UNIT 3 SYMMETRICAL FAULT ANALYSIS

Introduction

Short circuit study is one of the basic power system analysis problems. It is also known as fault analysis. When a fault occurs in a power system, bus voltages reduces and large current flows in the lines. This may cause damage to the equipments. Hence faulty section should be isolated from the rest of the network immediately on the occurrence of a fault. This can be achieved by providing relays and circuit breakers.

The calculation of currents in network elements for different types of faults occurring at different locations is called SHORT CIRCUIT STUDY. The results obtained from the short circuit study are used to find the relay settings and the circuit breaker ratings which are essential for power system protection.

Symmetrical short circuit on Synchronous Machine

The selection of a circuit breaker for a power system depends not only upon the current the breaker is to carry under normal operating conditions but also upon the maximum current it may have to carry momentarily and the current it may have to interrupt at the voltage of the line in which it is placed. In order to approach the problem of calculating the initial current we need to study the behaviour of a synchronous generator when it is short circuited.

When an ac voltage is applied suddenly across a series R-L circuit, the current which flows has two components 1. a steady state sinusoidally varying component of constant amplitude and 2. a non-periodic and exponentially decaying with a time constant of L/R. (which is also referred as the dc component current). The initial value of the dc component of current depends on the magnitude of the ac voltage when the circuit is closed.

A similar but more complex phenomenon occurs when a short circuit occurs suddenly across the terminals of a synchronous machine. A good way to analyze the effect of a three-phase short circuit at the terminals of a previously unloaded generator is to take an oscillogram of the current in one phase upon the occurrence of such fault. Since the voltages generated in the phases of a three-phase machine are displaced 120 electrical degrees from each other, the short circuit occurs at different points on the voltage wave of each phase. For this reason the unidirectional or dc transient component of current is different in each phase. If the dc component of current is eliminated from the current of each phase, the short circuit current plotted versus time will be as shown in Fig. 3.1

