

# **EE1025 POWER SYSTEM ANALYSIS**

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## **UNIT I – POWER SYSTEM OVERVIEW (6 hours)**

**Power scenario in India, Power system components, Representation. Single line diagram, per unit quantities, p.u. impedance diagram, Network graph, Bus incidence matrix, Primitive parameters, Bus admittance matrix using singular method, Formation of bus admittance matrix of large power network, Representation of off nominal transformer**

## **UNIT II – POWER FLOW ANALYSIS (10 hours)**

**Bus classification, Formulation of Power Flow problems, Power flow solution using Gauss Seidel method, Handling of Voltage controlled buses, Power Flow Solution by Newton Raphson method, Fast Decoupled Power Flow Solution.**

## **UNIT III – SYMMETRICAL FAULT ANALYSIS (9 hours)**

**Symmetrical short circuit on Synchronous Machine, Bus Impedance matrix building algorithm, Symmetrical fault analysis through bus impedance matrix, Selection of circuit breaker, Fault level, Current limiting reactors.**

#### **UNIT IV– UNSYMMETRICAL FAULT ANALYSIS (10 hours)**

**Symmetrical components, Sequence impedance, Sequence networks, Analysis of unsymmetrical fault at generator terminals, Use of bus impedance matrix for analyzing unsymmetrical fault occurring at any point in a power system.**

#### **UNIT V– POWER SYSTEM STABILITY (10 hours)**

**Introduction to stability studies, Swing equation, Swing curve, Equal area criterion, Critical clearing angle and time, Modified Euler's method, Fourth order Runge Kutta method, Multi-machine transient stability.**

#### **TEXT BOOKS**

- 1. John.J.Grainger, William D. Stevenson, “Power System Analysis”, Tata Mc Graw Hill Publishing company, New Delhi, 2003.**
- 2. Nagarath I.J. and Kothari D.P. “Modern Power System Analysis”, Fourth Edition, Tata Mc Graw Hill Publishing company, New Delhi, 2011.**

## **REFERENCES**

- 1. Hadi Sadat, “Power System Analysis”, Tata Mc Graw Hill Publishing company, New Delhi, 2002.**
- 2. Pai M.A. “Computer Techniques in Power System Analysis”, Tata Mc Graw Hill Publishing Company, New Delhi, 2003.**
- 3. Abhijit Chakrabarti and Sunita Halder, “Power System Analysis Operation and Control”, PHI Learning Private Limited, New Delhi, 2011.**
- 4. Arthur R and Vijay Vittal, “Power Systems Analysis”, Dorling Kinderley (India) Private Limited, New Delhi, 2012. EE1025POWER SYSTEM ANALYSIS**

# **UNIT 1 POWER SYSTEM OVERVIEW**

## **POWER SCENARIO IN INDIA**

Today's power systems are very huge in terms of Installed capacity, Energy generated, Transmission and Distribution system, Number of customers and Total investment. Installed capacity in India exceeds 275 GW with annual energy generated energy exceeding 1138 Billion KWh ( $1138 \times 10^{12}$  KWh). The power system feeds a very large number of domestic, commercial, industrial, agriculture and other customers. Operation and control of such a big interconnected power system is really challenging task and it cannot be done manually. Therefore power systems are controlled by using powerful computers installed at Energy Control Centers.

### **Levels of power system operation**

The total power system in India is not being operated in totally integrated manner. However, attempts are being made to achieve full integration.

National Load Dispatching Center, having headquarters at New Delhi, is the coordinating agency.

**There are FIVE Regional Loading Dispatching Centers as described below.**

- 1. Northern region load dispatching center at New Delhi covering states of HP, J and K, Haryana, Punjab, Rajasthan, UP, Delhi, Uttaranchal and Chandigarh.**
- 2. Western region load dispatching center at Mumbai covering Gujarat, MP, Maharashtra, Chhattisgarh, Goa, Daman Diu, Dadra and Nagar Haveli.**
- 3. Southern region load dispatching center at Bangalore covering AP, Karnataka, Tamil Nadu, Kerala, Pondicherry and Lakshadweep.**
- 4. Eastern region load dispatching center at Kolkata covering WB, Bihar, Orissa, Sikkim and Jharkhand.**
- 5. North eastern load dispatching center at Shillong covering Assam, Manipur, Meghalaya, Nagaland, Tripura, Arunachal Pradesh and Mizoram.**

**The third level of consists of state load dispatching stations in each state capital.**



**Installed capacity in different regions are as follows:**

<b>Northern</b>	<b>73 GW</b>
<b>Western</b>	<b>100 GW</b>
<b>Southern</b>	<b>66 GW</b>
<b>Eastern</b>	<b>33 GW</b>
<b>North eastern</b>	<b>4 GW</b>

**ALL INDIA INSTALLED CAPACITY (IN MW) OF POWER STATIONS (As on July 2015)**

Region	Modewise breakup						Grand Total
	Coal	Gas	Diesel	Nuclear	Hydro	Renewable	
Northern Region	40943	5331	0	1620	17796	7511	73203
Western Region	67029	10815	0	1840	7447	13005	100137
Southern Region	30842	4963	917	2320	11398	15245	65685
Eastern Region	28582	190	0	0	4113	434	33320
North Eastern Region	310	1662	36	0	1242	262	3513
Islands	0	0	40	0	0	11	51
<b>ALL INDIA</b>	<b>167707</b>	<b>22962</b>	<b>993</b>	<b>5780</b>	<b>41997</b>	<b>36470</b>	<b>275912</b>

## **POWER SYSTEM COMPONENTS AND THEIR REPRESENTATION**

**Major components in power system that are to be modelled for different system problems are:**

- 1. Synchronous generators**
- 2. Transmission lines**
- 3. Transformers**
- 4. Loads**

## **Synchronous generator**

**Synchronous generator is a device which converts mechanical energy to electrical energy. The required mechanical energy is supplied by a prime mover which usually a steam or hydraulic turbine. The stator and rotor are the two principal parts of a synchronous generator. The stationary part which is essentially a hollow cylinder is called the stator or armature. It has longitudinal slots in which the coils of armature winding are placed. This winding carries the current supplied to an electrical load when it is functioning as an individual generator. The armature winding carries the current supplied to the system when it is synchronised to the grid. The rotor of the synchronous generator is mounted on the shaft and rotated inside the hollow stator. The winding on the rotor is called the field winding and is supplied with dc current.**

**The very high magnetomotive force (mmf) produced by the current in the field winding combines with the mmf produced by the current in the armature winding. The resultant flux across the air gap between the stator and the rotor generates voltage in the coils of the armature winding.**

The rotor flux  $\Phi_f$  is the only one to be considered when the armature current is zero. The flux  $\Phi_f$  generates the no-load voltage  $E_{a0}$  which we shall designate here as  $E_f$ . The flux  $\Phi_{ar}$  due to armature reaction mmf will be in phase with the current  $i_a$ . The sum of  $\Phi_f$  and  $\Phi_{ar}$ , is  $\Phi_r$ , the resultant flux which generates the voltage  $E_r$  in the coil winding composing phase  $a$ . The phasor diagram for phase  $a$  is shown in Fig. 1.1. Voltages  $E_f$  and  $E_{ar}$  lag the fluxes  $\Phi_f$  and  $\Phi_{ar}$  which generate them by  $90^\circ$ . The resultant flux  $\Phi_r$  is the flux across the air gap of the machine and generates  $E_r$  in the stator. Voltage  $E_r$  lags the flux  $\Phi_r$  by  $90^\circ$ . Similar phasor diagram can be drawn for phase  $b$  and phase  $c$  also.

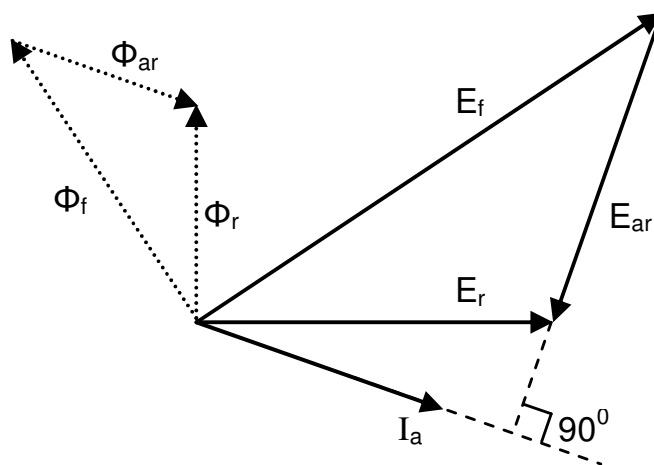
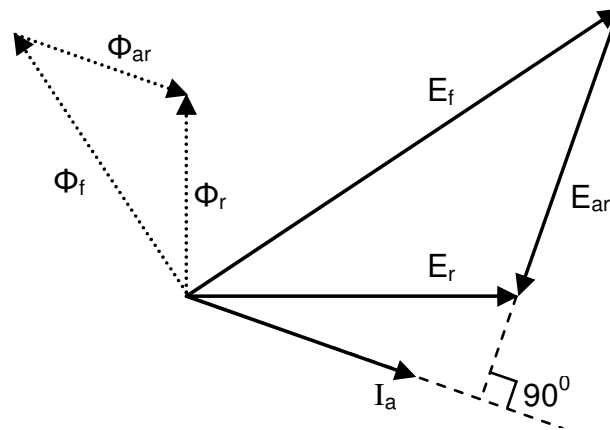


Fig. 1.1 Phasor diagram for phase  $a$  of a synchronous generator



**Fig. 1.1 Phasor diagram for phase *a* of a synchronous generator**

In Fig. 1.1 we note that  $E_{ar}$  is lagging  $I_a$  by  $90^\circ$ . The magnitude of  $E_{ar}$  is determined by  $\Phi_{ar}$  which in turn is proportional to  $|I_a|$  since it is the result of armature current. So we can specify an inductive reactance  $X_{ar}$  such that

$$E_{ar} = -j X_{ar} I_a \quad (1.1)$$

Equation (1.1) defines  $E_{ar}$  so that it has the proper phase angle with respect to  $I_a$ . Knowing that the voltage generated in phase *a* by the air gap flux is  $E_r$  we can write

$$E_r = E_f + E_{ar} = E_f - j X_{ar} I_a \quad (1.2)$$

The terminal voltage  $V_t$  will be less than  $E_r$  by the voltage drop due to armature current times the armature leakage reactance  $X_\ell$  if the armature resistance is neglected, Then

$$V_t = E_r - j X_\ell I_a \quad (1.3)$$

Use of Eq. (1.2) in the above results

$$V_t = E_f - j X_{ar} I_a - j X_\ell I_a \quad (1.4)$$

$$\text{Thus } V_t = E_f - j X_s I_a \quad (1.5)$$

where  $X_s$ , called the synchronous reactance, is equal to  $X_{ar} + X_\ell$ . If the resistance of the armature  $R_a$  is to be considered, Eq. (1.5) becomes

$$V_t = E_f - j X_s I_a - R_a I_a = E_f - (R_a + j X_s) I_a \quad (1.6)$$

Armature resistance  $R_a$  is usually so much smaller than  $X_s$  that its omission is acceptable.

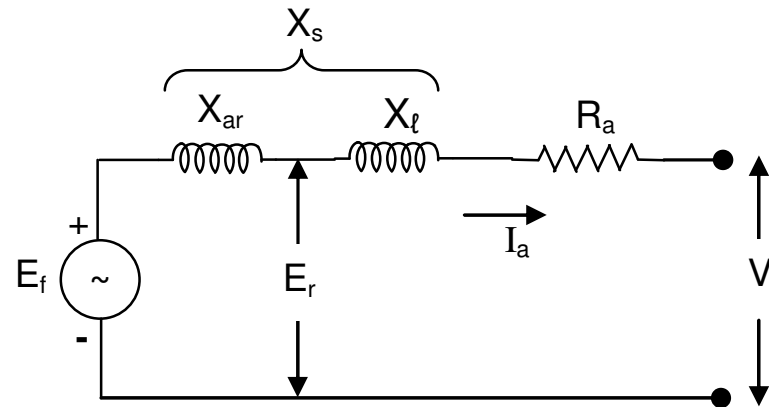


Fig. 1.2 Equivalent circuit of an ac generator

Now we have arrived at a relationship which allows us to represent the ac generator by the simple but very useful equivalent circuit shown in Fig. 1.2 which corresponds to Eq. (1.6).

Generally modelling of components depends on the study. The model of ac generator discussed is used for steady state analysis.

While performing short circuit analysis for transient and sub-transient period, the steady state reactance of the generator will be replaced by transient reactance or sub-transient reactance of the ac generator.

While conducting power flow analysis, generators are modelled as equivalent complex power injection.



## Modelling of transmission lines

Transmission lines are represented by a two-port  $\pi$ -model whose parameters correspond to the positive sequence equivalent circuit of the transmission lines. A transmission line with a series impedance of  $R + j X$  and total line charging susceptance of  $j B$ , will be modelled by the equivalent circuit shown in Figure 1.3.

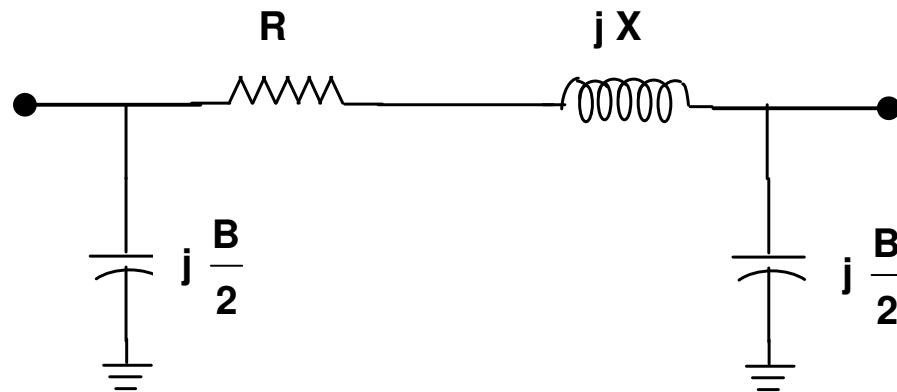


Fig. 1.3 Equivalent circuit for a transmission line

## Modelling of transformer

Transformer is a static device used to step-up or step-down the voltage level. Power transformers and distribution transformers are used power system network.

