

**HEAT TRANSFER**  
**Module I**  
**HEAT CONDUCTION**

**Course material Adapted from:**

1. Warren. L, McCabe, Julian. C. Smith and Peter Harriott, “Unit Operations of Chemical Engineering”, 7th Edn., McGraw Hill International Edition, NewYork 2005.
2. Holman. J.P., “Heat Transfer”, 9th Edn., Tata McGraw Hill Book Co., New Delhi, 2008.
3. R.C.Sachdeva, “Fundamentals of Engineering Heat and Mass Transfer”, 4th Edition, New Age International Publishers, 2010
4. [www.che.utexas.edu/course/che360/lecture\\_notes/chapter\\_2.ppt](http://www.che.utexas.edu/course/che360/lecture_notes/chapter_2.ppt)

**CONTENTS**

Introduction to various modes of heat transfer, Fourier’s law of heat conduction, effect of temperature on thermal conductivity, steady-state conduction, compound resistances in series, heat flow through a cylinder, and critical radius of insulation in pipes. Introduction to unsteady state conduction.

## Introduction

Practically all the operations that are carried out by the chemical engineers involve the production or absorption of energy in the form of heat. The study of temperature distribution and heat transfer is of great importance to engineers because of its almost universal occurrence in many branches of science and engineering. The first step in the optimal design of heat exchangers such as boilers, heaters, refrigerators and radiators is a detailed analysis of heat transfer. This is essential to determine the feasibility and cost of the undertaking, as well as the size of equipment required to transfer a specified amount of heat in a given time.

### Difference between thermodynamics and heat transfer

#### Thermodynamic tells us

- (i) How much heat is transferred
- (ii) How much work is done
- (iii) Final state of the system

#### Heat transfer tells us:

- (i) How much heat is transferred (with what modes)
- (ii) At what rate heat is transferred
- (iii) Temperature distribution inside the body



## 1.1. INTRODUCTION TO VARIOUS MODES OF HEAT TRANSFER

The various modes of heat transfer are (i) conduction (ii) convection (iii) radiation.

### Conduction

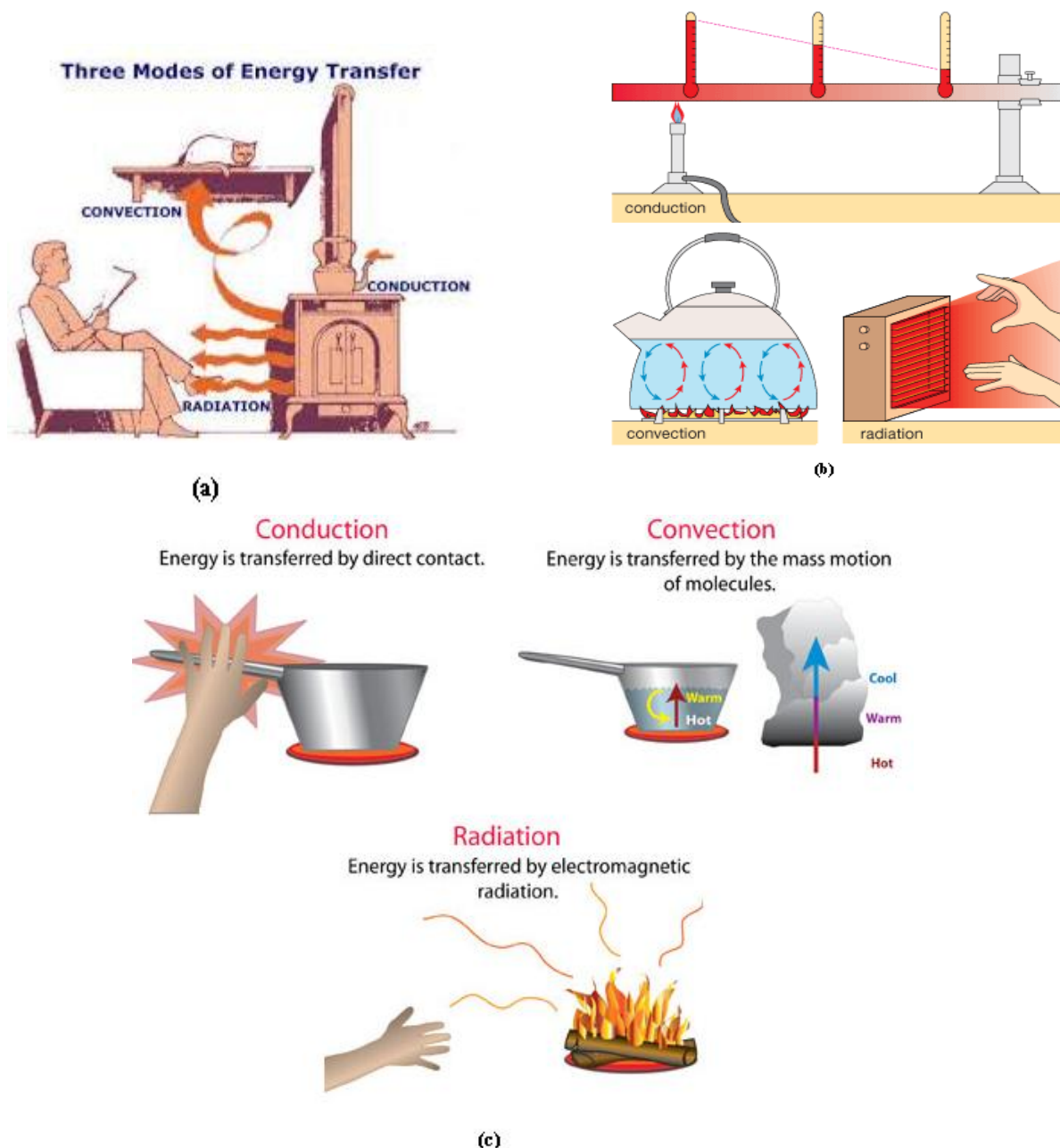
Heat transfer by the actual but invisible movement of molecules within the continuous substance due to temperature gradient is known as conduction.

### Convection

When a current or macroscopic particle of fluid crosses a specific surface, it carries with it a definite quantity of enthalpy. Such a flow of enthalpy is called convection. Convection is the mode of heat transfer in which the heat flow is associated with the movement of fluid.

### Radiation

Transfer of energy through space by electromagnetic waves is known as radiation.