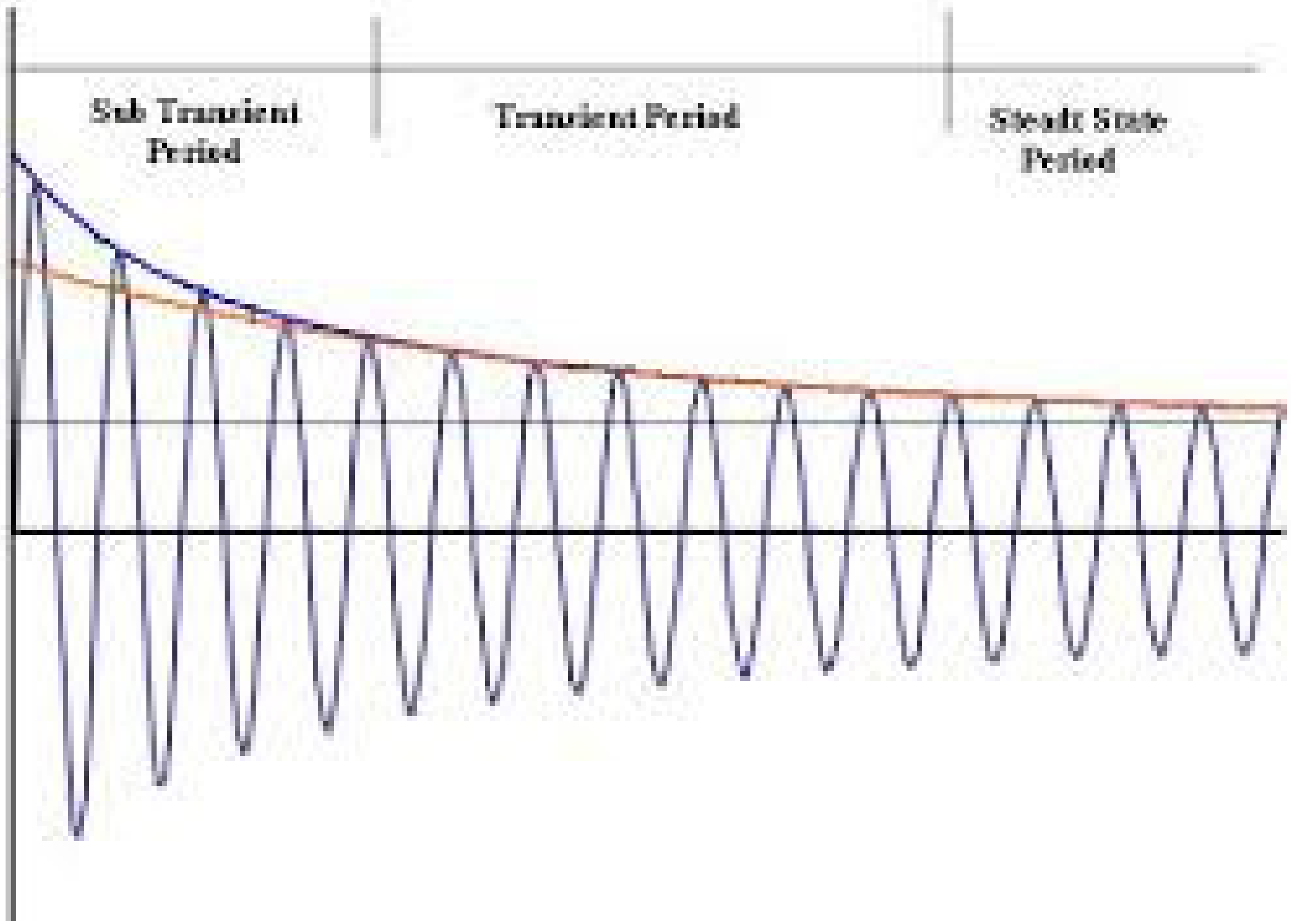


Fig. 3.1 Short circuit current of the synchronous generator

In the synchronous generator, generally the reduction of the air gap flux is caused by the mmf of the current in the armature which is known as the armature reaction effect. At the instant prior to short circuit, the no load armature current is very small resulting negligible armature reaction effect and maximum air gap flux. When there is a sudden increase of stator current on short circuit, the air gap flux cannot change instantaneously due to eddy currents flowing in the rotor and damper circuits, which oppose this change. Since, the stator mmf is unable to establish any armature reaction, the reactance due to armature reaction is negligible and the initial reactance is very small and almost equal to armature leakage reactance alone. This results in very large initial current as seen from Fig. 3.1. This period is referred as subtransient period.

After a few cycles, the eddy current in the damper circuit and eventually in the field circuit decays to some extent and the air gap flux reduces due to partial armature reaction effect resulting in reduction in short circuit current as seen in Fig. 3.1 Now the machine said to function in the transient period.

After another few cycles, the eddy current in the damper circuit fully decays allowing reduction in air gap flux due to armature reaction effect. Now the steady state condition is reached as seen from Fig. 3.1.

The rms value of the current determined by the intercept of the current envelope with zero time is called the subtransient current $|I''|$. Direct axis subtransient reactance X_d'' is $|E_g| / |I''|$ where $|E_g|$ is the rms phase voltage at no load.

The rms value of the current determined by the intercept of the current envelope leaving first few cycles with zero time is called the transient current $|I'|$. Direct axis transient reactance X_d' is $|E_g| / |I'|$.

The rms value of the steady state short circuit current is |I|. Direct axis reactance X_d is $|E_g| / |I|$.

The currents and reactances discussed above are defined by the following equations.

$$|I| = \frac{0a}{\sqrt{2}} = \frac{|E_g|}{X_d} \quad (3.1)$$

$$|I'| = \frac{0b}{\sqrt{2}} = \frac{|E_g|}{X_d'} \quad (3.2)$$

$$|I''| = \frac{0c}{\sqrt{2}} = \frac{|E_g|}{X_d''} \quad (3.3)$$

The subtransient current $|I''|$ is much larger than the steady state current $|I|$ because the decrease in air gap flux caused by the armature current cannot take place immediately. So large voltage is induces in the armature winding just after the fault occurs than exists after steady state is reached. However, we account for the difference in induced voltage by using different reactances in series with the no load voltage E_g to calculate currents for subtransient, transient, and steady state conditions.