While arriving the equivalent circuit of a transformer if we refer all quantities to either the primary or the secondary side of the transformer. For instance, if we refer all voltages, currents and impedances to the primary side, the equivalent circuit will be as shown in Fig. 1.4 where  $a$  is the ratio of primary rated voltage to secondary rated voltage.

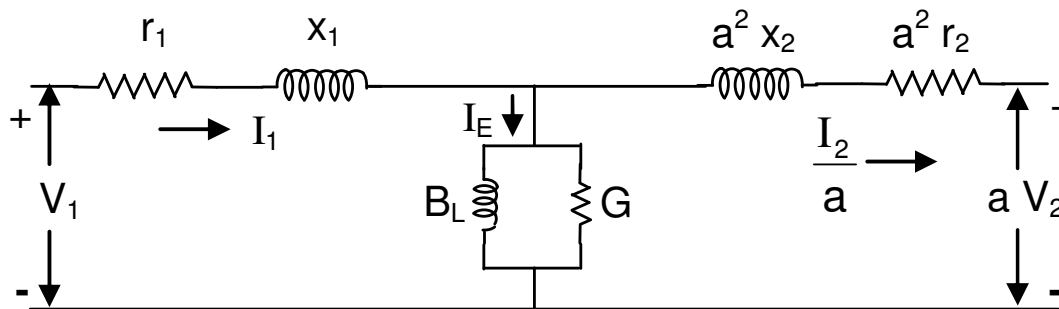


Fig. 1.4 Transformer model – referred to primary

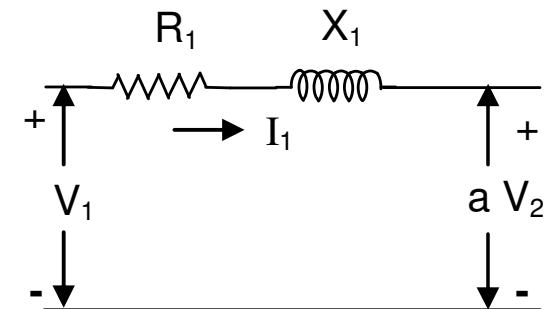
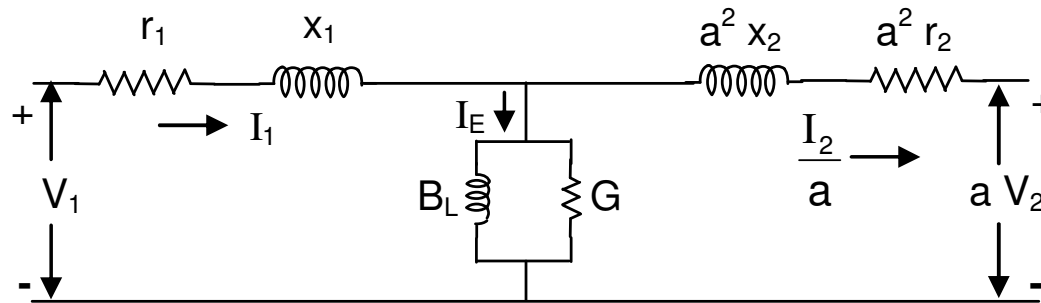
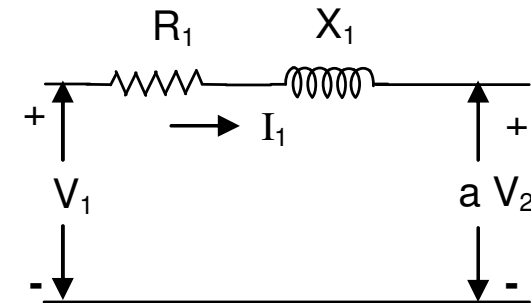


Fig. 1.5 Simplified Transformer model



**Fig. 1.4 Transformer model – referred to primary**



**Fig. 1.5 Simplified Transformer model**

Very often we neglect magnetizing current because it is so small as compared to the usual load currents. To further simplify the circuit we let

$$R_1 = r_1 + a^2 r_2 \quad (1.7)$$

and  $X_1 = x_1 + a^2 x_2 \quad (1.8)$

to obtain the equivalent circuit of Fig 1.5. All impedances and voltages in the part of the circuit connected to the secondary terminals must now be referred to the primary side.

## **Modelling of loads**

Contrary to synchronous generator, loads will absorb real and reactive power. In power flow study, loads are modelled as equivalent complex power injections namely –  $(P_L + j Q_L)$ .

In case of transient stability study, loads are included as shunt admittances at the corresponding buses. Shunt admittance corresponding to load  $(P_L + j Q_L)$  can be obtained as

$$y_L = \frac{P_L - jQ_L}{|V|^2}$$

Loads are neglected in short circuit study as the prefault currents are very small compared to fault currents.

## SINGLE LINE OR ONE-LINE DIAGRAM

Electric power systems are supplied by three-phase generators. Ideally, the generators are supplying power to balanced three phase loads. Fig.1.6 shows a star connected generator supplying star connected balanced load.

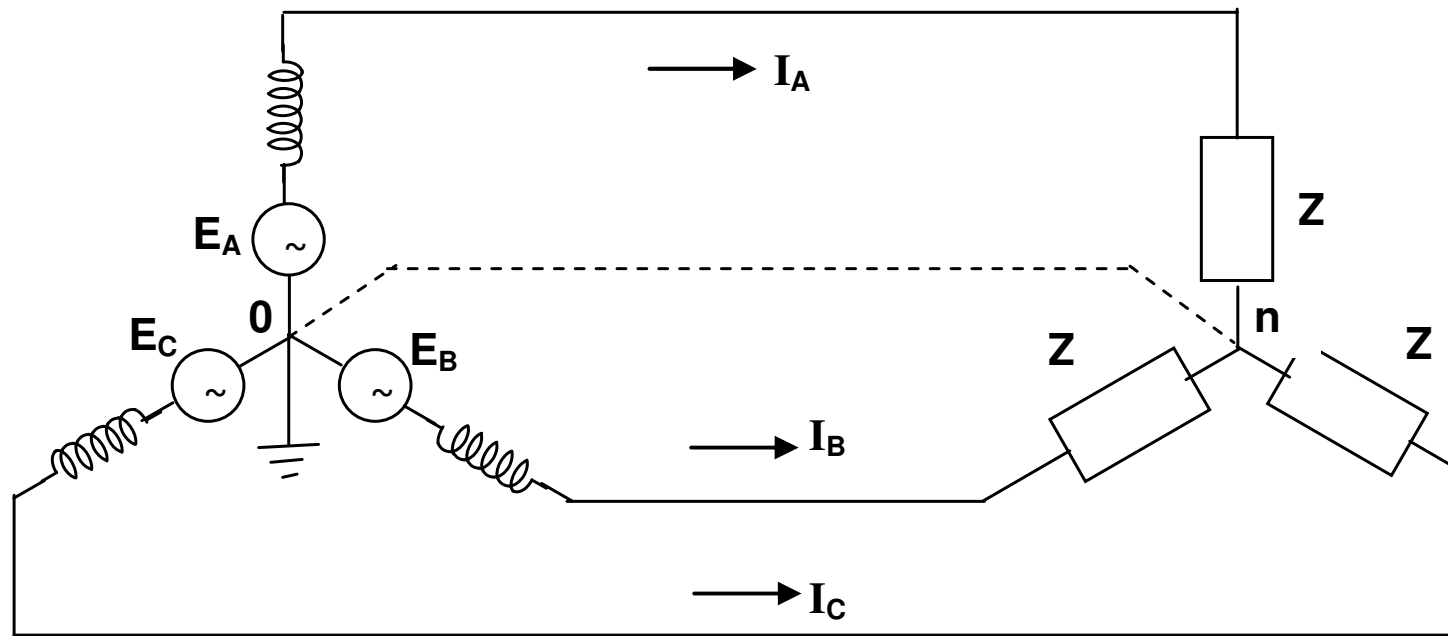


Fig. 1.6 Y- connected generator supplying balanced Y- connected load

A balanced three-phase system is always solved as a single-phase circuit composed of one of the three lines and the neutral return. Single-phase circuit of three-phase system considered above is shown in Fig. 1.7.

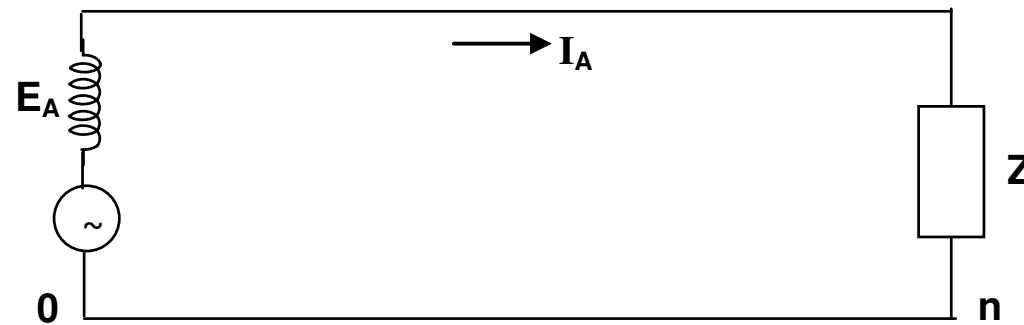


Fig. 1.7 Single-phase circuit

Often the diagram is simplified further by omitting the neutral and by indicating the component parts by standard symbols rather than by their equivalent circuits. Such a simplified diagram of electric system is called a **one-line diagram**. It is also called as single line diagram. The one-line diagram of the simple three-phase system considered above is shown in Fig. 1.8

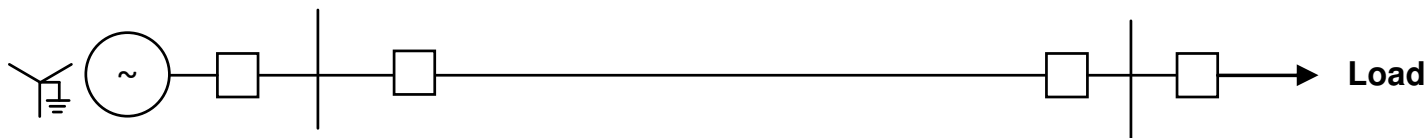
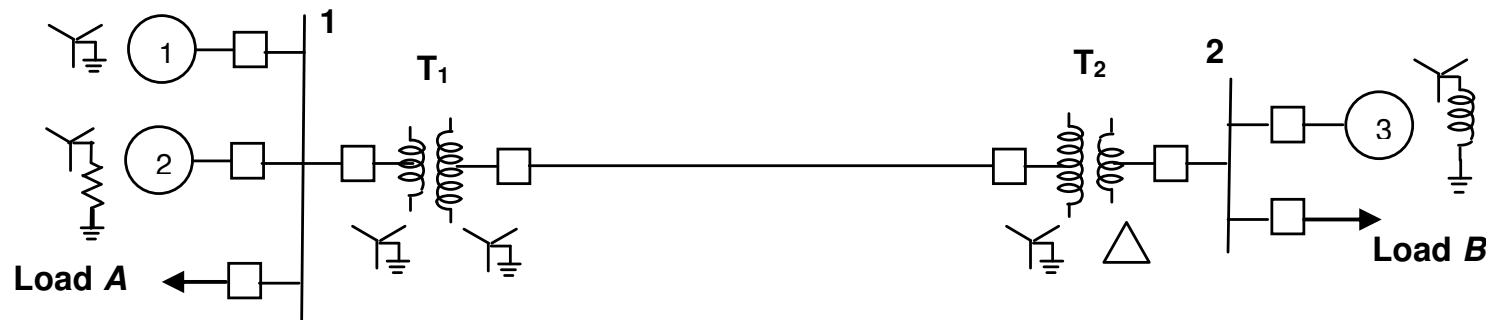


Fig. 1.8 One-line diagram

**Fig. 1.9 shows the one-line diagram of a sample power system.**



**Fig. 1.9 One-line diagram of a sample power system**

**This system has two generators, one solidly grounded and the other grounded through a resistor, that are connected to a bus 1. A transmission line is fed through a step-up transformer installed at bus 1. A step-down transformer is connected at the other end of the transmission line. At its low voltage side a generator, grounded through a reactor, is connected at bus 2. Load A and load B are connected at buses 1 and 2 respectively.**

**On the one-line diagram information about the loads, the ratings of the generators and transformers, and reactances of different components of the circuit are often given.**

## Impedance and reactance diagram

In order to calculate the performance of a power system under load condition or upon the occurrence of a fault, the one line diagram is used to draw the single-phase or per phase equivalent circuit of the system.

Refer the one-line diagram of a sample power system shown in Fig. 1.10.



Fig. 1.10 One-line diagram of a sample power system

Fig.1.11 combines the equivalent circuits for the various components to form the **per-phase impedance diagram** of the system



The impedance diagram does not include the current limiting impedances shown in the one-line diagram because no current flows in the ground under balanced condition.