**Fig.1. Modes of Heat Transfer**

## 1.2. APPLICATIONS OF HEAT TRANSFER

Energy production and conversion	-steam power plant, solar energy conversion etc.
Refrigeration and air-conditioning	
Domestic applications	-ovens, stoves, toaster
	Cooling of electronic equipment
Manufacturing / materials processing	-welding, casting, soldering, laser machining
Automobiles / aircraft design	

### 1.3. Conduction

It is the transfer of internal energy by microscopic diffusion and collisions of particles or quasi-particles within a body due to a temperature gradient. The microscopically diffusing and colliding objects include molecules, electrons, atoms, and phonons. They transfer disorganized microscopic kinetic and potential energy, which are jointly known as internal energy. Conduction can only take place within an object or material, or between two objects that are in direct or indirect contact with each other. On a microscopic scale, heat conduction occurs as hot, rapidly moving or vibrating atoms and molecules interact with neighboring atoms and molecules, transferring some of their energy (heat) to these neighboring particles. In other words, heat is transferred by conduction when adjacent atoms vibrate against one another, or as electrons move from one atom to another.

### 1.4. FOURIER'S LAW OF HEAT CONDUCTION

According to Fourier's law of heat conduction, heat flux is proportional to temperature gradient.

$$Q = -kA \frac{dT}{dx}$$

Where Q = Rate of heat transfer, W

k = Thermal conductivity, W/mK or W/m °C

A = Heat transfer area, m<sup>2</sup>

$\frac{dT}{dx}$  = Temperature gradient, °C/m



### EFFECT OF TEMPERATURE ON THERMAL CONDUCTIVITY

Thermal conductivity is the physical property of the substance. It depends upon temperature gradient. For pure metals thermal conductivity decreases with increase in temperature. For gases and insulators thermal conductivity increases with increase in temperature.

For small ranges of temperature, k may be considered constant. For larger temperature ranges, thermal conductivity can be approximated by an equation of the form  $k = a + bT$ , where a and b are empirical constants.

## Steady-State Conduction

It is the form of conduction which happens when the temperature difference driving the conduction is constant so that after an equilibrium time, the spatial distribution of temperatures (temperature field) in the conducting object does not change any further. In steady state conduction, the amount of heat entering a section is equal to amount of heat coming out.

## Unsteady state conduction

It is the form of conduction which happens when the temperature difference driving the conduction is not constant so that after an equilibrium time, the spatial distribution of temperatures (temperature field) in the conducting object changes as a function of time.

### 1.5. HEAT TRANSFER THROUGH A PLANE WALL

Let us consider a plane wall of thickness  $L$ , thermal conductivity  $k$ , inside surface temperature  $T_i$ , outside surface temperature  $T_o$ . Let  $Q$  be the rate of heat transferred through the plane wall.

By Fourier's law of heat conduction

$$Q = -kA \frac{dT}{dx} \quad (1)$$

$$Q \int_0^L dx = -kA \int_{T_i}^{T_o} dT \quad (2)$$

Integrating eqn(2), we get

$$Q = \frac{kA(T_i - T_o)}{L} \quad (3)$$

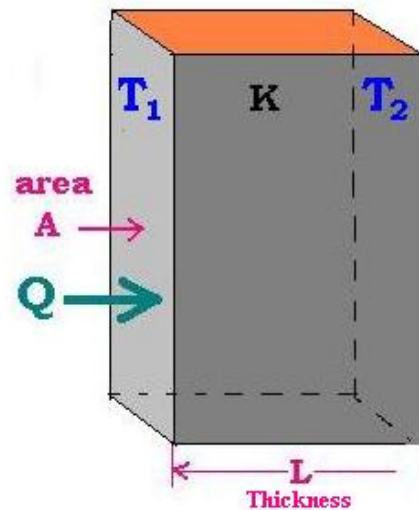


Fig. 2. Plane wall

### 1.6. HEAT TRANSFER THROUGH A HOLLOW CYLINDER

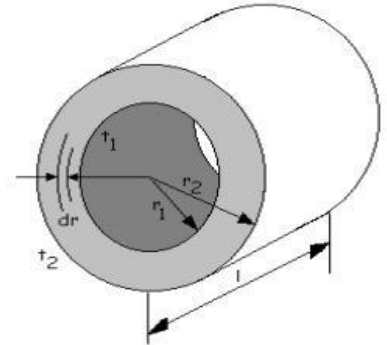
Let us consider a hollow cylinder as shown in fig.3. The inside radius of the cylinder is  $r_1$ , the outside radius is  $r_2$ , and the length of the cylinder is  $L$ . The thermal

conductivity of the material of which the cylinder is made is  $k$ . The temperature of the outside surface is  $T_2$ , and that of the inside surface is  $T_1$ .

By Fourier's law of heat conduction

$$Q = -kA \frac{dT}{dr} \quad (1)$$

$$Q \int_{r_1}^{r_2} \frac{dr}{r} = -k 2\pi L \int_{T_1}^{T_2} dT \quad (2)$$



**Fig.3. Hollow cylinder**

Integrating eqn(2), we get

$$Q = \frac{2\pi kL (T_1 - T_2)}{\ln\left(\frac{r_2}{r_1}\right)} \quad (3)$$

### LOGARITHMIC MEAN RADIUS AND ARITHMETIC MEAN RADIUS

Logarithmic mean radius is the radius that when applied to the integrated equation for a flat wall, will give the correct rate of heat flow through a thick walled cylinder. It is given by the expression

$$\bar{r}_L = \frac{r_o - r_i}{\ln\left(\frac{r_o}{r_i}\right)}$$

Arithmetic mean radius is used for thin walled cylinder.

$$r_A = \frac{r_o + r_i}{2}$$

where,  $\bar{r}_L$  = log mean radius

$r_A$  = Arithmetic mean radius

$r_o$  = outer radius

$r_i$  = inner radius

## 1.7. COMPOUND RESISTANCES IN SERIES

### (I) Heat Transfer Through A Composite Plane Wall

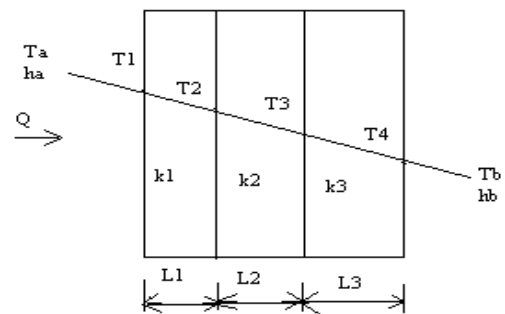
Let us consider a flat wall constructed of a series of 3 layers as shown in fig.4. Let the thickness of the layers be  $L_1$ ,  $L_2$ ,  $L_3$  and the average thermal conductivities of the materials of which the layers are made be  $k_1$ ,  $k_2$ ,  $k_3$  respectively. Let us consider a hot fluid at a temperature  $T_a$  and heat transfer coefficient  $h_a$  inside the wall and cold fluid at a temperature  $T_b$  and heat transfer coefficient  $h_b$  outside the wall. Let  $T_1$ ,  $T_2$ ,  $T_3$  and  $T_4$  be the interface temperatures. It is desired to derive an equation for calculating the rate of heat flow through the series of resistances.