Equations (3.1) to (3.3) indicate the method of determining fault current in a generator when its reactances are known. If the generator is unloaded when the fault occurs, the machine is represented by E_g in series with the proper reactance.

The resistance is taken into account if greater accuracy is desired. If there is impedance external to the generator between its terminals and the short circuit, the external impedance must be included in the circuit.

Example 3.1

Two synchronous generators are connected in parallel at the low voltage side of a three-phase Δ-Y transformer as shown in Fig. 3.2. Machine 1 is rated 50 MVA, 13.8 kV. Machine 2 is rated 25 MVA, 13.8 kV. Each generator has subtransient reactance, transient reactance and direct axis synchronous reactance of 25%, 40% and 100% respectively. The transformer is rated 75 MVA, 13.8Δ/69Y with a reactance of 10%. Before the fault occurs, the voltage on high voltage side of the transformer is 66 kV. The transformer is unloaded and there is no circulating current between the generators.

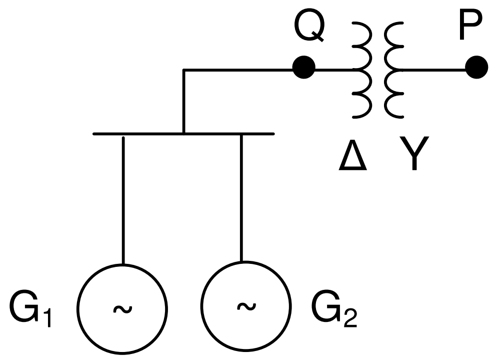


Fig. 3.2 One-line diagram for Example 3.1

(a) Find the current supplied by the generators.

(b) A three-phase short circuit occurs at P. Determine the subtransient, transient and steady state short circuit current in each generator.

(c) A three-phase short circuit occurs at Q. Determine the subtransient, transient and steady state short circuit current in each generator.

Select a base of 75 MVA and 69 kV in the high tension circuit.

Summarize the results in a tabular form.

Solution

Base voltage at the low tension circuit = 13.8 kV

Prefault voltage at the LV side = $\frac{13.8}{69}$ X 66 = 13.2 kV

Base current at the LV side = $\frac{75 \times 10^3}{\sqrt{3} \times 13.8}$ = 3137.77 amp.

On the selected base

Generator 1: $X_d'' = 0.25 \times \frac{75}{50} = 0.375$ p.u. $X_d' = 0.4 \times \frac{75}{50} = 0.6$ p.u.

$X_d = 1.0 \times \frac{75}{50} = 1.5$ p.u. $E_{g1} = \frac{13.2}{13.8} = 0.9565$ p.u.

Generator 2: $X_d'' = 0.25 \times \frac{75}{25} = 0.75$ p.u. $X_d' = 0.4 \times \frac{75}{25} = 1.2$ p.u.

$X_d = 1.0 \times \frac{75}{25} = 3.0$ p.u. $E_{g2} = \frac{13.2}{13.8} = 0.9565$ p.u.

Transformer:

$X = 0.1$ **p.u.**

(a) Transformer is unloaded. Therefore

$I_{g1} = I_{g2} = 0$

(b) per unit subtransient reactance diagram is shown in Fig. 3.3.