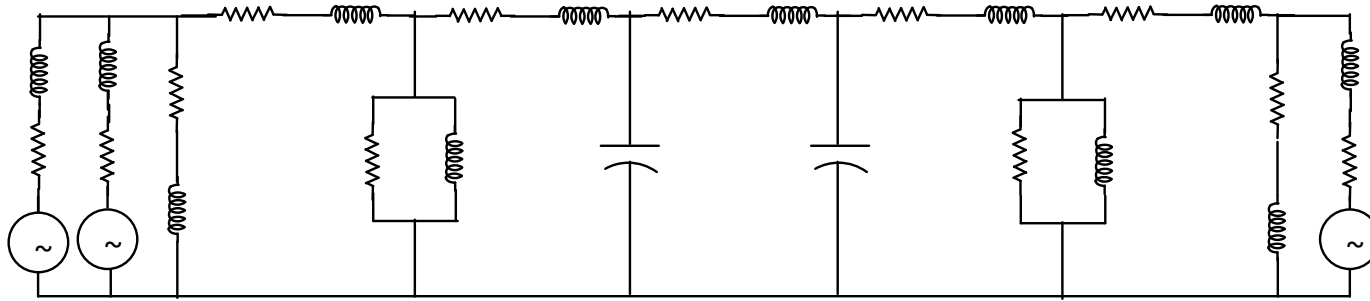


Fig. 1.11 Per-phase impedance diagram

Since the inductive reactance of different system component is much larger than its resistance generally, the resistances are omitted. Static loads have little effect on the total line current during the fault and are usually omitted. The resulting diagram as shown in Fig. 1.12 and is called as per-phase reactance diagram of the system.

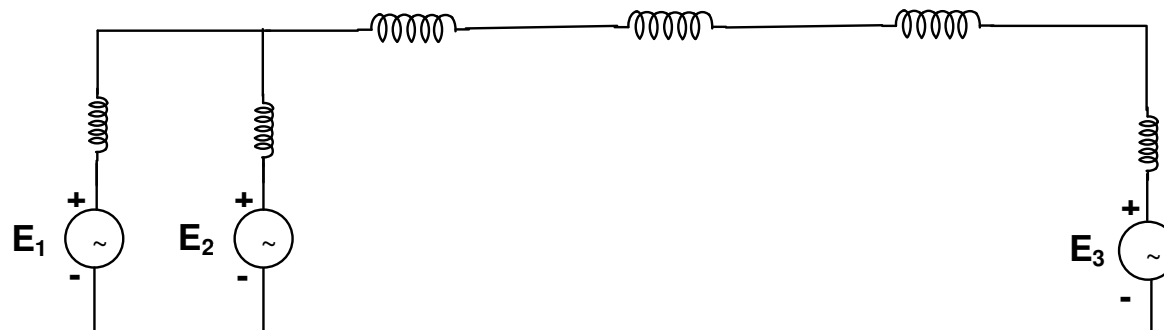


Fig. 1.12 Per-phase reactance diagram

## **Per-unit quantities**

**Absolute values may not give the full significance of quantities. Consider the marks scored by a student in three subjects as 10, 40 and 75. Many of you may be tempted to say that he is poor in subject 1, average in subject 2 and good in subject 3. That is true only when the base for all the marks is 100. If the bases are 10, 50 and 100 for the three subjects respectively then his marks in percentage are 100, 80 and 95. Now only we can correctly decide about his performance.**

**One consumer complained that his supply voltage is 340 V and another said he is getting supply with 10.45 kV. We cannot immediately say who is severely affected. Once we know their base voltages are 400 V and 11 kV, we find that they are getting 85 % and 95% of the standard voltage and hence the problem of the first consumer is more severe.**

**In many situation absolute values have no meaning and thus there is a need to specify base quantities.**

**In many situation absolute values have no meaning and thus there is a need to specify base quantities.**

$$\text{Percentage} = \frac{\text{actual value}}{\text{base}} \times 100$$

$$\text{Per-unit quantity} = \frac{\text{percentage}}{100} = \frac{\text{actual value}}{\text{base}}$$

**In power system we shall deal with voltage, current, impedance and apparent power. When they are large values, we may use kV, ampere, ohm and kVA as their units. It is to be noted that out of the four quantities voltage, current, impedance and apparent power if we specify two quantities, other two quantities can be calculated.**

**Generally, base volt-ampere in MVA and base voltage in kV are specified.**

**For a single-phase system, the following formulas relate the various quantities.**

$$\begin{aligned}
 \text{Base current, A} &= \frac{\text{base VA}}{\text{base voltage, V}} = \frac{\text{base MVA} \times 10^6}{\text{base voltage, kV} \times 1000} \\
 &= \frac{\text{base MVA} \times 1000}{\text{base voltage, kV}}
 \end{aligned}
 \tag{1.9}$$

$$\text{Base impedance, } \Omega = \frac{\text{base voltage, V}}{\text{base current, A}} = \frac{\text{base voltage, kV} \times 1000}{\text{base current, A}}
 \tag{1.10}$$

Substituting eq. (1.9) in the above

$$\text{Base impedance, } \Omega = \frac{(\text{base voltage, kV})^2 \times 1000}{\text{base MVA} \times 1000} \quad \text{Thus}$$

$$\text{Base impedance, } \Omega = \frac{(\text{base voltage, kV})^2}{\text{base MVA}}
 \tag{1.11}$$

Since power factor is a dimensionless quantity

$$\text{Base power, MW} = \text{base MVA}
 \tag{1.12}$$

$$\text{Per - unit impedance of an element} = \frac{\text{actual impedance, } \Omega}{\text{base impedance, } \Omega} \quad (1.13)$$

With the specified base voltage in kV and base volt ampere in MVA,

$$\text{Base impedance, } \Omega = \frac{(\text{base voltage, kV})^2}{\text{base MVA}} \quad (1.14)$$

$$\text{Per-unit impedance} = \text{actual impedance} \times \frac{\text{Base MVA}}{(\text{Base voltage, kV})^2} \quad (1.15)$$

For three-phase system, when base voltage is specified it is line base voltage and the specified MVA is three phase MVA. Now let us consider a three phase system. Let *Base voltage, kV* and *Base MVA* are specified. Then single-phase base voltage, kV = *Base voltage, kV* /  $\sqrt{3}$  and single-phase base MVA = *Base MVA* / 3. Substituting these in eq. (1.14)

$$\text{Base impedance, } \Omega = \frac{[\text{Base voltage, kV} / \sqrt{3}]^2}{\text{Base MVA} / 3} = \frac{(\text{Base voltage, kV})^2}{\text{Base MVA}} \quad (1.16)$$

$$\text{Base impedance, } \Omega = \frac{[\text{Base voltage, kV} / \sqrt{3}]^2}{\text{Base MVA} / 3} = \frac{(\text{Base voltage, kV})^2}{\text{Base MVA}} \quad (1.16)$$

$$\text{Per-unit impedance} = \frac{\text{actual impedance}}{\text{base impedance}}; \text{ Therefore}$$

$$\text{Per-unit impedance} = \text{actual impedance} \times \frac{\text{Base MVA}}{(\text{Base voltage, kV})^2} \quad (1.17)$$

It is to be noted that eqs.(1.16) and (1.17) are much similar to eqs.(1.14) and (1.15). Thus the formulas for base impedance and per-unit impedance in terms of the base voltage in kV and base MVA are same for single phase and three phase systems.

### **Per-unit quantities on a different base**

Sometimes, knowing the per-unit impedance of a component based on a particular base values, we need to find the per-unit value of that component based on some other base values. From eq.(1.9) It is to be noted that the per-unit impedance is directly proportional to base MVA and inversely proportional to (base kV)<sup>2</sup>. Therefore, to change from per-unit impedance on a given base to per-unit impedance on a new base, the following equation applies:

$$\text{Per-unit } Z_{\text{new}} = \text{per-unit } Z_{\text{given}} \frac{\text{base MVA}_{\text{new}}}{\text{base MVA}_{\text{given}}} \times \left( \frac{\text{base kV}_{\text{given}}}{\text{base kV}_{\text{new}}} \right)^2 \quad (1.18)$$

### **EXAMPLE 1.1**

**A three phase 500 MVA, 22 kV generator has winding reactance of 1.065  $\Omega$ . Find its per-unit reactance.**

#### **Solution**

$$\text{Base impedance} = \frac{22^2}{500} = 0.968 \, \Omega \quad ; \quad \text{Per-unit reactance} = \frac{1.065}{0.968} = 1.1002$$

$$\text{Using eq.(1.7), per – unit reactance} = 1.065 \times \frac{500}{22^2} = 1.1002$$

### **EXAMPLE 1.2**

**The reactance of a generator is given as 0.25 per-unit based on the generator's of 18 kV, 500 MVA. Find its per-unit reactance on a base of 20 kV, 100 MVA.**

#### **Solution**

$$\text{New per-unit reactance} = 0.25 \times \frac{100}{500} \times \left(\frac{18}{20}\right)^2 = 0.0405$$



### **EXAMPLE 1.3**

**A single phase 9.6 kVA, 500 V / 1.5 kV transformer has a leakage reactance of  $1.302 \Omega$  with respect to primary side. Find its per-unit reactance with respect to primary and secondary sides.**

#### **Solution**

***With respect to Primary:***    Per-unit impedance =  $1.302 \times \frac{0.0096}{(0.5)^2} = 0.05$

***With respect to Secondary:***    Leakage reactance =  $1.302 \times \left(\frac{1.5}{0.5}\right)^2 = 11.718 \Omega$

$$\text{Per-unit impedance} = 11.718 \times \frac{0.0096}{(1.5)^2} = 0.05$$

#### ***Conclusion***

**Per-unit impedance of the transformer is same referred to primary as well as secondary.**

## **Advantages of per-unit calculation**

**Manufacturers usually specify the impedance of a piece of apparatus in percent or per-unit on the base of the name plate rating.**

**The per-unit impedances of machines of same type and widely different rating usually lie within narrow range although the ohmic values differ much.**

**For a transformer, when impedance in ohm is specified, it must be clearly mentioned whether it is with respect to primary or secondary. The per-unit impedance of the transformer, once expressed on proper base, is the same referred to either side.**

**The way in which the three-phase transformers are connected does not affect the per-unit impedances although the transformer connection does determine the relation between the voltage bases on the two sides of the transformer.**

### EXAMPLE 1.4

A 300 MVA, 20 kV three-phase generator has a sub-transient reactance of 20%. The generator supplies a number of synchronous motors over 64-km transmission line having transformers at both ends, as shown in Fig. 1.7. The motors, all rated 13.2 kV, are represented by just two equivalent motors. Rated inputs to the motors are 200 MVA and 100 MVA for  $M_1$  and  $M_2$ , respectively. For both motors  $X'' = 20\%$ . The three phase transformer  $T_1$  is rated 350 MVA, 230/20 kV with leakage reactance of 10%. Transformer  $T_2$  is composed of three single-phase transformers each rated 127/13.2 kV, 100 MVA with leakage reactance of 10%. Series reactance of the transmission line is  $0.5\Omega/\text{km}$ . Draw the impedance diagram, with all impedances marked in per-unit. Select the generator rating as base in the generator circuit.

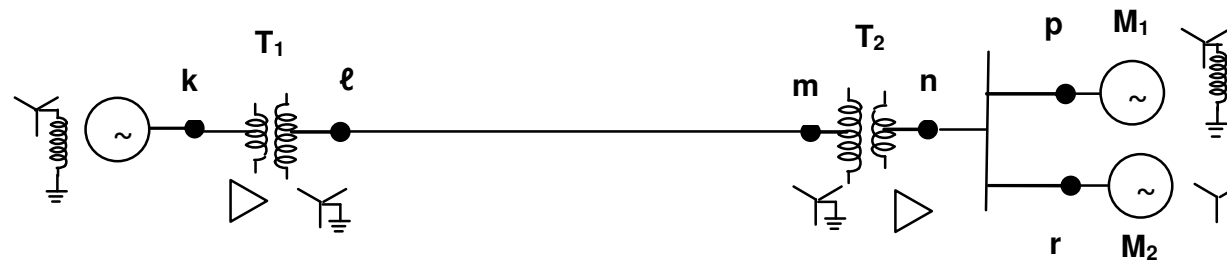
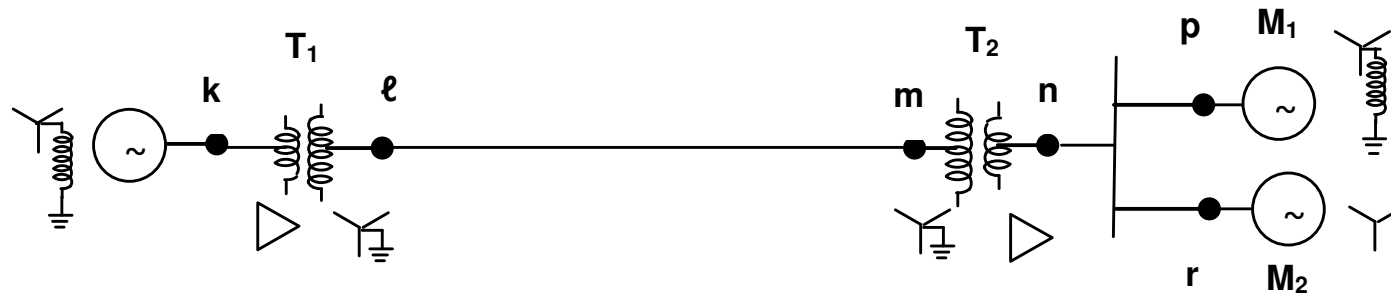


Fig. 1.13 One-line diagram for Example 1.4



**Fig. 1.13 One-line diagram for Example 1.4**

### **Solution**

**Base MVA = 300;     Base voltage at generator side = 20 kV**

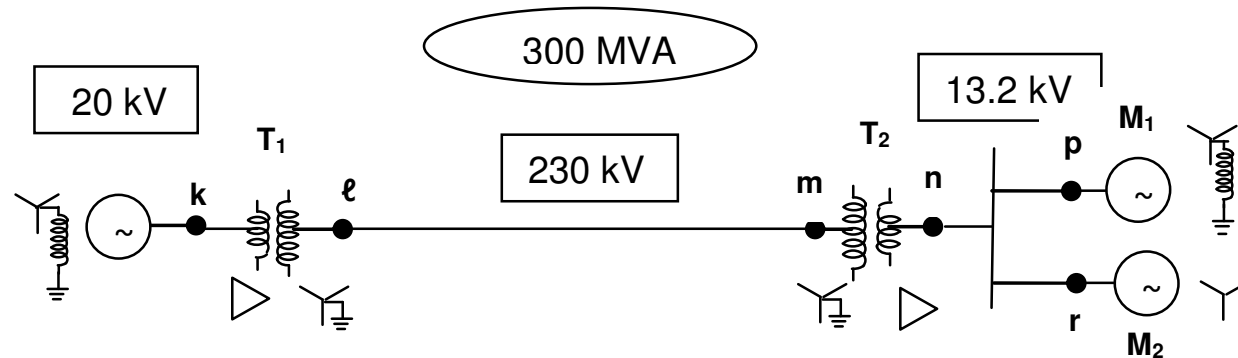
**Base voltage in transmission line = 230 kV**

**Line to line voltages of transformer  $T_2$  :  $\sqrt{3} \times 127 / 13.2 = 220 / 13.2$  kV**

**Base voltage at motor side =  $230 \times \frac{13.2}{220} = 13.8$  kV**

**Base MVA and base voltages at different sections are marked.**

Base MVA and base voltages at different sections are marked.