**Rate of heat flow from the hot fluid to the inner surface of the wall**

**By Newton's law of cooling**

$$Q = h_a A (T_a - T_1) \quad (1)$$

By rearranging eqn(1), we get



**Fig.4.Composite wall**

$$Q = \frac{(T_a - T_1)}{\frac{1}{h_a A}}$$

**Rate of the heat flow through the I layer**

**By Fourier's law of heat conduction**

$$Q = \frac{k_1 A (T_1 - T_2)}{L_1} \quad (2)$$

By rearranging eqn(2), we get

$$Q = \frac{(T_1 - T_2)}{\frac{L_1}{k_1 A}}$$

**Rate of the heat flow through the II layer**

**By Fourier's law of heat conduction**

$$Q = \frac{k_2 A (T_2 - T_3)}{L_2} \quad (3)$$

By rearranging eqn(3),we get

$$Q = \frac{(T_2 - T_3)}{\frac{L_2}{k_2 A}}$$

**Rate of the heat flow through the III layer**

**By Fourier's law of heat conduction**

$$Q = \frac{k_3 A (T_3 - T_4)}{L_3} \quad (4)$$

By rearranging eqn(4),we get

$$Q = \frac{(T_3 - T_4)}{\frac{L_3}{k_3 A}}$$

**Rate of heat flow from the outer surface of the wall to the cold fluid**

**By Newton's law of cooling**

$$Q = h_b A (T_4 - T_b) \quad (5)$$

By rearranging eqn(5),we get

$$Q = \frac{(T_4 - T_b)}{\frac{1}{h_b A}}$$

$$\text{Overall rate of heat flow} = \frac{\text{overall temperature drop}}{\text{overall thermal resistance}}$$

**Overall rate of heat flow**

$$Q = \frac{(T_a - T_b)}{\frac{1}{h_a A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A} + \frac{1}{h_b A}} \quad (6)$$

## (ii) Heat Transfer Through Coaxial Cylinders

Let us consider coaxial cylinders constructed of a series of 3 layers as shown in fig.5. Let  $r_1$ ,  $r_2$ ,  $r_3$  and  $r_4$  be the radii of the cylinders and the average thermal conductivities of the materials of which the layers are made be  $k_1$ ,  $k_2$ ,  $k_3$  respectively. Let us consider a hot fluid at a temperature  $T_a$  and heat transfer coefficient  $h_a$  inside the cylinder and cold fluid at a temperature  $T_b$  and heat transfer coefficient  $h_b$  outside the cylinder. Let  $T_1$ ,  $T_2$ ,  $T_3$  and  $T_4$  be the interface temperatures. It is desired to derive an equation for calculating the rate of heat flow through the series of resistances.

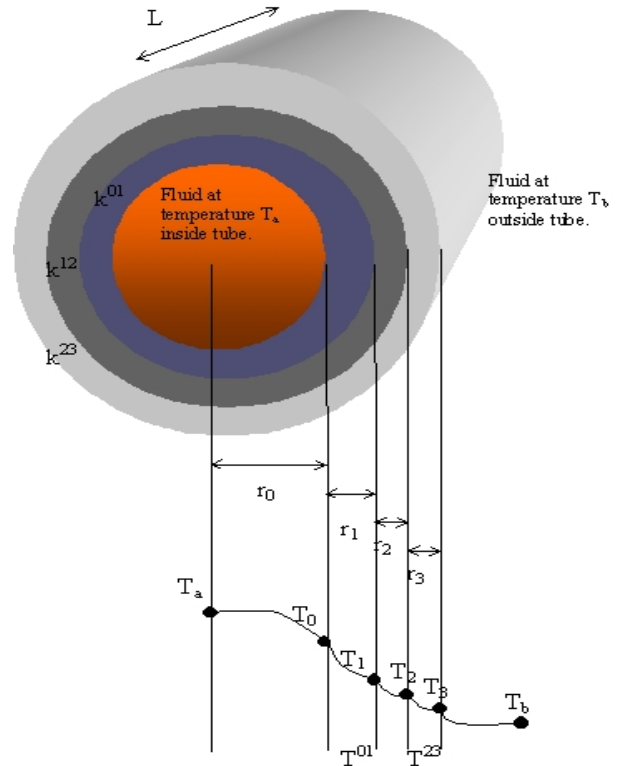


Fig.5.Coaxial cylinders

**Rate of heat flow from the hot fluid to the inner surface of the wall**

**By Newton's law of cooling**

$$Q = h_a A (T_a - T_1) \quad (1)$$

$$Q = h_a 2\pi r_1 L (T_a - T_1)$$

By rearranging eqn(1), we get

$$Q = \frac{(T_a - T_1)}{\frac{1}{h_a 2\pi r_1 L}}$$

**Rate of the heat flow through the I layer**

**By Fourier's law of heat conduction**

$$Q = \frac{2\pi k_1 L (T_1 - T_2)}{\ln\left(\frac{r_2}{r_1}\right)} \quad (2)$$

By rearranging eqn(2), we get

$$Q = \frac{(T_1 - T_2)}{\frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k_1 L}}$$

**Rate of the heat flow through the II layer**

**By Fourier's law of heat conduction**

$$Q = \frac{2\pi k_2 L (T_2 - T_3)}{\ln\left(\frac{r_3}{r_2}\right)} \quad (3)$$

By rearranging eqn(3),we get

$$Q = \frac{(T_2 - T_3)}{\frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi k_2 L}}$$

**Rate of the heat flow through the III layer**

**By Fourier's law of heat conduction**

$$Q = \frac{2\pi k_3 L (T_3 - T_4)}{\ln\left(\frac{r_4}{r_3}\right)} \quad (4)$$

By rearranging eqn(4),we get

$$Q = \frac{(T_3 - T_4)}{\frac{\ln\left(\frac{r_4}{r_3}\right)}{2\pi k_3 L}}$$

**Rate of heat flow from the outer surface of the wall to the cold fluid**

**By Newton's law of cooling**

$$Q = h_b A (T_4 - T_b) \quad (5)$$

$$Q = h_b 2\pi r_4 L (T_4 - T_b)$$

By rearranging eqn(5),we get

$$Q = \frac{(T_4 - T_b)}{\frac{1}{h_b 2\pi r_4 L}}$$

$$\text{Overall rate of heat flow} = \frac{\text{overall temperature drop}}{\text{overall thermal resistance}}$$