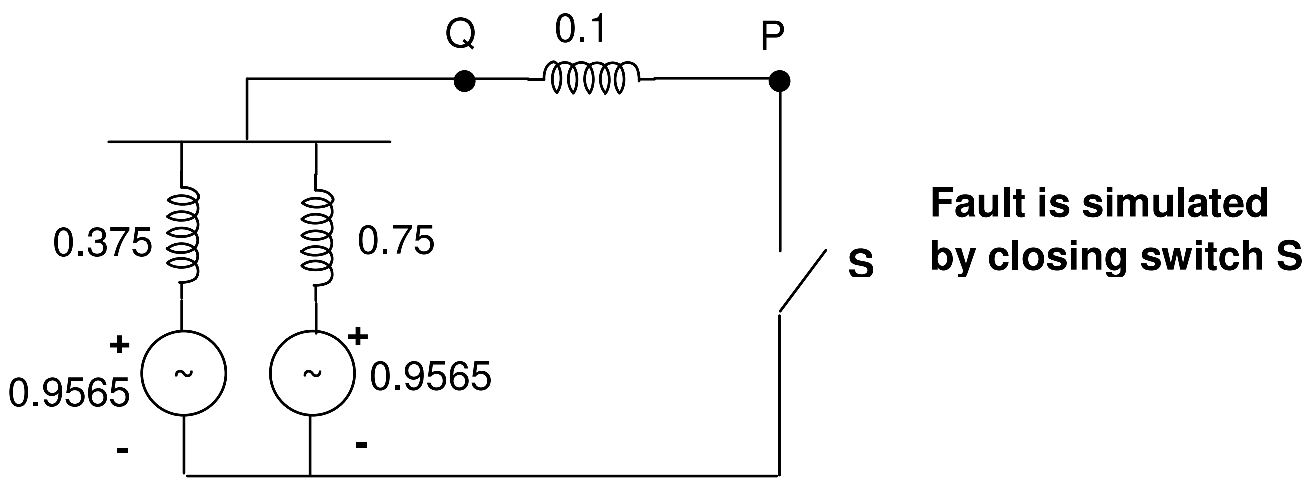


Fig. 3.3 per unit subtransient reactance diagram

Using Thevenin's equivalent above reactance diagram for the faulted condition can be reduced as shown in Fig. 3.4.

$$\frac{0.375 \times 0.75}{0.375 + 0.75} = 0.25 \text{ p.u.}$$

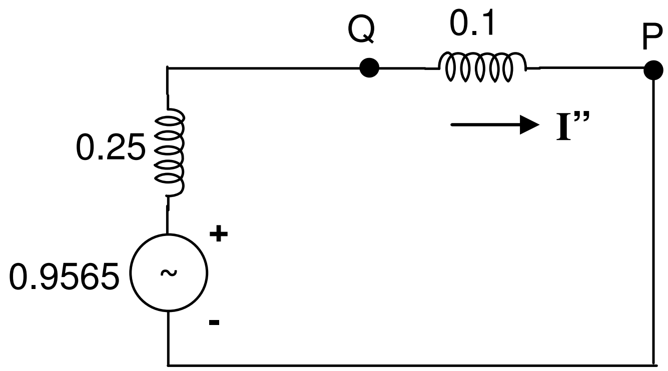


Fig. 3.4 Reduced per unit subtransient reactance diagram

Subtransient current $I'' = \frac{0.9565}{j0.35} = -j\,2.7329$ **p.u.**

Voltage at Q = j 0.1 x (- j 2.7329) = 0.27329 p.u.

Referring to Fig. 3.3

Current supplied by generator 1 $= \frac{0.9565 - 0.27329}{j0.375} = -j\,1.8219$ **p.u.**

Subtransient current in machine 1 $|I_1''| = 5716.7$ **A**

Current supplied by generator 2 $= \frac{0.9565 - 0.27329}{j0.75} = -j\,0.9109$ **p.u.**

Subtransient current in machine 2 $|I_2''| = 2858.3$ **A**

per unit transient reactance diagram is shown in Fig. 3.5.