Per-unit reactance of generator = 0.2

$$\text{Per-unit reactance of transformer } T_1 = 0.1 \times \frac{300}{350} = 0.0857$$

$$\text{Per-unit reactance of transmission line} = 0.5 \times 64 \times \frac{300}{230^2} = 0.1825$$

$$\text{Per-unit reactance of transformer } T_2 = 0.1 \times \left(\frac{220}{230}\right)^2 = 0.0915$$

$$\text{Per-unit reactance of motor } M_1 = 0.2 \times \frac{300}{200} \times \left(\frac{13.2}{13.8}\right)^2 = 0.2745$$

$$\text{Per-unit reactance of motor } M_2 = 0.2 \times \frac{300}{100} \times \left(\frac{13.2}{13.8}\right)^2 = 0.549$$

Per-unit reactance of generator = 0.2

$$\text{Per-unit reactance of transformer } T_1 = 0.1 \times \frac{300}{350} = 0.0857$$

$$\text{Per-unit reactance of transmission line} = 0.5 \times 64 \times \frac{300}{230^2} = 0.1825$$

$$\text{Per-unit reactance of transformer } T_2 = 0.1 \times \left(\frac{220}{230}\right)^2 = 0.0915$$

$$\text{Per-unit reactance of motor } M_1 = 0.2 \times \frac{300}{200} \times \left(\frac{13.2}{13.8}\right)^2 = 0.2745$$

$$\text{Per-unit reactance of motor } M_2 = 0.2 \times \frac{300}{100} \times \left(\frac{13.2}{13.8}\right)^2 = 0.549$$

Per-unit impedance diagram is shown in Fig. 1.14

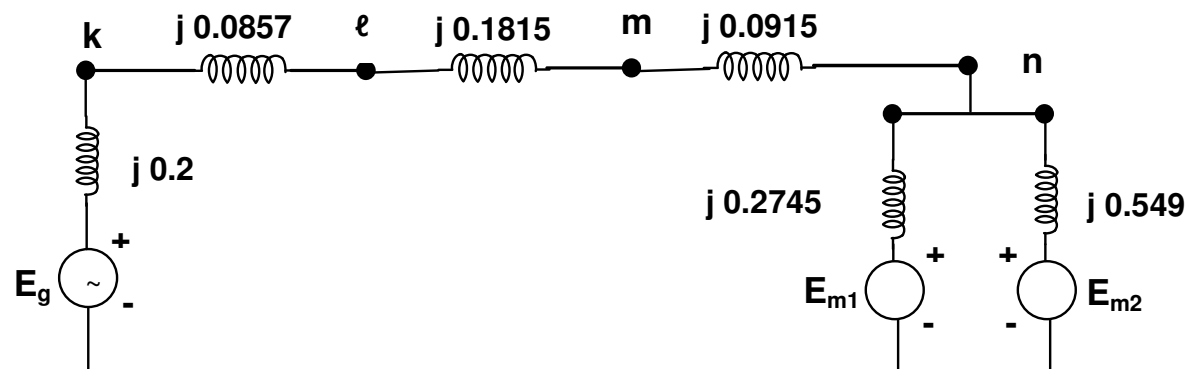


Fig. 1.14 Per-unit impedance diagram

### **EXAMPLE 1.5**

A transformer rated 200 MVA, 345Y / 20.5 Δ kV connected at the receiving end of a transmission line feeds a balanced load rated 180 MVA, 22.5 kV, 0.8 power factor. Determine

- (a) the rating of each of three single-phase transformers which when properly connected will be equivalent to the above three-phase transformer and
- (b) the complex impedance of the load in per-unit, if the base in the transmission line is 100 MVA, 345 kV.

### **Solution**

- (a) Rating of each single-phase transformer:  $200/3$  MVA,  $(345/\sqrt{3}) / 20.5$  kV

i.e 66.7 MVA, 199.2 / 20.5 kV

(b) Load  $|Z| = \frac{|V|^2}{|S|} = \frac{22.5^2}{180} = 2.81 \Omega \quad (|S| = |V||I| \text{ and } |Z| = \frac{|V|}{|I|})$

Base MVA = 100; Base voltage at the load side = 20.5 kV

$$\text{Load in per-unit} = 2.81 \times \frac{100}{20.5^2} \angle 36.87^\circ = 0.669 \angle 36.87^\circ = 0.5352 + j0.4014$$

## NETWORK GRAPH

A power network is essentially an interconnection of several two-terminal components such as generators, transformers, transmission lines, motors and loads. Each element has impedance. The voltage across the element is called element voltage and the current flowing through the element is called the element current. A set of components when they are connected form a Primitive network.

A representation of a power system and the corresponding oriented graph are shown in Fig. 1.15.

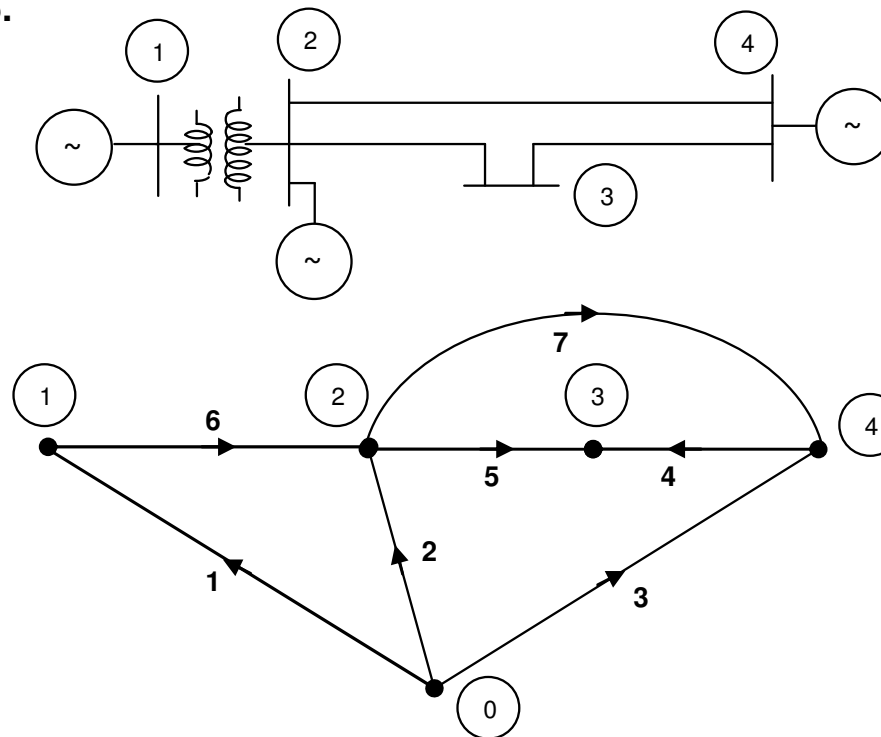


Fig. 1.15 A power system network and corresponding oriented graph



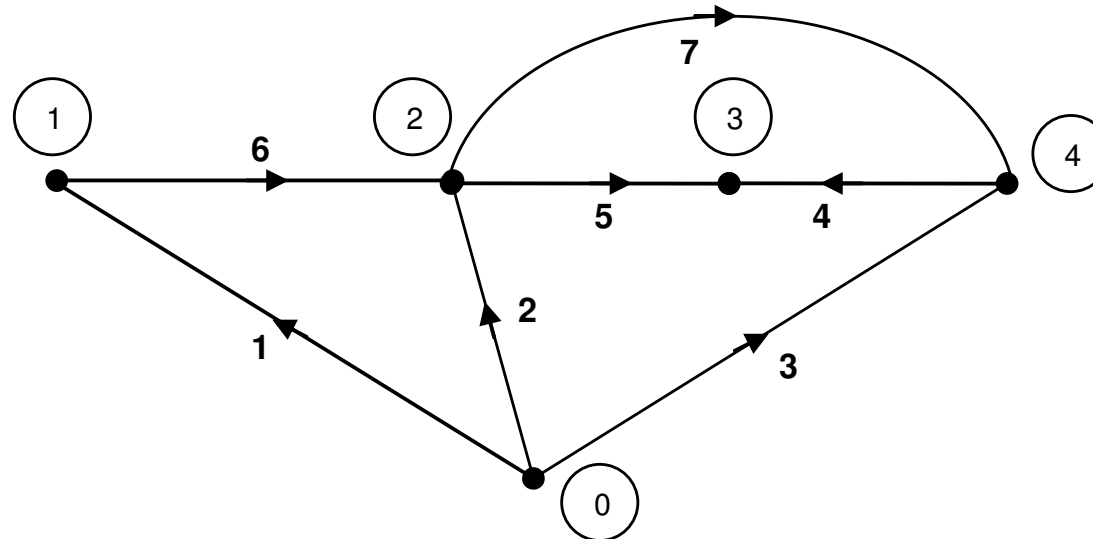
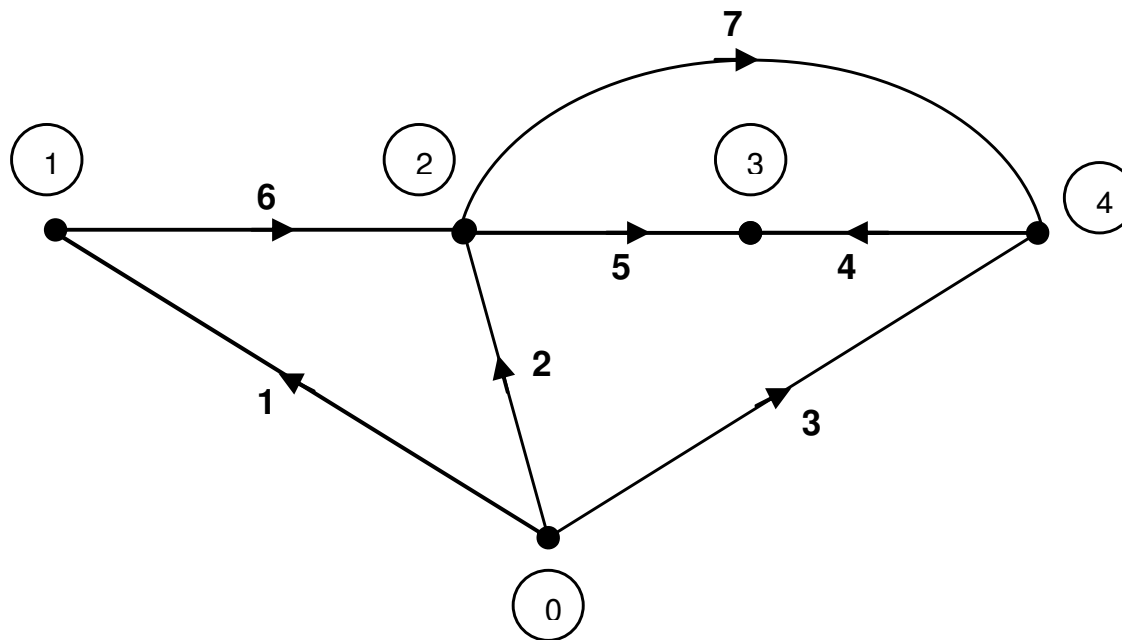


Fig. 1.15 A power system network and corresponding oriented graph

Connectivity various elements to form the network can be shown by the bus incidence matrix **A**. For above system, this matrix is obtained as

$$\mathbf{A} = \begin{array}{c} \begin{array}{c} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \\ \textcircled{4} \end{array} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline -1 & & & & & 1 & \\ & -1 & & & 1 & -1 & 1 \\ & & & -1 & -1 & & \\ & & -1 & 1 & & & -1 \end{array} \end{array} \quad (1.19)$$