### Overall rate of heat flow

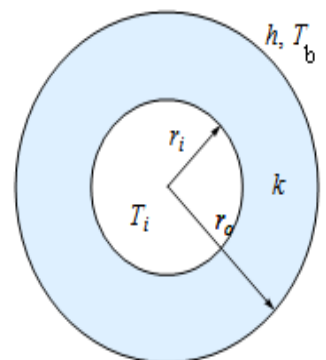
$$Q = \frac{2\pi L (T_a - T_b)}{\frac{1}{h_a r_1} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{k_1} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{k_2} + \frac{\ln\left(\frac{r_4}{r_3}\right)}{k_3} + \frac{1}{h_b r_4}} \quad (6)$$

## 1.8. Insulation

The addition of insulation material on a surface reduces the amount of heat flow to the ambient. There are certain instances in which the addition of insulation to the outside surface of cylindrical or spherical walls does not reduce the heat loss. Under certain circumstances it actually increases the heat loss up to a certain thickness of insulation. It is a well known fact that the rate of heat transfer will approach zero if an infinite amount of insulation is added. This means that there must be a value of radius for which rate of heat transfer is maximum. This value is known as the critical radius of insulation,  $r_c$ .

### 1.8.1. CRITICAL RADIUS OF INSULATION IN PIPES

Let us consider an insulating layer in the form of a hollow cylinder of length  $L$ . Let  $r_i$  and  $r_o$  be the inner and outer radii of insulation. The thermal conductivity of the material of which the layer is made be  $k$ . Let the inside surface of insulation be at a temperature  $T_i$ , and the outside surface at a temperature  $T_o$  be dissipating heat by convection to the surroundings at a temperature  $T_b$  with a heat transfer coefficient  $h$ . Then the rate of heat transfer  $Q$  through this insulation layer is



**Fig.6. Insulation layer**

$$Q = \frac{2\pi L (T_i - T_b)}{\frac{\ln\left(\frac{r_o}{r_i}\right)}{k} + \frac{1}{hr_o}} \quad (1)$$

The value of critical radius  $r_c$ , that is  $r_o$  for which  $Q$  is a maximum may be obtained by equating  $dQ/dr_o$  to zero.

$$\frac{dQ}{dr_o} = \frac{0 - (T_i - T_b) \left[ \frac{1}{2\pi k L r_o} - \frac{1}{2\pi h L r_o^2} \right]}{\left[ \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi k L} + \frac{1}{2\pi h L r_o} \right]^2} \quad (2)$$

$(T_i - T_b) \neq 0$  (Since it is the driving force)

$$\therefore \frac{1}{2\pi k L r_o} - \frac{1}{2\pi h L r_o^2} = 0 \quad (3)$$

$$r_o = \frac{k}{h} = r_c$$

The radius at which the rate of heat transfer is maximum is known as the critical radius of insulation.

### 1.9.VARIABLE THERMAL CONDUCTIVITY

Let us a hollow cylinder. The inside radius of the cylinder is  $r_i$ , the outside radius is  $r_o$ , and the length of the cylinder is  $L$ . The thermal conductivity of the material of which the cylinder is varies with temperature as  $k = k_o (1 + \beta T)$ . The temperature of the outside surface is  $T_o$ , and that of the inside surface is  $T_i$ .

By Fourier's law of heat conduction

$$Q = -kA \frac{dT}{dr} \quad (1)$$

$$Q = -k_o (1 + \beta T) A \frac{dT}{dr}$$

$$Q \int_{r_i}^{r_o} \frac{dr}{r} = -k_o 2\pi L \int_{T_i}^{T_o} (1 + \beta T) dT \quad (2)$$

$$Q \ln \frac{r_o}{r_i} = k_o 2\pi L \left( 1 + \beta \frac{[T_i + T_o]}{2} \right) (T_i - T_o)$$

$$Q = \frac{k_o 2\pi L \left( 1 + \beta \frac{[T_i + T_o]}{2} \right) (T_i - T_o)}{\ln \frac{r_o}{r_i}} \quad (3)$$

### 1.10. Introduction to Unsteady state heat transfer

A solid body is said to be in a steady state if its temperature does not vary with time. If however there is an abrupt change in its surface temperature or environment it takes some time before the body to attain an equilibrium temperature or steady state. During this interim period the temperature varies with time and the body is said to be in an unsteady or transient state.

The analysis of unsteady state heat transfer is of great interest to engineers because of its widespread occurrence such as in boiler tubes, rocket nozzles, automobile engines, cooling of IC engines, cooling and freezing of food, heat treatment of metals by quenching, etc. For practical purposes it is necessary to know the time taken to attain a certain temperature when the environment suddenly changes. The solution of an unsteady state problem will be more complex than that of steady state one because of the presence of another variable time,  $t$ .

Transient heat conduction problems can be divided into periodic heat flow and non periodic heat flow problems. Periodic heat flow problems are those in which the temperature varies on a regular basis, eg., the variation of temperature of the surface of the earth during a twenty four hour period.. In the non periodic type, the temperature at any point within the system varies non linearly with time.

#### **1.10.1. Systems with negligible internal resistance – Lumped Heat Analysis**

Heat transfer in heating and cooling of a body is dependent upon both the internal and surface resistances. The simplest unsteady state problem is one in which the internal resistance is negligible, that is, the convective resistance at the surface boundary is very large when compared to the internal resistance due to conduction. In other words, the solid has an infinite thermal conductivity so that there is no variation of temperature inside the solid and temperature is a function of time only. This situation cannot exist in reality because all the solids have a finite thermal conductivity and there will always be a temperature gradient inside whenever heat is added or removed. Problems such as heat treatment of metals by quenching, time response of thermocouples and thermometers, etc can be analysed by this idealization of negligible internal resistance. The process in which the internal resistance is ignored being negligible in comparison with its surface resistance is called the Newtonian heating and cooling process. In Newtonian heating and cooling process the temperature throughout the solid is considered to be uniform at a given time. Such an analysis is called the lumped heat capacity analysis.

#### **1.10.2. Systems with negligible surface resistance**

Another class of transient problems met with in practice is one in which the surface resistance is negligible compared to the overall resistance. This amounts to saying that the convective heat transfer coefficient at the surface is infinity. For such a process the surface temperature remains constant for all the time and its value is equal to that of ambient temperature.