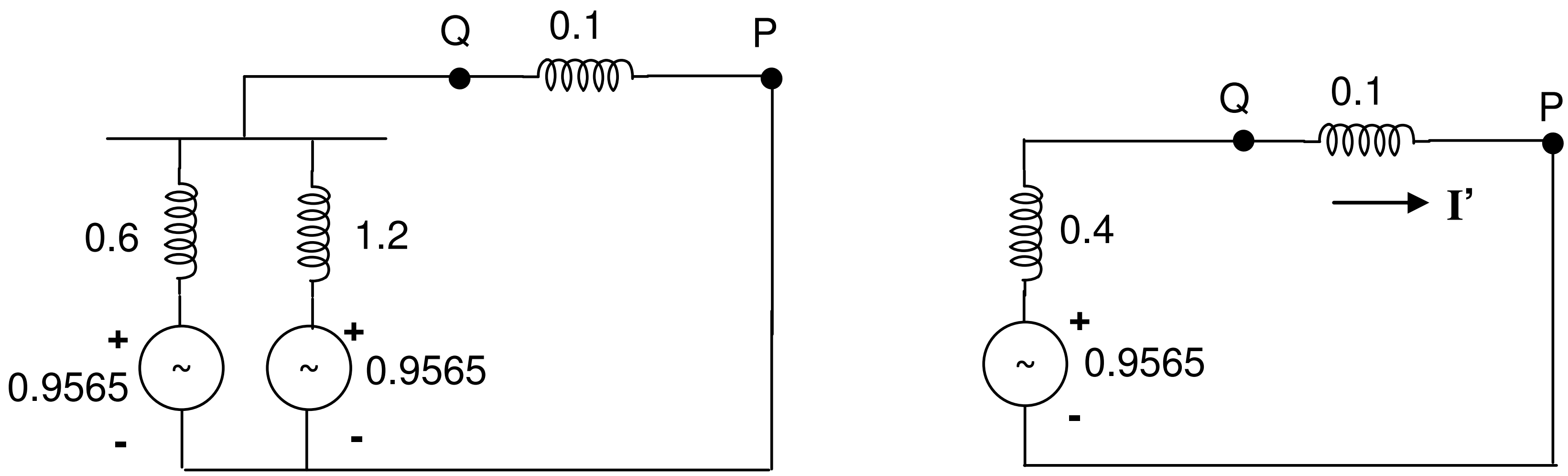


Fig. 3.5 per unit transient reactance diagram

Subtransient current $I' = \dfrac{0.9565}{j0.5} = -\,j\,1.913$ **p.u.**

Voltage at Q = j 0.1 x (- j 1.913) = 0.1913 p.u.

Referring to Fig. 3.5

Current supplied by generator 1 $= \dfrac{0.9565 - 0.1913}{j0.6} = -\,j\,1.275$ **p.u.**

Transient current in machine 1 $|I_1'| = 4001.7$ **A**

Current supplied by generator 2 $= \dfrac{0.9565 - 0.1913}{j1.2} = -\,j\,0.6377$ **p.u.**

Transient current in machine 2 $|I_2'| = 2000.9$ **A**

per unit direct axis reactance diagram is shown in Fig. 3.6.

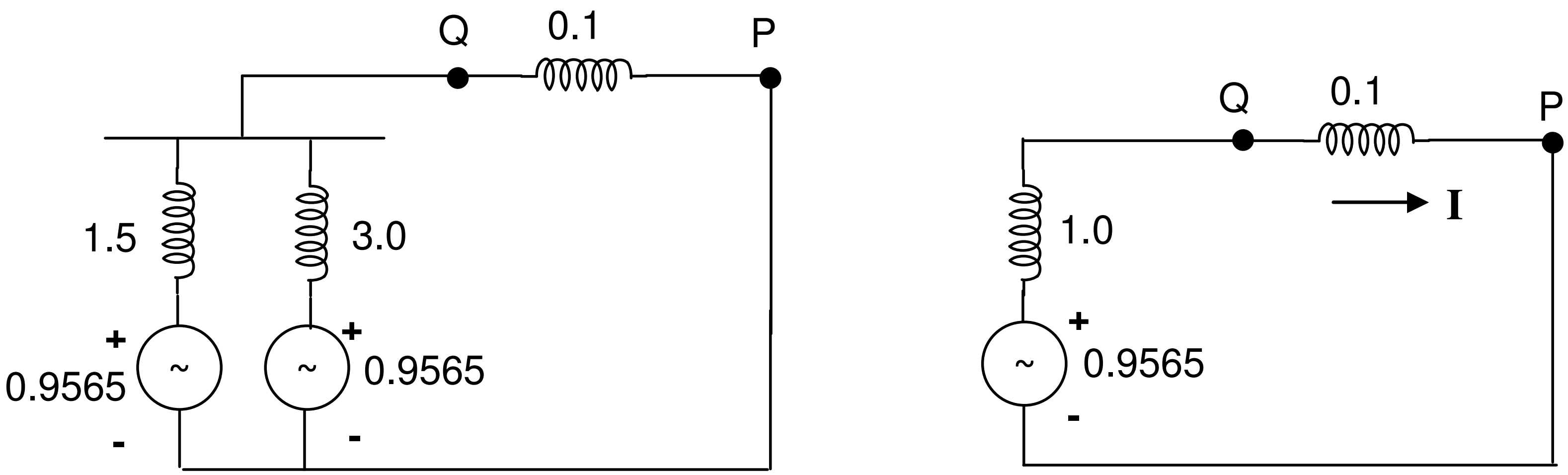


Fig. 3.6 per unit direct axis reactance diagram

Steady state short circuit current $I = \dfrac{0.9565}{j1.1} = -j\,0.8695$ p.u.