Element voltages are referred as  $v_1, v_2, v_3, v_4, v_5, v_6$  and  $v_7$ . Element currents are referred as  $i_1, i_2, i_3, i_4, i_5, i_6$  and  $i_7$ . In power system problems quite often we make use of bus voltages and bus currents. For the above network, the bus voltages are  $V_1, V_2, V_3$  and  $V_4$ . The bus voltages are always measured with respect to the ground bus. The bus currents are designated as  $I_1, I_2, I_3$ , and  $I_4$ . The element voltages are related to bus voltages as:



$$v_1 = -V_1$$

$$v_2 = -V_2$$

$$v_3 = -V_4$$

$$v_4 = V_4 - V_3$$

$$v_5 = V_2 - V_3$$

$$v_6 = V_1 - V_2$$

$$v_7 = V_2 - V_4$$

$$v_1 = -V_1$$

$$v_2 = -V_2$$

$$v_3 = -V_4$$

$$v_4 = V_4 - V_3$$

$$v_5 = V_2 - V_3$$

$$v_6 = V_1 - V_2$$

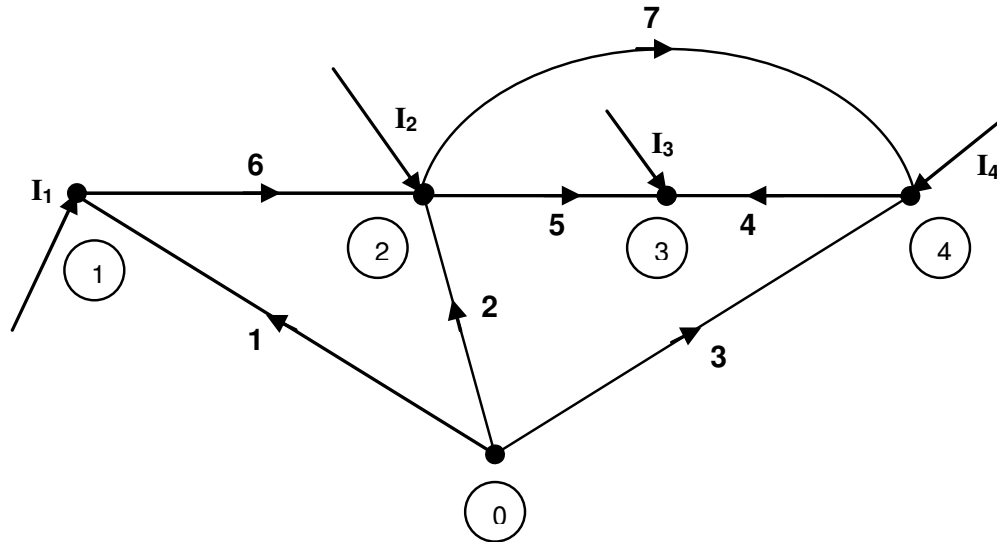
$$v_7 = V_2 - V_4$$

Expressing the relation in matrix form

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{bmatrix} = \begin{bmatrix} -1 & & & \\ & -1 & & \\ & & & -1 \\ & & -1 & 1 \\ & 1 & -1 & \\ 1 & -1 & & \\ & 1 & & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} \quad (1.20)$$

$$\text{Thus } v = A^T V_{\text{bus}} \quad (1.21)$$

The element currents are related to bus currents as:



$$I_1 = -i_1 + i_6$$

$$I_2 = -i_2 + i_5 - i_6 + i_7$$

$$I_3 = -i_4 - i_5$$

$$I_4 = -i_3 + i_4 - i_7$$

Expressing the relation in matrix form

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} -1 & & & & & 1 & \\ & -1 & & & 1 & -1 & 1 \\ & & & -1 & -1 & & \\ & & -1 & 1 & & & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \end{bmatrix}$$

Thus  $I_{bus} = A i$

(1.22)

## PRIMITIVE PARAMETERS

For a single element the performance equation is

$$V_1 = Z_1 i_1$$

For network with two elements the performance equations are

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

Here  $Z_{11}$  and  $Z_{22}$  are the self impedances of elements 1 and 2 and  $Z_{12}$  is the mutual impedance between elements 1 and 2. For the seven element network shown in Fig. 1.15 element voltages, element currents are related as

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} & Z_{15} & Z_{16} & Z_{17} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} & Z_{25} & Z_{26} & Z_{27} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} & Z_{35} & Z_{36} & Z_{37} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} & Z_{45} & Z_{46} & Z_{47} \\ Z_{51} & Z_{52} & Z_{53} & Z_{54} & Z_{55} & Z_{56} & Z_{57} \\ Z_{61} & Z_{62} & Z_{63} & Z_{64} & Z_{65} & Z_{66} & Z_{67} \\ Z_{71} & Z_{72} & Z_{73} & Z_{74} & Z_{75} & Z_{76} & Z_{77} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \end{bmatrix} \quad (1.23)$$

Here  $z_{ii}$  is the self impedance of element  $i$  and  $z_{ij}$  is the mutual impedance between elements  $i$  and  $j$ . In matrix notation the above can be written as

$$\mathbf{v} = \mathbf{z} \mathbf{i} \quad (1.24)$$

Here  $\mathbf{z}$  is known as primitive impedance matrix. The inverse form of above is

$$\mathbf{i} = \mathbf{y} \mathbf{v} \quad (1.25)$$

In the above  $\mathbf{y}$  is called as primitive admittance matrix. Matrices  $\mathbf{z}$  and  $\mathbf{y}$  are inverses of each other. Similar to the above two relations, in terms of bus frame

$$\mathbf{V}_{bus} = \mathbf{Z}_{bus} \mathbf{I}_{bus} \quad (1.26)$$

Here  $\mathbf{V}_{bus}$  is the bus voltage vector,  $\mathbf{I}_{bus}$  is the bus current vector and  $\mathbf{Z}_{bus}$  is the bus impedance matrix. The inverse form of above is

$$\mathbf{I}_{bus} = \mathbf{Y}_{bus} \mathbf{V}_{bus} \quad (1.27)$$

Here  $\mathbf{Y}_{bus}$  is known as bus admittance matrix. Matrices  $\mathbf{Z}_{bus}$  and  $\mathbf{Y}_{bus}$  are inverses of each other.

## Derivation of bus admittance matrix

It was shown that

$$\mathbf{v} = \mathbf{A}^T \mathbf{V}_{bus} \quad (1.21)$$

$$\mathbf{I}_{bus} = \mathbf{A} \mathbf{i} \quad (1.22)$$

$$\mathbf{i} = \mathbf{y} \mathbf{v} \quad (1.25)$$

Substituting eq. (1.21) in the above

$$\mathbf{i} = \mathbf{y} \mathbf{A}^T \mathbf{V}_{bus} \quad (1.28)$$

Substituting the above in eq. (1.22)

$$\mathbf{I}_{bus} = \mathbf{A} \mathbf{y} \mathbf{A}^T \mathbf{V}_{bus} \quad (1.29)$$

Comparing eqs. (1.27) and (2.11)

$$\mathbf{Y}_{bus} = \mathbf{A} \mathbf{y} \mathbf{A}^T \quad (1.30)$$

**This is a very general formula for bus admittance matrix and admits mutual coupling between elements.**

In several power system problems mutual couplings will have negligible effect and often omitted. In that case the primitive impedance matrix  $z$  and the primitive admittance matrix  $y$  are diagonal and  $Y_{bus}$  can be obtained by inspection. This is illustrated through the seven-element network considered earlier. When mutual couplings are neglected

$$[y] = \begin{bmatrix} y_{11} & & & & & & \\ & y_{22} & & & & & \\ & & y_{33} & & & & \\ & & & y_{44} & & & \\ & & & & y_{55} & & \\ & & & & & y_{66} & \\ & & & & & & y_{77} \end{bmatrix} \quad (1.31)$$

Bus incidence matrix  $A$  is

-1					1	
	-1			1	-1	1
			-1	-1		
		-1	1			-1



$$Y_{bus} = A \, y \, A^T$$

= A

y<sub>11</sub>

y<sub>22</sub>

y<sub>33</sub>

y<sub>44</sub>

y<sub>55</sub>

y<sub>66</sub>

y<sub>77</sub>

-1			
	-1		
			-1
		-1	1
	1	-1	
1	-1		
	1		-1

=

-1					1	
	-1			1	-1	1
			-1	-1		
		-1	1			-1

-y <sub>11</sub>			
	-y <sub>22</sub>		
			-y <sub>33</sub>
		-y <sub>44</sub>	y <sub>44</sub>
	y <sub>55</sub>	-y <sub>55</sub>	
y <sub>66</sub>	-y <sub>66</sub>		
	y <sub>77</sub>		-y <sub>77</sub>

=

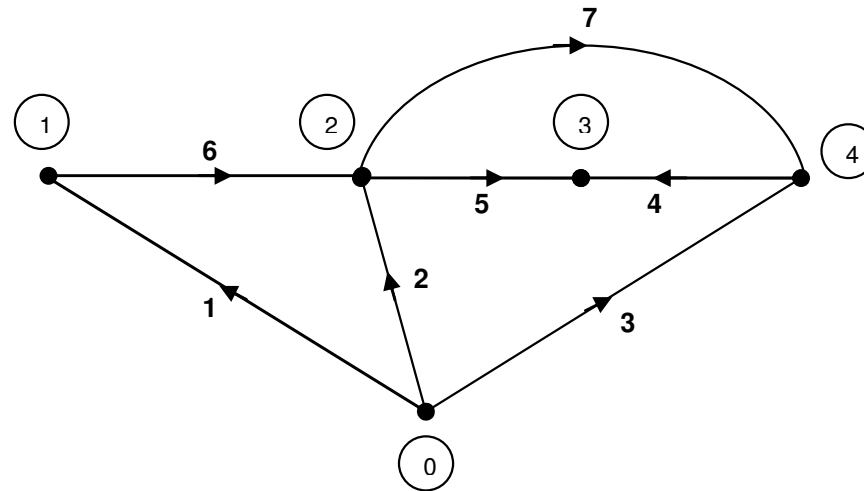
<b>-1</b>					<b>1</b>	
	<b>-1</b>			<b>1</b>	<b>-1</b>	<b>1</b>
			<b>-1</b>	<b>-1</b>		
		<b>-1</b>	<b>1</b>			<b>-1</b>

<b>-y<sub>11</sub></b>			
	<b>-y<sub>22</sub></b>		
			<b>-y<sub>33</sub></b>
		<b>-y<sub>44</sub></b>	<b>y<sub>44</sub></b>
	<b>y<sub>55</sub></b>	<b>-y<sub>55</sub></b>	
<b>y<sub>66</sub></b>	<b>-y<sub>66</sub></b>		
	<b>y<sub>77</sub></b>		<b>-y<sub>77</sub></b>

**Y<sub>bus</sub> =**

	①	②	③	④
①	<b>y<sub>11</sub> + y<sub>66</sub></b>	<b>- y<sub>66</sub></b>	<b>0</b>	<b>0</b>
②	<b>- y<sub>66</sub></b>	<b>y<sub>22</sub> + y<sub>55</sub> + y<sub>66</sub> + y<sub>77</sub></b>	<b>- y<sub>55</sub></b>	<b>-y<sub>77</sub></b>
③	<b>0</b>	<b>- y<sub>55</sub></b>	<b>y<sub>44</sub> + y<sub>55</sub></b>	<b>- y<sub>44</sub></b>
④	<b>0</b>	<b>-y<sub>77</sub></b>	<b>- y<sub>44</sub></b>	<b>y<sub>33</sub> + y<sub>44</sub> + y<sub>77</sub></b>

	1	2	3	4
1	$y_{11} + y_{66}$	$-y_{66}$	0	0
2	$-y_{66}$	$y_{22} + y_{55} + y_{66} + y_{77}$	$-y_{55}$	$-y_{77}$
3	0	$-y_{55}$	$y_{44} + y_{55}$	$-y_{44}$
4	0	$-y_{77}$	$-y_{44}$	$y_{33} + y_{44} + y_{77}$



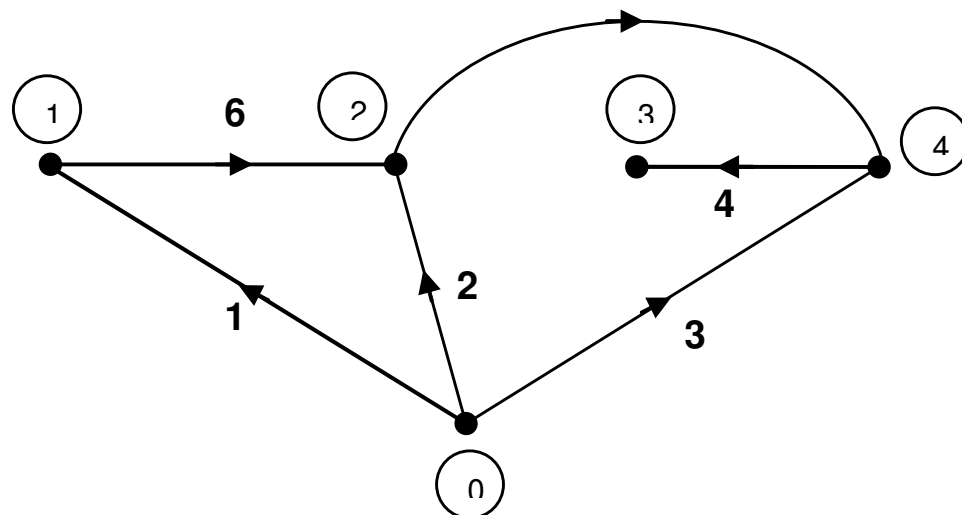
When there is no mutual coupling, the rules to form the elements of  $Y_{bus}$  are:

- The diagonal element  $Y_{ii}$  equals the sum of the admittances directly connected to bus  $i$ .
- The off-diagonal element  $Y_{ij}$  equals the negative of the admittance connected between buses  $i$  and  $j$ . If there is no element between buses  $i$  and  $j$ , then  $Y_{ij}$  equals to zero.