**P.No. 1.** A furnace wall consists of two layers, 22.5cm of fire brick( $k=1.2\text{kcal/hr m }^{\circ}\text{C}$ ) and 12.5cm of insulating brick ( $k=0.15\text{kcal/hr m }^{\circ}\text{C}$ ) . The temperature inside the furnace is  $1650^{\circ}\text{C}$  and the inside heat transfer coefficient is  $60\text{kcal/hr m }^{\circ}\text{C}$  . The temperature of the surrounding atmosphere is  $27^{\circ}\text{C}$  and the outside heat transfer coefficient is  $10\text{kcal/hr m}^2^{\circ}\text{C}$  . Determine the rate of heat of loss per square meter of the wall.

**Solution:**

$$L_1 - 22.5 \times 10^{-2} \text{ m}$$

$$L_2 - 12.5 \times 10^{-2} \text{ m}$$

$$k_1 - 1.2 \text{ kcal/hr m }^{\circ}\text{C}$$

$$k_2 - 0.15 \text{ kcal/hr m }^{\circ}\text{C}$$

$$h_a - 60 \text{ kcal/hr m}^2^{\circ}\text{C}$$

$$h_b - 10\text{kcal/hr m}^2^{\circ}\text{C}$$

$$T_a - 1650^{\circ}\text{C}$$

$$T_b - 27^{\circ}\text{C}$$

$$Q = \frac{(T_a - T_b)}{\frac{1}{h_a A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{1}{h_b A}}$$

$$Q / A = 1426.8 \text{ W / m}^2$$

**P.No.2.** A pipe carrying steam at  $220^{\circ}\text{C}$  has an I.D. of 15cm. The convection coefficient on the inside wall is  $60\text{W/m}^2\text{K}$  . The pipe wall thickness is 15mm and the thermal conductivity is  $35\text{W/mK}$  . The outside is exposed to a chemical at  $130^{\circ}\text{C}$  with a convection coefficient of  $15\text{W/m}^2\text{K}$  . If the pipe wall is covered with two insulation layers, the first 3cm thickness with  $k=0.12\text{W/mK}$  and the second 4cm thickness with  $k=0.35\text{W/mK}$  . Determine the rate of heat transfer.

**Solution :**

$$r_1 - 75 \times 10^{-3} \text{ m}$$

$$r_2 - 90 \times 10^{-3} \text{ m}$$

$$r_3 - 120 \times 10^{-3} \text{ m}$$

$$r_4 - 160 \times 10^{-3} \text{ m}$$

$$k_1 - 35 \text{ W / m K}$$

$$k_2 = 0.12 \text{ W / m K}$$

$$k_3 = 0.35 \text{ W / m K}$$

$$h_a = 60 \text{ W / m}^2 \text{ K}$$

$$h_b = 15 \text{ W / m}^2 \text{ K}$$

$$Q = \frac{2\pi L (T_a - T_b)}{\frac{1}{h_a r_1} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{k_1} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{k_2} + \frac{\ln\left(\frac{r_4}{r_3}\right)}{k_3} + \frac{1}{h_b r_4}}$$

$$Q = 146.32 \text{ W}$$

## ASSIGNMENT:

### Composite Wall

1. A composite wall consists of a 17cm thick firebrick layer ( $k = 1.1 \text{ W/m } ^\circ\text{C}$ ) and a 13cm thick ordinary brick layer ( $k = 0.70 \text{ W/m } ^\circ\text{C}$ ). The inside and outside surface temperatures of the walls are  $400^\circ\text{C}$  and  $45^\circ\text{C}$  respectively. Calculate the heat loss per unit area of the wall. Also calculate the temperature between the ordinary brick and the firebrick layers.
2. A steam boiler furnace is made of a layer of fireclay ( $k = 0.533 \text{ W/m K}$ ) 12.5cm thick and a layer of red brick ( $k = 0.7 \text{ W/mK}$ ) 50cm thick. If the wall temperature inside the boiler furnace is  $1100^\circ\text{C}$  and that on the outside wall is  $50^\circ\text{C}$ , determine the amount of heat loss per square meter of the furnace wall.
3. The wall of a cold storage consists of three layers, an outer layer of ordinary brick of 25cm thick, a middle layer of cork, 10cm thick, and inner layer of cement, 6cm thick. The thermal conductivities of the materials are:  $k_{\text{brick}} = 0.7 \text{ W/m } ^\circ\text{C}$ ,  $k_{\text{cork}} = 0.043 \text{ W/m } ^\circ\text{C}$ ,  $k_{\text{cement}} = 0.72 \text{ W/m } ^\circ\text{C}$ . The temperature of the outer surface of the wall is  $30^\circ\text{C}$ , and the inner is  $-15^\circ\text{C}$ . Calculate rate of heat transfer per unit area of the wall and interface temperatures.
4. A furnace wall consists of 23cm of refractory fire clay brick ( $k=1\text{W/m } ^\circ\text{C}$ ) 11.5 cm of the silica brick ( $k=0.188\text{W/m } ^\circ\text{C}$ ) and 6mm of iron plate ( $k=45\text{W/m } ^\circ\text{C}$ ). The fire side of the refractory is at  $1150^\circ\text{C}$  and outside surface of steel is  $32^\circ\text{C}$ . Determine the heat loss.
5. A furnace wall is made of inside silica brick ( $k=1.858\text{W/m } ^\circ\text{C}$ ) and outside magnetite brick ( $k=5.8\text{W/m } ^\circ\text{C}$ ). The thickness of the silica brick is 12cm and that of magnetite brick is 20cm. The temperature of silica brick surface inside the furnace is  $300^\circ\text{C}$  and at the outside surface of magnetite is  $130^\circ\text{C}$ . Find the heat loss per square meter of the furnace wall.
6. The composite wall of a furnace consists of an inner layer of silica brick, 15cm thick ( $k=1.04 \text{ W/m } ^\circ\text{C}$ ), and an outer layer of insulating brick, 20cm thick ( $k=0.2 \text{ W/m } ^\circ\text{C}$ ). The inside temperature of the furnace is  $800^\circ\text{C}$  and the interface temperature is  $705^\circ\text{C}$ . Calculate the rate of heat loss per unit area of the furnace wall.
7. A wall of 0.5m thickness is to be constructed from a material which has an average thermal conductivity of  $1.4\text{W/mK}$ . The wall is to be insulated with a material having an average thermal conducting of  $0.35\text{W/mK}$ , so that the heat loss per square meter will not exceed  $1450\text{W}$ . Assuming that the inner and outer surface temperatures are  $1200^\circ\text{C}$  and  $15^\circ\text{C}$  respectively, calculate the thickness of insulation required.
8. A wall 2cm thick is to be constructed from a material which has an average thermal conductivity of  $1.3 \text{ W/m } ^\circ\text{C}$ . The wall is to be insulated with a material having an average thermal conductivity of  $0.35\text{W/m } ^\circ\text{C}$ , so that the heat loss per square meter will not exceed  $1830\text{W}$ . Assuming that the inner and outer surface temperatures of the insulated wall are  $1300$  and  $30^\circ\text{C}$ , calculate the thickness of the insulation required.
9. A composite wall is made of two layers of 0.30m and 0.15m thickness with surfaces held at  $600^\circ\text{C}$  and  $20^\circ\text{C}$  respectively. If the conductivities are 20 and  $50\text{W/mK}$  determine the heat conducted. In order to restrict the heat loss to  $5\text{kW/m}^2$  another layer of 0.15m thickness. Determine the thermal conductivity required.