Voltage at Q = $j\,0.1 \times (-j\,0.8695) = 0.08695$ p.u.

Referring to Fig. 3.6

Current supplied by generator 1 $= \dfrac{0.9565 - 0.08695}{j1.5} = -j\,0.5797$ p.u.

Steady state short circuit current in machine 1 $|I_1| = 1819$ A

Current supplied by generator 2 $= \dfrac{0.9565 - 0.08695}{j3.0} = -j\,0.2899$ p.u.

Steady state short circuit current in machine 2 $|I_2| = 909.48$ A

(b) Fault occurs at point Q.

per unit subtransient reactance diagram is shown in Fig. 3.7.

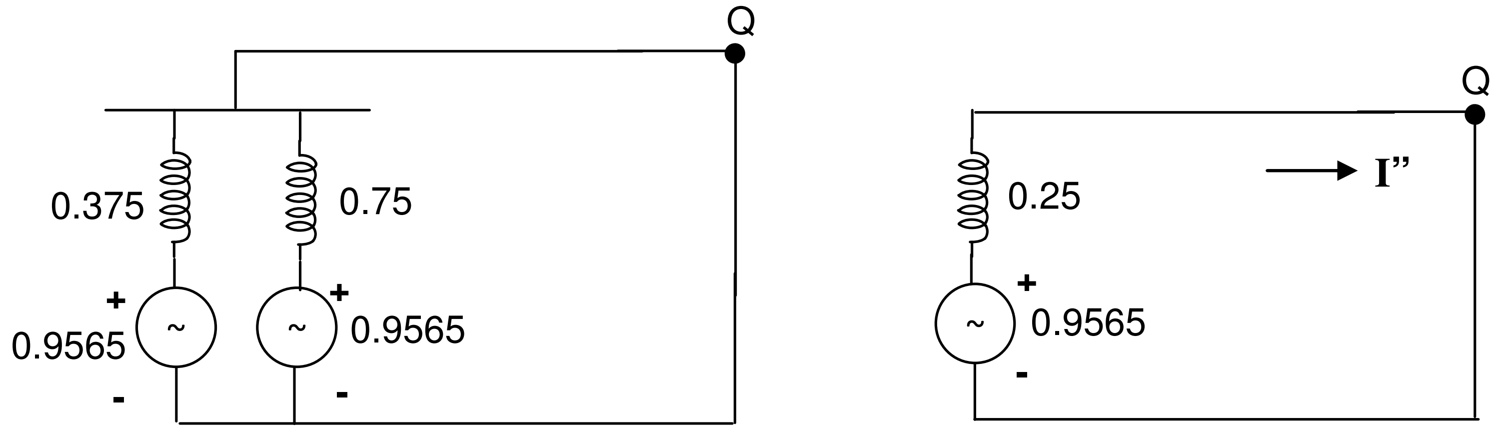


Fig. 3.7 per unit subtransient reactance diagram

Subtransient current $I'' = \dfrac{0.9565}{j0.25} = -\,j\,3.826$ **p.u.**

Current supplied by generator 1 $= \dfrac{0.9565 - 0}{j0.375} = -\,j\,2.5507$ **p.u.**

Subtransient current in machine 1 $|I_1''| = 8003.4$ **A**

Current supplied by generator 2 $= \dfrac{0.9565 - 0}{j0.75} = -\,j\,1.2753$ **p.u.**

Subtransient current in machine 2 $|I_2''| = 4001.7$ **A**

per unit transient reactance diagram is shown in Fig. 3.8.