Bus admittance matrix can be constructed by adding the elements one by one. Separating the entries corresponding to the element 5 that is connected between buses 2 and 3 the above  $Y_{bus}$  can be written as

$$Y_{bus} = \begin{array}{c|cccc} & \textcircled{1} & \textcircled{2} & \textcircled{3} & \textcircled{4} \\ \hline \textcircled{1} & \mathbf{y_{11} + y_{66}} & \mathbf{- y_{66}} & \mathbf{0} & \mathbf{0} \\ \textcircled{2} & \mathbf{- y_{66}} & \mathbf{y_{22} + y_{66} + y_{77}} & \mathbf{0} & \mathbf{-y_{77}} \\ \textcircled{3} & \mathbf{0} & \mathbf{0} & \mathbf{y_{44}} & \mathbf{- y_{44}} \\ \textcircled{4} & \mathbf{0} & \mathbf{-y_{77}} & \mathbf{- y_{44}} & \mathbf{y_{33} + y_{44} + y_{77}} \end{array}$$

$$+ \begin{array}{c|cccc} & \textcircled{1} & \textcircled{2} & \textcircled{3} & \textcircled{4} \\ \hline \textcircled{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \textcircled{2} & \mathbf{0} & \mathbf{y_{55}} & \mathbf{- y_{55}} & \mathbf{0} \\ \textcircled{3} & \mathbf{0} & \mathbf{- y_{55}} & \mathbf{y_{55}} & \mathbf{0} \\ \textcircled{4} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{array}$$



**It can be inferred that the effect of adding element 5 between buses 2 and 3 is to add admittance  $y_{55}$  to elements  $Y_{bus}(2,2)$  and  $Y_{bus}(3,3)$  and add  $-y_{55}$  to elements  $Y_{bus}(2,3)$  and  $Y_{bus}(3,2)$ . To construct the bus admittance matrix  $Y_{bus}$ , initially all the elements are set to zero; then network elements are added one by one, each time four elements of  $Y_{bus}$  are modified.**

**Calculation of bus admittance matrix by adding elements one by one can be extended even the network contains coupled group. This is illustrated in the following examples.**

### Example 1.6

Consider the power network shown in Fig. 1.16. The ground bus is marked as 0. Grounding impedances at buses 1, 2, and 3 are  $j0.6 \Omega$ ,  $j0.4 \Omega$  and  $j0.5 \Omega$  respectively. Impedances of the elements 3-4, 2-3, 1-2 and 2-4 are  $j0.25 \Omega$ ,  $j0.2 \Omega$ ,  $0.2 \Omega$  and  $j0.5 \Omega$ . The mutual impedance between elements 2-3 and 2-4 is  $j0.1 \Omega$ . Obtain the bus admittance matrix of the power network.

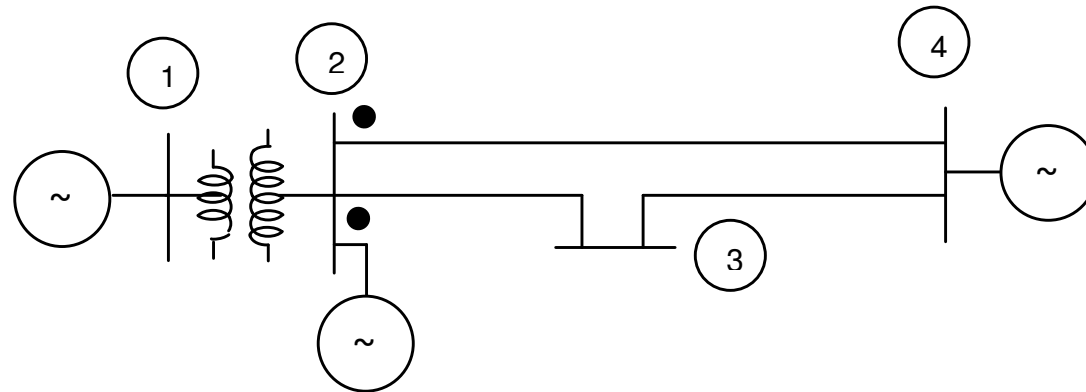


Fig. 1.16 Power network – Example 1.6

### Solution METHOD 1

The oriented graph of the network, with impedances marked is shown in Fig. 1.17.

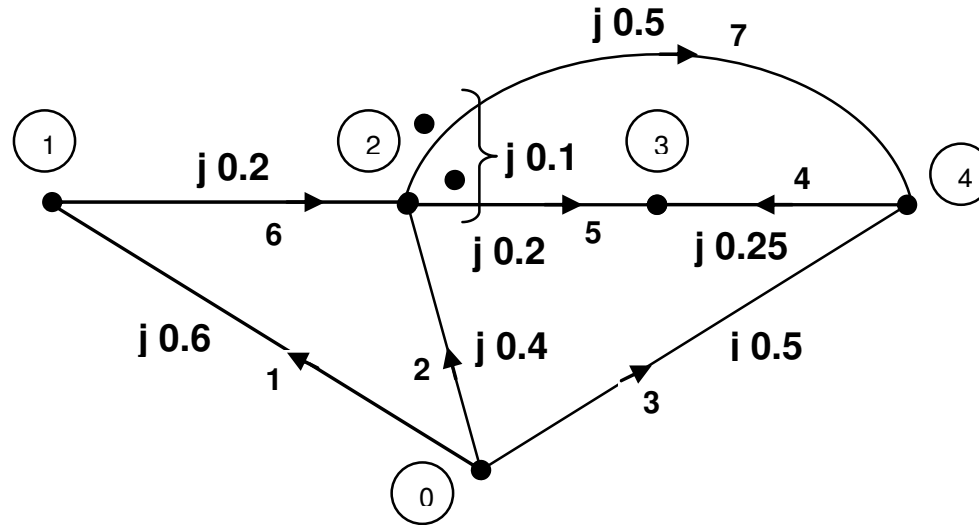


Fig. 1.17 Data for Example 1.6

Primitive impedance matrix is:

	1	2	3	4	5	6	7
1	0.6						
2		0.4					
3			0.5				
4				0.25			
5					0.2		0.1
6						0.2	
7					0.1		0.5

## Inverting the primitive impedance matrix

$y = -j$

	1	2	3	4	5	6	7
1	1.6667						
2		2.5					
3			2.0				
4				4.0			
5					5.5556		-1.1111
6						5.0	
7					-1.1111		2.2222

Bus incidence matrix A is:

A =

-1					1	
	-1			1	-1	1
			-1	-1		
		-1	1			-1



Bus admittance matrix  $Y_{bus} = A y A^T$

$Y_{bus} = -j A$

1.6667						
	2.5					
		2.0				
			4.0			
				5.5556		-1.1111
					5.0	
				-1.1111		2.2222

-1			
	-1		
			-1
		-1	1
	1	-1	
1	-1		
	1		-1

$Y_{bus} = -j$

-1					1	
	-1			1	-1	1
			-1	-1		
		-1	1			-1

-1.6667			
	- 2.5		
			- 2.0
		- 4.0	4.0
	4.4444	- 5.5556	1.1111
5.0	- 5.0		
	1.1111	1.1111	- 2.2222

$$Y_{bus} = -j$$

<b>-1</b>					<b>1</b>	
	<b>-1</b>			<b>1</b>	<b>-1</b>	<b>1</b>
			<b>-1</b>	<b>-1</b>		
		<b>-1</b>	<b>1</b>			<b>-1</b>

<b>-1.6667</b>			
	<b>- 2.5</b>		
			<b>- 2.0</b>
		<b>- 4.0</b>	<b>4.0</b>
	<b>4.4444</b>	<b>- 5.5556</b>	<b>1.1111</b>
<b>5.0</b>	<b>- 5.0</b>		
	<b>1.1111</b>	<b>1.1111</b>	<b>- 2.2222</b>

$$Y_{bus} =$$

	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
<b>1</b>	<b>- j6.6667</b>	<b>j5.0</b>	<b>0</b>	<b>0</b>
<b>2</b>	<b>j5.0</b>	<b>- j13.0556</b>	<b>j4.4444</b>	<b>j1.1111</b>
<b>3</b>	<b>0</b>	<b>j4.4444</b>	<b>- j9.5556</b>	<b>j5.1111</b>
<b>4</b>	<b>0</b>	<b>j1.1111</b>	<b>j5.1111</b>	<b>- j8.2222</b>

## METHOD 2

Consider the coupled group alone. Its primitive impedance matrix is

$$\mathbf{z} = \mathbf{j} \begin{array}{c} \begin{array}{cc} & \begin{array}{c} 5 \qquad 7 \end{array} \\ \begin{array}{c} 5 \\ 7 \end{array} & \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.5 \end{bmatrix} \end{array} \quad \text{Its inverse is } \mathbf{y} = -\mathbf{j} \begin{array}{c} \begin{array}{cc} & \begin{array}{c} 5 \qquad 7 \end{array} \\ \begin{array}{c} 5 \\ 7 \end{array} & \begin{bmatrix} 5.5555 & -1.1111 \\ -1.1111 & 2.2222 \end{bmatrix} \end{array}$$

Corresponding bus incidence matrix is:

$$\mathbf{A} = \begin{array}{c} \begin{array}{cc} & \begin{array}{c} 5 \qquad 7 \end{array} \\ \begin{array}{c} \textcircled{2} \\ \textcircled{3} \\ \textcircled{4} \end{array} & \begin{bmatrix} 1 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} \end{array} ;$$

Bus admittance matrix of the coupled group is  $\mathbf{A} \mathbf{y} \mathbf{A}^T$

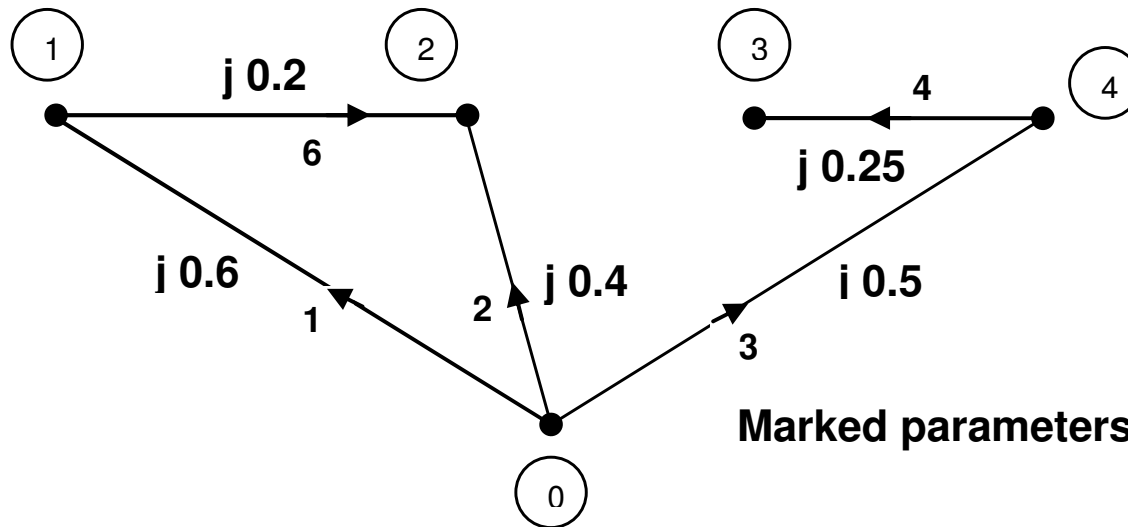
**Bus admittance matrix of the coupled group is  $A y A^T$**

$$A y = -j \begin{matrix} \textcircled{2} \\ \textcircled{3} \\ \textcircled{4} \end{matrix} \begin{matrix} 5 & 7 \\ \left[ \begin{array}{cc} 4.4444 & 1.1111 \\ -5.5555 & 1.1111 \\ 1.1111 & -2.2222 \end{array} \right] \end{matrix}$$

**Bus admittance matrix of the coupled group is**

$$\begin{matrix} \textcircled{2} \\ \textcircled{3} \\ \textcircled{4} \end{matrix} \begin{matrix} \textcircled{2} & \textcircled{3} & \textcircled{4} \\ \left[ \begin{array}{ccc} -j5.5555 & j4.4444 & j1.1111 \\ j4.4444 & -j5.5555 & j1.1111 \\ j1.1111 & j1.1111 & -j2.2222 \end{array} \right] \end{matrix}$$

The network excluding the coupled is considered.



Marked parameters are impedances.

Its bus admittance obtained by adding the elements one by one is:

$$\begin{array}{c}
 \textcircled{1} \\
 \textcircled{2} \\
 \textcircled{3} \\
 \textcircled{4}
 \end{array}
 \begin{array}{c}
 \textcircled{1} \quad \textcircled{2} \quad \textcircled{3} \quad \textcircled{4} \\
 \left[ \begin{array}{cccc}
 -j6.6667 & j5 & 0 & 0 \\
 j5 & -j7.5 & 0 & 0 \\
 0 & 0 & -j4 & j4 \\
 0 & 0 & j4 & -j6
 \end{array} \right]
 \end{array}$$

**Final bus admittance is obtained by adding the corresponding elements in the above two matrices. Thus**

$$\mathbf{Y}_{\text{bus}} = \begin{array}{c} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \\ \textcircled{4} \end{array} \begin{array}{c} \textcircled{1} \quad \textcircled{2} \quad \textcircled{3} \quad \textcircled{4} \\ \begin{array}{|c|c|c|c|} \hline -j6.6667 & j5.0 & 0 & 0 \\ \hline j5.0 & -j13.0556 & j4.4444 & j1.1111 \\ \hline 0 & j4.4444 & -j9.5556 & j5.1111 \\ \hline 0 & j1.1111 & j5.1111 & -j8.2222 \\ \hline \end{array} \end{array}$$