10. A furnace wall consists of 200mm of refractory fireclay brick, 100mm of kaolin brick, and 6 mm of steel plate. The fire side of the refractory is at  $1150^{\circ}\text{C}$ , and the outside of the steel is at  $30^{\circ}\text{C}$ . (a) Calculate the heat loss per square meter cross section of the wall. (b) Determine the interface temperatures.  $k_{\text{fireclay}} = 1.520 \text{ W/m}^{\circ}\text{C}$ ;  $k_{\text{kaolin brick}} = 0.138 \text{ W/m}^{\circ}\text{C}$ ;  $k_{\text{steel}} = 45 \text{ W/m}^{\circ}\text{C}$ .
11. An exterior wall of a house may be approximated by a 10cm layer of common brick ( $k=0.7\text{W/m}^{\circ}\text{C}$ ) followed by a 3.75cm layer of gypsum plaster ( $k=0.48\text{W/m}^{\circ}\text{C}$ ). What thickness of loosely packed rock-wool insulation ( $k=0.065\text{W/m}^{\circ}\text{C}$ ) should be added to reduce the heat loss through the wall by 80%?
12. The door of a cold storage plant is made from two 6mm thick glass sheets ( $k = 0.75\text{W/mK}$ ) separated by a uniform air gap ( $k = 0.02\text{W/mK}$ ) of 2mm. The temperature of the air inside the room is  $-20^{\circ}\text{C}$  and the ambient air temperature is  $30^{\circ}\text{C}$ . Assuming the heat transfer coefficient between glass and air to be  $23.26 \text{ W/m}^2\text{K}$ , determine the rate of heat loss into the room per unit area of the door. Neglect convection effect in the air gap.

### **Coaxial cylinders**

1. A thickwalled tube of stainless steel ( $k= 19 \text{ W/m}^{\circ}\text{C}$ ) with 2cm ID and 4cm OD is covered with a 3cm layer of asbestos insulation ( $k= 0.2 \text{ W/m}^{\circ}\text{C}$ ). If the inside wall temperature of the pipe is maintained at  $600^{\circ}\text{C}$  and outside wall temperature is maintained at  $100^{\circ}\text{C}$ , calculate the heat loss per meter length. Also calculate the tube insulation interface temperature.
2. A hot steam pipe having an inside surface temperature of  $250^{\circ}\text{C}$  has an inside diameter of 8cm and a wall thickness of 5.5mm. It is covered with a 9cm layer of insulation having  $k=0.5 \text{ W/m}^{\circ}\text{C}$ , followed by a 4cm layer of insulation having  $k = 0.25 \text{ W/m}^{\circ}\text{C}$ . The outside temperature of insulation is  $20^{\circ}\text{C}$ . Calculate the heat lost per meter of length. Assume  $k=47 \text{ W/m}^{\circ}\text{C}$  for the pipe.
3. A cylindrical hot gas duct, 0.5m inside radius, has an inner layer of fireclay brick ( $k = 1.3 \text{ W/m}^{\circ}\text{C}$ ) of 0.27m thickness. The outer layer, 0.14m thick is made of a special brick ( $k=0.92 \text{ W/m}^{\circ}\text{C}$ ). The brickwork is enclosed by an outer steel cover which has a temperature of  $65^{\circ}\text{C}$ . The inside temperature of the composite cylindrical wall of the duct is  $400^{\circ}\text{C}$ . Neglecting the thermal resistance of the steel cover, calculate the rate of heat loss per meter of the duct and also the interface temperature between the ceramic layers.
4. A 10cm O.D steam pipe carrying saturated steam at temperature  $195^{\circ}\text{C}$  is lagged to 20cm diameter with magnesia ( $k= 0.07 \text{ W/mK}$ ) and further lagged with laminated asbestos ( $k=0.08 \text{ W/mK}$ ) to 25cm diameter. The whole pipe is further protected by a layer of canvas. If the temperature under the canvas is  $20^{\circ}\text{C}$ , calculate the rate of heat loss on 150m length of pipe.
5. A pipe of I.D 15.4cm and O.D 16.8cm carries saturated steam at temperature  $190^{\circ}\text{C}$ . The thermal conductivity of the pipe wall is  $51 \text{ W/mK}$ . The pipe is insulated with a 10cm thick fibre glass blanket ( $k=0.072 \text{ W/mK}$ ). If the outer surface temperature is  $41^{\circ}\text{C}$ , calculate the rate of heat loss over a 10 m section of the pipe.
6. A steel pipe 33.4mm outer diameter, 3.38mm wall thickness carries saturated steam at  $121^{\circ}\text{C}$ . Pipe is insulated with 50mm layer of magnesia pipe covering and outside this magnesia is 75 mm layer of cork. Inside temperature of cork is at  $32^{\circ}\text{C}$ . Calculate the heat loss per meter length of the pipe and the temperature at

the boundaries between metal and magnesia and between magnesia and cork.  
Data: steel  $k=45\text{W/m}^\circ\text{C}$  ; Mg,  $k=0.0588\text{W/m}^\circ\text{C}$  , Cork,  $k=0.0519\text{W/m}^\circ\text{C}$  .