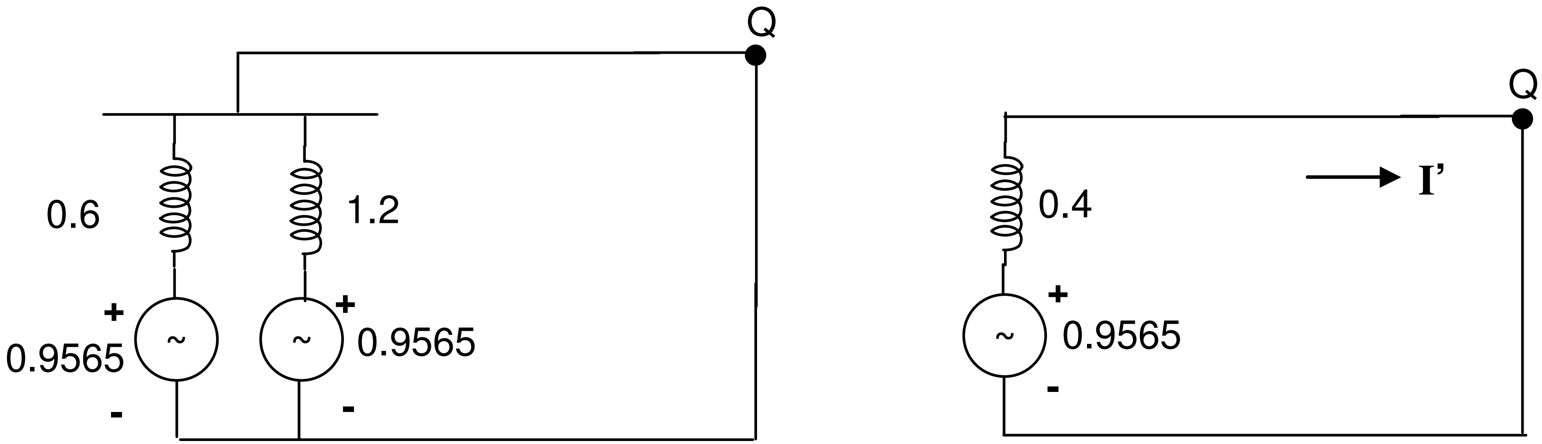


Fig. 3.8 per unit transient reactance diagram

Transient current $I' = \dfrac{0.9565}{j0.4} = -\,j\,2.3913$ p.u.

Current supplied by generator 1 $= \dfrac{0.9565 - 0}{j0.6} = -\,j\,1.5942$ p.u.

Transient current in machine 1 $|I_1'| = 5002.2$ A

Current supplied by generator 2 $= \dfrac{0.9565 - 0}{j1.2} = -\,j\,0.7971$ p.u.

Subtransient current in machine 2 $|I_2'| = 2501.1$ A per unit direct axis transient reactance diagram is shown in Fig. 3.9.

