 vehicle has a first-stage mass of 10.018 kg, whereas the RP-1 first stage has a mass of 13.256. From Isakowitz [1991] we can deduce that typical stage densities are 256 kg/m^3 for H_2/O_2 (from the S-1C stage) and 655 kg/m^3 for RP-1/O_2 (from the Ariane-5 core stage). Using these numbers, we find that the volume of the H_2/O_2 stage is 0.04 m^3 and the volume of the RP-1/O_2 stage is 0.02 m^3. The H_2/O_2 stage is twice as big despite its being lighter. These numbers validate our previous discussion concerning why we would choose a lower specific impulse but denser propellant for a first stage, as was done for the Saturn-V.

Table C.4. Results of Analysis for Two-Stage Vehicles. The vehicle made up completely of propellants with high specific impulse outperforms all others. A two-stage, all-solid vehicle seems impractical. Remember, we have normalized our vehicle masses by assuming a 1-kg payload. For other payloads, multiply these numbers by the payload mass to get actual mass.

	All H_2O_2	RP-1 and H_2	N_2H_4 and H_2	All Solids
Stage 1 - I_{sp} (s)	410	290	290	260
Stage 2 - I_{sp} (s)	435	435	435	290
Stage 1 - Inert-mass fraction	0.095	0.070	0.050	0.100
Stage 2 - Inert-mass fraction	0.100	0.100	0.100	0.080
Stage 1 - Δv (m/s)	4140	2610	2880	3780
Stage 2 - Δv (m/s)	4860	6390	6120	5220
Stage 1 - Propellant mass (kg)	9.066	12.328	12.558	63.179
Stage 1 - Inert mass (kg)	0.952	0.928	0.661	7.020
Stage 2 - Propellant mass (kg)	2.668	5.648	4.956	9.708
Stage 2 - Inert mass (kg)	0.296	0.628	0.551	0.844
Initial vehicle mass (kg)	**14.106**	**20.531**	**19.726**	**81.752**
Payload mass/Initial mass	7.1 %	4.9 %	5.1%	1.2 %

Performing similar analysis for three stages further lowers the masses of the vehicles. We find an initial mass for the all-H_2/O_2 vehicle of 12.312 kg and 47.356 kg for the all-solids vehicle. However, for both the RP-1 and hydrazine first-stage vehicles, we find that optimizing drives the first stage Δv to zero. This means that a two-stage vehicle using propellants with higher specific impulse is lighter than a three-stage vehicle using one stage with a low specific impulse.

The mass of the all-solid vehicle is still quite high compared to the one using liquids. This observation explains why existing vehicles, such as Scout and Pegasus, have so many stages.

Conclusions

We have shown why staging can be a valuable tool, presented an example of how staging can help in a launch mission (while hopefully dispelling some misconceptions), and shown how to size a vehicle. However, we have limited our discussion to fairly conventional approaches. It is very easy to quibble over the design numbers chosen here, but a sensitivity analysis shows that our basic conclusions do not change much if we vary specific impulse by 10 seconds or inert-mass fraction by a few percent.

We choose the launch mission as an example, but this type of analysis applies to any mission that requires a large Δv. Another example is transferring from low-Earth orbit to geostationary orbit. We typically use two stages for this mission—one stage for the perigee kick and another stage for the apogee kick in a Hohmann transfer. In the orbit-transfer example, the payoffs and sizing are a bit more straightforward because we are not trying to accelerate continuously. Other examples include lunar or planetary missions.

Many studies deal with optimizing missions by minimizing initial vehicle mass. But it is almost meaningless to minimize mass without including the cost of the minimization. As previously mentioned, doubling or tripling the number of stages may double or triple the cost. As designers, we would much rather try to get away with designing fewer stages. Each additional stage drastically increases the amount of work we must do and the probability that we might fail.

References

Isakowitz, Steven J. 1991. *International Reference Guide to Space Launch Systems.* Washington, DC: American Institute of Aeronautics and Astronautics.

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