7. A multilayer cylindrical wall of a furnace is constructed of 4.5cm layer of insulating brick with thermal conductivity of  $0.081\text{W/mK}$  followed by a 9cm layer of common brick with thermal conductivity of  $0.0812\text{W/mK}$  . The inner wall temperature is  $2500^\circ\text{C}$  and outer wall temperature is  $70^\circ\text{C}$  .What is the heat loss through the wall, when the inner diameter of the furnace is 1.2m and the length of the wall is 1m?
8. A steel pipe (I.D. 4.14cm and O.D. 4.74cm) carries steam at  $450^\circ\text{C}$ . The steel pipe is covered with a 2.5cm layer of an insulating material ( $k=0.09\text{kcal/hr m}^\circ\text{C}$ ) .This is covered with a 5cm layer of another insulating material ( $k=0.06\text{kcal/hr m}^\circ\text{C}$ ) . If the temperature of the outermost insulation layer is  $60^\circ\text{C}$ , calculate the heat loss in kcal/hr per meter length of pipe and the layer contact temperatures. Neglect resistance of the steam film and assume  $k$  for the steel pipe as  $36\text{kcal/hr m}^\circ\text{C}$ .
9. A steel pipe line ( $k=50\text{W/mK}$ ) of 100mm and O.D.110mm is to be covered with two layers of insulation each having a thickness of 50mm .The thermal conductivity of the first insulation material is  $0.06\text{W/mK}$  , and that of the second is  $0.12\text{W/mK}$  . Calculate the loss of heat per meter length of pipe and the interface temperature between the two layers of insulation when the temperature of the inside tube surface is  $250^\circ\text{C}$  and that of the outside surface of the insulation is  $50^\circ\text{C}$ .

### **Combined Heat Transfer**

1. A furnace wall consists of two layers, 22.5cm of fire brick ( $k=1.2\text{kcal/hr m}^\circ\text{C}$ ) and 12.5cm of insulating brick ( $k=0.15\text{kcal/hr m}^\circ\text{C}$ ) . The temperature inside the furnace is  $1650^\circ\text{C}$  and the inside heat transfer coefficient is  $60\text{Kcal/hr m}^{2^\circ}\text{C}$ . The temperature of the surrounding atmosphere is  $27^\circ\text{C}$  and the outside heat transfer coefficient is  $10\text{kcal/hr m}^{2^\circ}\text{C}$  .Neglecting the thermal resistance of the mortar joints determine the rate of heat of loss per square meter of the wall.
2. The inner dimensions of a freezer cabinet are 60cm x 60cm x 50cm(height). The cabinet walls consist of two 2mm thick enameled sheet steel ( $k=40\text{W/mK}$ ) walls separated by a 4cm layer of fiberglass( $k=0.049\text{W/mK}$ ) insulation. The inside temperature is to be maintained at  $-15^\circ\text{C}$  and the outside temperature on a hot summer day is  $45^\circ\text{C}$ .Calculate the rate of heat transfer assuming heat transfer coefficient of  $10\text{W/m}^2\text{K}$  both on the inside and outside of the cabinet. Also calculate the outer surface temperature of the cabinet.
3. A steel tube having  $k=46\text{W/m}^\circ\text{C}$  has an inside diameter of 3cm and wall thickness of 2mm. A fluid flows on the inside of the tube producing a convection coefficient of  $1500\text{W/m}^2\text{C}$  on the inside surface, while a second fluid flows across the outside of the tube producing a convection coefficient of  $197\text{W/m}^2\text{C}$  on the outside tube surface. The inside fluid temperature is  $223^\circ\text{C}$  while the outside fluid temperature is  $57^\circ\text{C}$ . Calculate the heat lost by the tube per meter of length.
4. A steam pipe is covered with two layers of insulation, the first layer being 3cm thick and second 5cm. The pipe is made of steel( $k=58\text{W/mK}$ ) having an I.D of 160mm and O.D of 170mm. The inside and outside film coefficients are 30 and  $5.8\text{W/m}^2\text{K}$  respectively. Calculate the heat lost per metre of pipe if the steam temperature is  $300^\circ\text{C}$  and the air temperature is  $50^\circ\text{C}$ . The thermal conductivity of the two insulating materials are 0.17 and  $0.093\text{W/mK}$  respectively.
5. A steel tube ( $k=43.26\text{W/mK}$ ) of 5.08cm ID and 7.62 OD is covered with a 2.54cm layer of asbestos insulation( $k=0.208\text{W/mK}$ ). The inside surface of the tube receives heat by convection from a hot gas at a temperature of  $316^\circ\text{C}$  with the heat transfer coefficient  $284\text{W/m}^2\text{K}$ ,while the outer surface of the insulation is

exposed to the ambient air at  $38^{\circ}\text{C}$  with the heat transfer coefficient of  $17\text{ W/m}^2\text{K}$ . Calculate the loss of heat to ambient air for 3m length of the tube and also the interface temperatures.

6. A multilayer cylindrical wall of a furnace is constructed of 4.5cm layer of insulating brick with thermal conductivity of  $0.081\text{ W/mK}$  followed by a 9cm layer of common brick with thermal conductivity of  $0.0812\text{ W/m K}$ . The inner wall temperature is  $2500^{\circ}\text{C}$  and outer wall temperature is  $70^{\circ}\text{C}$ . (a) Calculate the heat loss through the wall, when the inner diameter of the furnace is 1.2m and the length of the wall is 1m. (b) Determine the interface temperatures.
7. A steel pipe having an I.D. 52.50mm and an O.D. 60.33mm and  $k=39.7\text{ kcal/hr m}^{\circ}\text{C}$  carries steam at  $150^{\circ}\text{C}$ . It is lagged with 12.7mm thick rock wool of thermal conductivity  $0.049\text{ kcal/hr m}^{\circ}\text{C}$  and the surrounding air is at  $20^{\circ}\text{C}$ . If the heat transfer coefficient from the insulated pipe to the surrounding air is  $6\text{ kcal/hr m}^2\text{C}$ , what will be the (a) heat loss per meter length of pipe (b) temperature at the boundaries between the pipe wall and rock wool and (c) between the rock wool and surrounding air.
8. A pipe carrying steam at  $230^{\circ}\text{C}$  has an internal diameter of 12cm and the pipe thickness is 7.5 mm. The conductivity of the pipe material is  $49\text{ W/m K}$ . The convective heat transfer coefficient on the inside is  $85\text{ W/m}^2\text{ K}$ . The pipe is insulated by one layer of insulation of 5 cm thickness having conductivity of  $0.15\text{ W/m K}$ . The outside is exposed to air at  $25^{\circ}\text{C}$  with a convection coefficient of  $18\text{ W/m}^2\text{K}$ . Determine the heat loss for 5m length and the interface temperatures.
9. A 5cm diameter steel pipe is covered with a 1cm layer of insulating material having  $k = 0.22\text{ W/m}^{\circ}\text{C}$  followed by a 3 cm thick layer of another insulating material having  $k = 0.06\text{ W/m}^{\circ}\text{C}$ . The entire assembly is exposed to a convection surrounding condition of  $h = 60\text{ W/m}^2\text{C}$  and  $T = 15^{\circ}\text{C}$ . The outside surface temperature of steel is  $400^{\circ}\text{C}$ . calculate the heat lost by the pipe insulation assembly for a pipe length of 20m.
9. A steel pipe ( $k= 44\text{ W/mK}$ ) of 5.08cm I.D and 7.62cm O.D is covered with a 2.54cm layer of asbestos insulation ( $k=0.208\text{ W/mK}$ ). The inside surface of the pipe receives heat from the hot gas at a temperature of  $316^{\circ}\text{C}$  with the heat transfer coefficient  $284\text{ W/m}^2\text{K}$ , while the outer surface of the insulation is exposed to the ambient air at  $38^{\circ}\text{C}$  with the heat transfer coefficient of  $17\text{ W/m}^2\text{K}$ . Calculate the heat loss to ambient air for 3m length of the pipe and also calculate the interface temperatures.
10. A pipe carrying steam at  $220^{\circ}\text{C}$  has an I.D. of 15cm. The convection coefficient on the inside wall is  $60\text{ W/m}^2\text{K}$ . The pipe wall thickness is 15mm and the thermal conductivity is  $35\text{ W/mK}$ . The outside is exposed to a chemical at  $130^{\circ}\text{C}$  with a convection coefficient of  $15\text{ W/m}^2\text{K}$ . If the pipe wall is covered with insulation layers, the first 3cm thickness with  $k=0.12\text{ W/mK}$  and the second 4cm thickness with  $k= 0.35\text{ W/mK}$ , determine the rate of heat transfer and interface temperature
11. A pipe carrying steam at  $230^{\circ}\text{C}$  has an internal diameter of 12cm and the pipe thickness is 7.5 mm. The conductivity of the pipe material is  $49\text{ W/m K}$ . The convective heat transfer coefficient on the inside is  $85\text{ W/m}^2\text{ K}$ . The pipe is insulated by one layer of insulation of 5 cm thickness having conductivity of  $0.15\text{ W/m K}$ . The outside is exposed to air at  $25^{\circ}\text{C}$  with a convection coefficient of  $18\text{ W/m}^2\text{K}$ . Determine the heat loss for 5m length and the interface temperatures.