**This is same as that obtained by Method 1.**

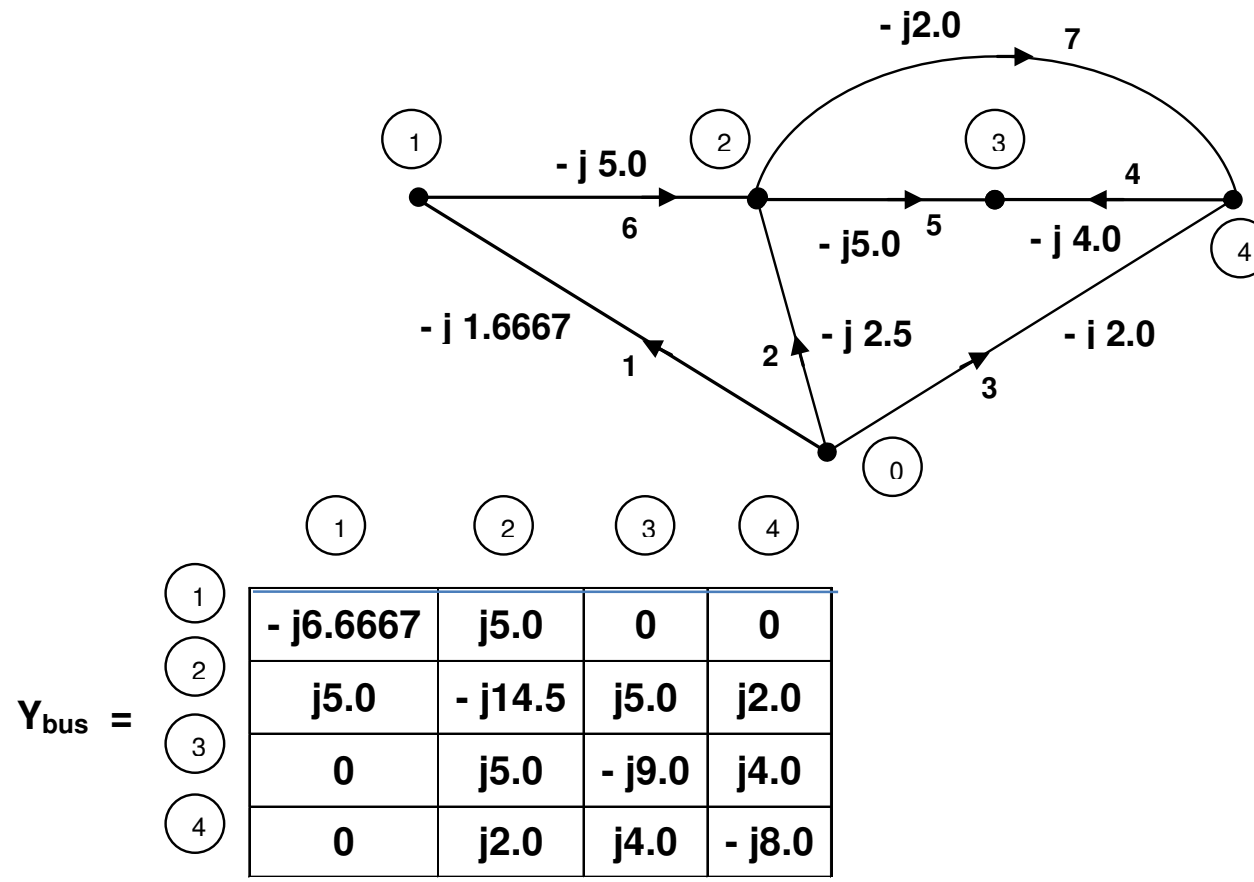
### Example 1.7

Neglect the mutual impedance and obtain  $Y_{bus}$  for the power network described in example 1.6 using the formulas for  $Y_{ii}$  and  $Y_{ij}$ .

### Solution

Admittances of elements 1 to 7 are

- j1.6667, - j2.5, - j2.0, - j4.0, - j5.0, - j5.0 and - j2.0. They are marked below.



### Example 1.8

Repeat previous example by adding elements one by one.

### Solution

Initially all the elements of  $Y_{bus}$  are set to zeros. Now add element 1:

$$Y_{bus} =$$

	1	2	3	4
1	-j1.6667	0	0	0
2	0	0	0	0
3	0	0	0	0
4	0	0	0	0

Next add element 2:



$$Y_{bus} = \begin{array}{c|cccc} \textcircled{1} & -j1.6667 & 0 & 0 & 0 \\ \textcircled{2} & 0 & -j2.5 & 0 & 0 \\ \textcircled{3} & 0 & 0 & 0 & 0 \\ \textcircled{4} & 0 & 0 & 0 & 0 \end{array}$$

Add element 3:

$$Y_{bus} = \begin{array}{c|cccc} & \textcircled{1} & \textcircled{2} & \textcircled{3} & \textcircled{4} \\ \textcircled{1} & -j1.6667 & 0 & 0 & 0 \\ \textcircled{2} & 0 & -j2.5 & 0 & 0 \\ & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & -j2.0 \end{array}$$

**Add element 4:**

		<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
<b>Y<sub>bus</sub> =</b>	<b>1</b>	<b>- j1.6667</b>	<b>0</b>	<b>0</b>	<b>0</b>
	<b>2</b>	<b>0</b>	<b>- j2.5</b>	<b>0</b>	<b>0</b>
	<b>3</b>	<b>0</b>	<b>0</b>	<b>- j4.0</b>	<b>j4.0</b>
	<b>4</b>	<b>0</b>	<b>0</b>	<b>j4.0</b>	<b>- j6.0</b>

**Add element 5:**

		<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
<b>Y<sub>bus</sub> =</b>	<b>1</b>	<b>- j1.6667</b>	<b>0</b>	<b>0</b>	<b>0</b>
	<b>2</b>	<b>0</b>	<b>- j7.5</b>	<b>j5.0</b>	<b>0</b>
	<b>3</b>	<b>0</b>	<b>j5.0</b>	<b>- j9.0</b>	<b>j4.0</b>
	<b>4</b>	<b>0</b>	<b>0</b>	<b>j4.0</b>	<b>- j6.0</b>

**Add element 6:**

		<div>1</div>	<div>2</div>	<div>3</div>	<div>4</div>
<b>Y<sub>bus</sub> =</b>	<div>1</div>	<b>- j6.6667</b>	<b>j5.0</b>	<b>0</b>	<b>0</b>
	<div>2</div>	<b>j5.0</b>	<b>- j12.5</b>	<b>j5.0</b>	<b>0</b>
	<div>3</div>	<b>0</b>	<b>j5.0</b>	<b>- j9.0</b>	<b>j4.0</b>
	<div>4</div>	<b>0</b>	<b>0</b>	<b>j4.0</b>	<b>- j6.0</b>

**Add element 7: Final bus admittance matrix**

		<div>1</div>	<div>2</div>	<div>3</div>	<div>4</div>
<b>Y<sub>bus</sub> =</b>	<div>1</div>	<b>- j6.6667</b>	<b>j5.0</b>	<b>0</b>	<b>0</b>
	<div>2</div>	<b>j5.0</b>	<b>- j14.5</b>	<b>j5.0</b>	<b>j2.0</b>
	<div>3</div>	<b>0</b>	<b>j5.0</b>	<b>- j9.0</b>	<b>j4.0</b>
	<div>4</div>	<b>0</b>	<b>j2.0</b>	<b>j4.0</b>	<b>- j8.0</b>

### Summary of methods to compute bus admittance matrix

When there is **no mutual couplings**  $Y_{bus}$  can be calculated by any one of the following methods.

i) Use  $Y_{bus} = A y A^T$

ii) Add one element at a time and modify  $Y_{bus}$ .

iii) Use the rules

- The diagonal element  $Y_{ii}$  equals the sum of the admittances directly connected to bus i.
- The off-diagonal element  $Y_{ij}$  equals the negative of the admittance connected between buses i and j. If there is no element between buses i and j, then  $Y_{ij}$  equals to zero.

When there is **one or more coupled groups**  $Y_{bus}$  can be calculated by any one of the following methods.

1. Use  $Y_{bus} = A y A^T$

2. Find bus admittance matrix of coupled groups using the corresponding bus incidence matrix and primitive admittance matrix. For the uncoupled portion, get the bus admittance matrix using any one of i), ii) and iii) mentioned above and finally add the corresponding elements.

## REPRESENTATION OF OFF-NOMINAL TAP SETTING TRANSFORMER

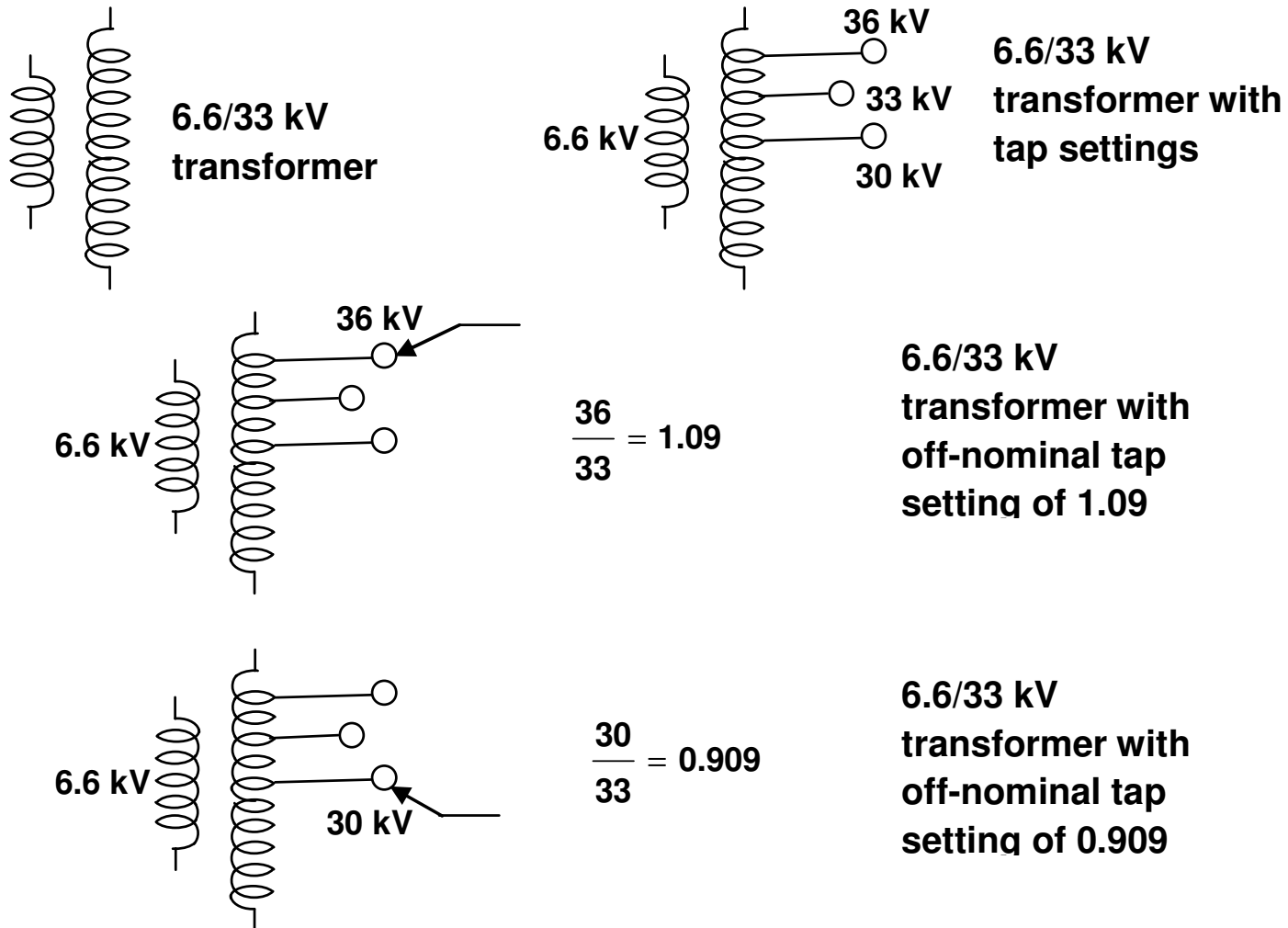


Fig. 1.18 Off-nominal tap setting transformer

Transformers with off-nominal tap settings can be modelled as the series combination of auto-transformer with transformer impedance as shown in Figure 1.19. The two transformer terminals k and m are commonly designated as the tap side and impedance side bus respectively.