### **Critical radius of insulation**

1. A tube of O.D. 2.5 cm is to be insulated with a layer of asbestos of thermal conductivity  $k = 0.2 \text{ W/m}^2\text{°C}$ . The conduction heat transfer coefficient from the surface of the asbestos to the ambient air is  $h_a = 12 \text{ W/m}^2\text{°C}$ . Calculate the critical radius of insulation.
2. A steam pipe 10 cm I.D. and 11 cm O.D. is covered with an insulating substance ( $k = 1 \text{ W/mK}$ ). The steam temperature and the ambient temperature are  $200^\circ\text{C}$  and  $20^\circ\text{C}$  respectively. If the convective heat transfer coefficient between the insulation surface and air is  $8 \text{ W/m}^2\text{K}$ , find the critical radius of insulation. For this value, calculate the heat loss per meter length of pipe and the outer surface temp. Neglect the resistance of the pipe material.
3. Calculate the critical radius of insulation for asbestos ( $k = 0.17 \text{ W/m}^2\text{°C}$ ) surrounding a pipe and exposed to room air at  $20^\circ\text{C}$  with  $h = 3.0 \text{ W/m}^2\text{°C}$ . Calculate the heat loss from a  $200^\circ\text{C}$ , 50 mm dia pipe covered with the critical radius of insulation and without insulation.
4. Evaluate the thickness of rubber insulation necessary in the case of a 10mm dia copper conductor to ensure max. heat transfer to the atmosphere, given the thermal conductivity of rubber as  $0.155 \text{ W/mK}$  and the surface coefficient as  $8.5 \text{ W/m}^2\text{K}$ . Estimate the max heat transfer rate per meter length of conductor if the temperature of rubber is not to exceed  $65^\circ\text{C}$  while the atmosphere is at  $30^\circ\text{C}$ .

### **Variable thermal conductivity**

1. A plane wall of fireclay brick of thickness 25cm is having temperatures  $1350^\circ\text{C}$  and  $50^\circ\text{C}$  on its two sides. The thermal conductivity of the fireclay brick is a function of temperature.  $k(\text{W/mK}) = 0.838 (1 + 0.0007T)$ . Calculate the rate of heat transfer.
2. A fire clay wall 20cm thick has its two surfaces maintained at  $1000^\circ\text{C}$  and  $200^\circ\text{C}$ . The thermal conductivity varies with temp. as  $k(\text{W/mK}) = 0.813 + 0.000582T$ . Calculate the rate of heat flow.
3. Calculate the heat loss per square metre of the surface area of a furnace wall 25cm thick. The inner and outer surface temperatures are  $400^\circ\text{C}$  and  $40^\circ\text{C}$  respectively. The variation of the thermal conductivity in  $\text{W/mK}$  with temperature in  $^\circ\text{C}$  is given by the following equation:  $K = 0.002T - 10^{-6}T^2$ .
4. Compute the heat loss per square meter surface area of a 40cm thick furnace wall having surface temp. of  $300^\circ\text{C}$  and  $50^\circ\text{C}$ , if the thermal conductivity  $k$  of the wall material is given by  $k = 0.005T - 5 \times 10^{-6} T^2$ ,  $T$  is in  $^\circ\text{C}$ .
5. The two faces of a slab at  $x = 0$  and  $x = L$  are kept at  $t_1$  and  $t_2^\circ\text{C}$  respectively. The 'k' of the material is given by as a temperature dependent value by  $k = k_0(t^2 - t_0^2)$  where  $t_0$  and  $k_0$  are constants. Deduce the expression for heat flow/unit area.
6. Calculate the rate of heat transfer in a slab of thickness 20 cm and area  $4.5 \text{ cm}^2$  when the two faces are maintained at  $200^\circ\text{C}$  and  $50^\circ\text{C}$ . The thermal conductivity,  $k$  can be expressed by the relation,  $k = 6.8 + 7.2 \times 10^{-3}T \text{ W/mK}$ .
7. Derive an expression for one dimensional heat transfer in a hollow cylinder. The thermal conductivity varies with temperature as  $k = k_0 (1 + \beta T)$
8. Derive an expression for one dimensional heat transfer in a hollow cylinder. The thermal conductivity varies with temperature as  $k = k_0 (1 + \alpha T + \beta T^2)$
9. A thick wall copper cylinder has an inside radius of 1cm and the outer radius of 2cm. The inner and outer surfaces are held at  $310^\circ\text{C}$  and  $290^\circ\text{C}$

respectively. Assume  $k$  varies with temperature as  $k(\text{W/mK}) = 317.9[1 - 9.25 \times 10^{-5}(T - 150)]$ . Determine the heat loss per unit length.

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