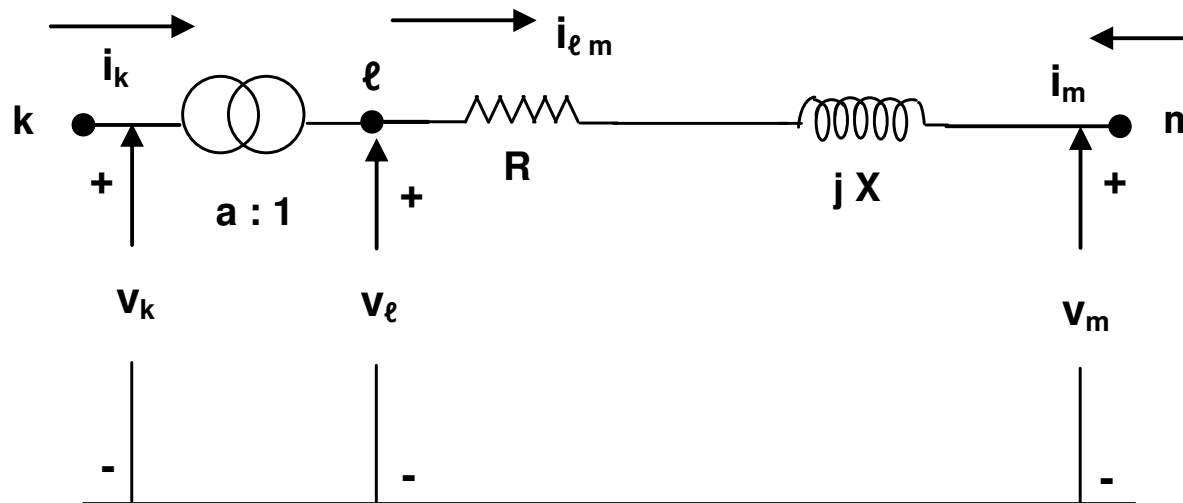
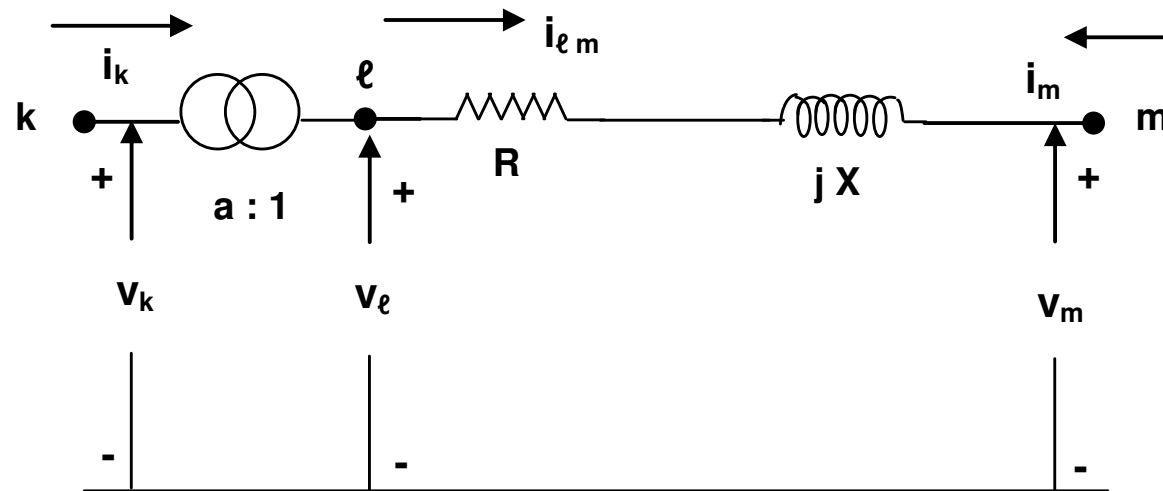
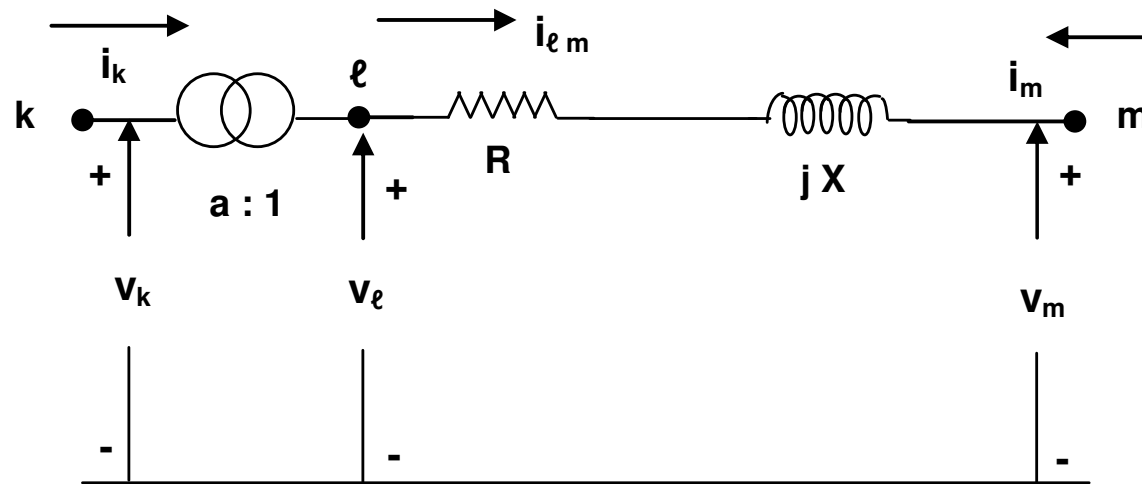


Fig. 1.19 Equivalent circuit for an off-nominal tap setting transformer



The nodal equations of the two port circuit of Fig. 1.19 can be derived by first expressing the current flows  $i_{l m}$  and  $i_m$  at each end of the series branch  $R + jX$  in terms of the terminal voltages  $v_l$  and  $v_m$ . Denoting the **admittance of this branch  $l$ - $m$  by  $y$** , the terminal current injection will be given by:

$$\begin{bmatrix} i_{l m} \\ i_m \end{bmatrix} = \begin{bmatrix} y & -y \\ -y & y \end{bmatrix} \begin{bmatrix} v_l \\ v_m \end{bmatrix} \quad (1.32)$$



$$\begin{bmatrix} i_{lm} \\ i_m \end{bmatrix} = \begin{bmatrix} y & -y \\ -y & y \end{bmatrix} \begin{bmatrix} v_l \\ v_m \end{bmatrix} \quad (1.32)$$

Knowing that  $v_k / v_l = a$  and  $i_{lm} / i_k = a$ , substituting for  $i_{lm}$  and  $v_l$  as

$$i_{lm} = a i_k; \quad v_l = v_k / a \quad \text{we get} \quad \begin{bmatrix} a i_k \\ i_m \end{bmatrix} = \begin{bmatrix} y & -y \\ -y & y \end{bmatrix} \begin{bmatrix} v_k / a \\ v_m \end{bmatrix}$$

The final form will be obtained as



$$\begin{bmatrix} \mathbf{i}_{\ell m} \\ \mathbf{i}_m \end{bmatrix} = \begin{bmatrix} \mathbf{y} & -\mathbf{y} \\ -\mathbf{y} & \mathbf{y} \end{bmatrix} \begin{bmatrix} \mathbf{v}_{\ell} \\ \mathbf{v}_m \end{bmatrix} \quad (1.32)$$

Knowing that  $\mathbf{v}_k / \mathbf{v}_{\ell} = \mathbf{a}$  and  $\mathbf{i}_{\ell m} / \mathbf{i}_k = \mathbf{a}$ , substituting for  $\mathbf{i}_{\ell m}$  and  $\mathbf{v}_{\ell}$  as

$$\mathbf{i}_{\ell m} = \mathbf{a} \mathbf{i}_k; \quad \mathbf{v}_{\ell} = \mathbf{v}_k / \mathbf{a} \quad \text{we get} \quad \begin{bmatrix} \mathbf{a} \mathbf{i}_k \\ \mathbf{i}_m \end{bmatrix} = \begin{bmatrix} \mathbf{y} & -\mathbf{y} \\ -\mathbf{y} & \mathbf{y} \end{bmatrix} \begin{bmatrix} \mathbf{v}_k / \mathbf{a} \\ \mathbf{v}_m \end{bmatrix}$$

The final form will be obtained as

$$\begin{bmatrix} \mathbf{i}_k \\ \mathbf{i}_m \end{bmatrix} = \begin{bmatrix} \frac{\mathbf{y}}{\mathbf{a}^2} & -\frac{\mathbf{y}}{\mathbf{a}} \\ -\frac{\mathbf{y}}{\mathbf{a}} & \mathbf{y} \end{bmatrix} \begin{bmatrix} \mathbf{v}_k \\ \mathbf{v}_m \end{bmatrix} \quad (1.33)$$

### Example 1.9

Consider the 4-bus power system whose one-line diagram is given in Fig. 1.20

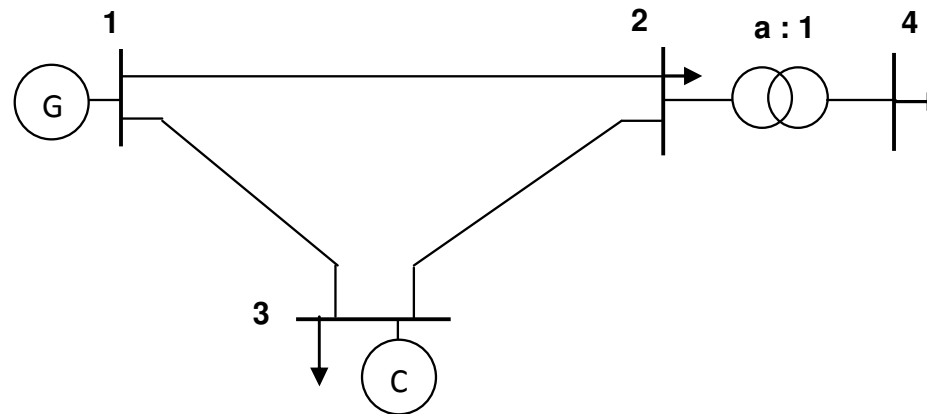


Fig. 1.20 One-line diagram of a 4-bus power system

Network data are listed below. Shunt capacitor of susceptance of  $j0.5$  pu is connected at bus 3.

From Bus	To Bus	R pu	X pu	Total line charging suscep.	Tap a	Tap side bus
1	2	0.02	0.06	0.2	---	---
1	3	0.02	0.06	0.25	---	---
2	3	0.05	0.10	0.00	---	---
2	4	0.00	0.08	0.00	0.98	2

Form the admittance matrix,  $Y$  for the entire system.

## Solution

The nodal equations for the transformer branch will be obtained by substituting for  $y$  and  $a$  in Equation (1.33):

$$\begin{bmatrix} \overset{\textcircled{2}}{i_2} \\ \overset{\textcircled{4}}{i_4} \end{bmatrix} = \begin{bmatrix} \overset{\textcircled{2}}{-j13.02} & \overset{\textcircled{4}}{j12.75} \\ \overset{\textcircled{4}}{j12.75} & -j12.50 \end{bmatrix} \begin{bmatrix} v_2 \\ v_4 \end{bmatrix}$$

Bus admittance matrix for the entire system can be obtained by including one branch at a time and expanding the admittance matrix to a 4x4 matrix:

$$Y_{\text{bus}} = \begin{array}{c} \begin{matrix} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \\ \textcircled{4} \end{matrix} \end{array} \begin{array}{c} \begin{matrix} \textcircled{1} & \textcircled{2} & \textcircled{3} & \textcircled{4} \end{matrix} \\ \begin{array}{|c|c|c|c|} \hline 10.00 - j29.77 & -5.00 + j15.00 & -5.00 + j15.00 & 0 \\ \hline -5.00 + j15.00 & 9.00 - j35.91 & -4.00 + j8.00 & j12.5 \\ \hline -5.00 + j15.00 & -4.00 + j8.00 & 9.00 - j22.37 & 0 \\ \hline 0 & j12.5 & 0 & -j12.50 \\ \hline \end{array} \end{array}$$

### Example 1.10

Using Matlab program obtain the bus admittance matrix of the transmission system with the following data.

#### Line data

Line No.	Between buses	Line Impedance	HLCA	Off nominal turns ratio
1	1 – 4	$0.08 + j 0.32$	$j 0.01$	---
2	1 – 6	$0.125 + j 0.5$	$j 0.015$	---
3	2 – 3	$0.8 + j 1.2$	0	---
4	2 – 5	$0.25 + j 0.75$	0	---
5	4 – 3	$j 0.125$	0	0.95
6	4 – 6	$0.1 + j 0.4$	$j 0.075$	---
7	6 – 5	$j 0.25$	0	1.05

#### Shunt capacitor data

Bus No. 4

Admittance  $j 0.05$

```
dip(['*****Formation of YBUS matrix *****'])
Nbus = 6; Nele = 7; Nsh = 1;
%
% Reading element data
%
Edata = [1 1 4 0.08+0.32i 0.01i 1;2 1 6 0.125+0.5i 0.015i 1;3 2 3 0.8+1.2i 0 1;...
         4 2 5 0.25+0.75i 0 1;5 4 3 0+0.125i 0 0.95;6 4 6 0.1+0.4i 0.075i 1;7 6 5 0+0.25i 0 1.05];
%
% Reading shunt data
%
Shdata = [1 4 0.05i];
```

```

%
% Displaying data
%
disp(['  Sl.No      From bus      To bus      Line Impedance      HLCA      ONTR'])
Edata
if Nsh~=0
    disp(['  Sl.No.      At bus      Shunt Admittance'])
    Shdata
end
%
% Formation of Ybus matrix
%
Ybus = zeros(Nbus,Nbus);

```

```

for k = 1:Nele
    p = Edata(k,2);
    q = Edata(k,3);
    yele = 1/Edata(k,4);
    Hlca = Edata(k,5);
    offa = Edata(k,6);
    offaa = offa*offa;
    Ybus(p,p) = Ybus(p,p) + yele/offaa + Hlca;
    Ybus(q,q) = Ybus(q,q) + yele + Hlca;
    Ybus(p,q) = Ybus(p,q) - yele/offa;
    Ybus(q,p) = Ybus(q,p) - yele/offa;
end
if Nsh~=0
    for i = 1:Nsh
        q = Shdata(i,2);
        yele = Shdata(i,3);
        Ybus(q,q) = Ybus(q,q) + yele;
    end
end
Ybus

```

\*\*\*\*\*Formation of YBUS matrix\*\*\*\*\*

Sl.No	From bus	To bus	Line Impedance	HLCA	ONTR
-------	----------	--------	----------------	------	------

Edata =

1.0000	1.0000	4.0000	$0.0800 + 0.3200i$	$0 + 0.0100i$	1.0000
2.0000	1.0000	6.0000	$0.1250 + 0.5000i$	$0 + 0.0150i$	1.0000
3.0000	2.0000	3.0000	$0.8000 + 1.2000i$	0	1.0000
4.0000	2.0000	5.0000	$0.2500 + 0.7500i$	0	1.0000
5.0000	4.0000	3.0000	$0 + 0.1250i$	0	0.9500
6.0000	4.0000	6.0000	$0.1000 + 0.4000i$	$0 + 0.0750i$	1.0000
7.0000	6.0000	5.0000	$0 + 0.2500i$	0	1.0500

Sl.No.	At bus	Shunt Admittance
--------	--------	------------------

Shdata =

1.0000	4.0000	$0 + 0.0500i$
--------	--------	---------------



Ybus =

$$\begin{bmatrix} 1.2059 - 4.7985i & 0 & 0 & -0.7353 + 2.9412i & 0 & -0.4706 + 1.8824i \\ 0 & 0.7846 - 1.7769i & -0.3846 + 0.5769i & 0 & -0.4000 + 1.2000i & 0 \\ 0 & -0.3846 + 0.5769i & 0.3846 - 8.5769i & 0 + 8.4211i & 0 & 0 \\ -0.7353 + 2.9412i & 0 & 0 + 8.4211i & 1.3235 - 14.0234i & 0 & -0.5882 + 2.3529i \\ 0 & -0.4000 + 1.2000i & 0 & 0 & 0.4000 - 5.2000i & 0 + 3.8095i \\ -0.4706 + 1.8824i & 0 & 0 & -0.5882 + 2.3529i & 0 + 3.8095i & 1.0588 - 7.7734i \end{bmatrix}$$