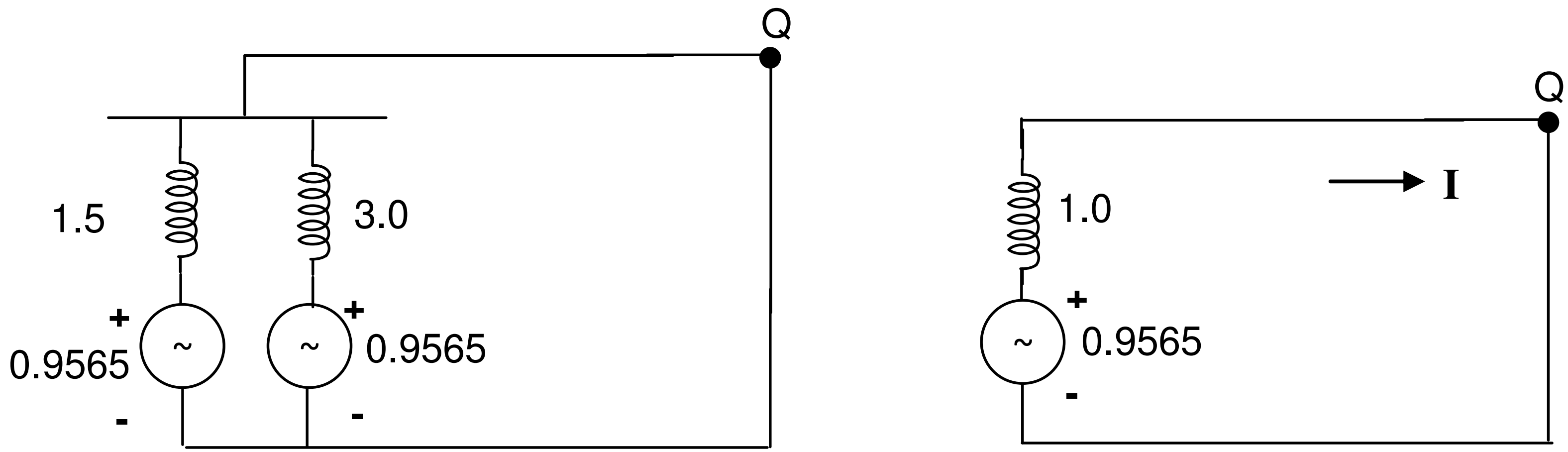


Fig. 3.9 per unit direct axis transient reactance diagram

Direct axis steady state short circuit current $I = \dfrac{0.9565}{j1.0} = -\,j\,0.9565$ **p.u.**

Current supplied by generator 1 $= \dfrac{0.9565 - 0}{j1.5} = -\,j\,0.6377$ **p.u.**

Steady state short current in machine 1 $|I_1| = 2000.9$ **A**

Current supplied by generator 2 $= \dfrac{0.9565 - 0}{j3.0} = -\,j\,0.3188$ **p.u.**

Steady state short circuit current in machine 2 $|I_2| = 1000.4$ **A**

Summary of results obtained:

In the prefault condition, since the transformer is not loaded $I_{g1} = I_{g2} = 0$

Fault occurs at the HV side of the transformer:

Subtransient		Transient		Steady state	
$\lvert I_1'' \rvert$	$\lvert I_2'' \rvert$	$\lvert I_1' \rvert$	$\lvert I_2' \rvert$	$\lvert I_1 \rvert$	$\lvert I_2 \rvert$
5717 A	2858 A	4002 A	2001 A	1819 A	909 A

Fault occurs at the LV side of the transformer i.e. at the generator terminals:

Subtransient		Transient		Steady state	
$\lvert I_1'' \rvert$	$\lvert I_2'' \rvert$	$\lvert I_1' \rvert$	$\lvert I_2' \rvert$	$\lvert I_1 \rvert$	$\lvert I_2 \rvert$
8003 A	4002 A	5002 A	2501 A	2001 A	1000 A

Bus Impedance matrix building algorithm

Bus impedance matrix Z_{bus} of a power network can be obtained by inverting the bus admittance matrix Y_{bus} , which is easy to construct. However, when the order of matrix is large, direct inversion requires more core storage and enormous computer time. Therefore inversion of Y_{bus} is prohibited for large size network.

Bus impedance matrix can be constructed by adding the network elements one after the other. Using impedance parameters, performance equations in bus frame of reference can be written as

$$E_{bus} = Z_{bus}\, I_{bus} \tag{3.4}$$

In the expanded form the above becomes

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} \tag{3.5}$$

$$
\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} \qquad (3.5)
$$

From this we can write

$$
E_p = Z_{p1}\, I_1 + Z_{p2}\, I_2 + \ldots\ldots. + Z_{pq}\, I_q + \ldots. + Z_{pN}\, I_N \qquad (3.6)
$$

From the above, it can be noted that with $I_q = 1$ p.u. other bus currents set to zero, $E_p = Z_{pq}$. Thus Z_{pq} can be obtained by measuring E_p when 1 p.u. current is injected at bus q and leaving the other bus currents as zero. In fact p and q can be varied from 1 to N.

While making measurements all the buses except one, are open circuited. Hence, the bus impedance parameters are called <u>open circuit impedances.</u> The diagonal elements in Z_{bus} are known as <u>driving point impedances</u>, while the off-diagonal elements are called <u>transfer impedances.</u>

While constructing Z_{bus} using building algorithm, elements are added one by one. At any stage, the added element may be a branch or a link as explained below.

Consider the sample power system shown in Fig. 3.10.

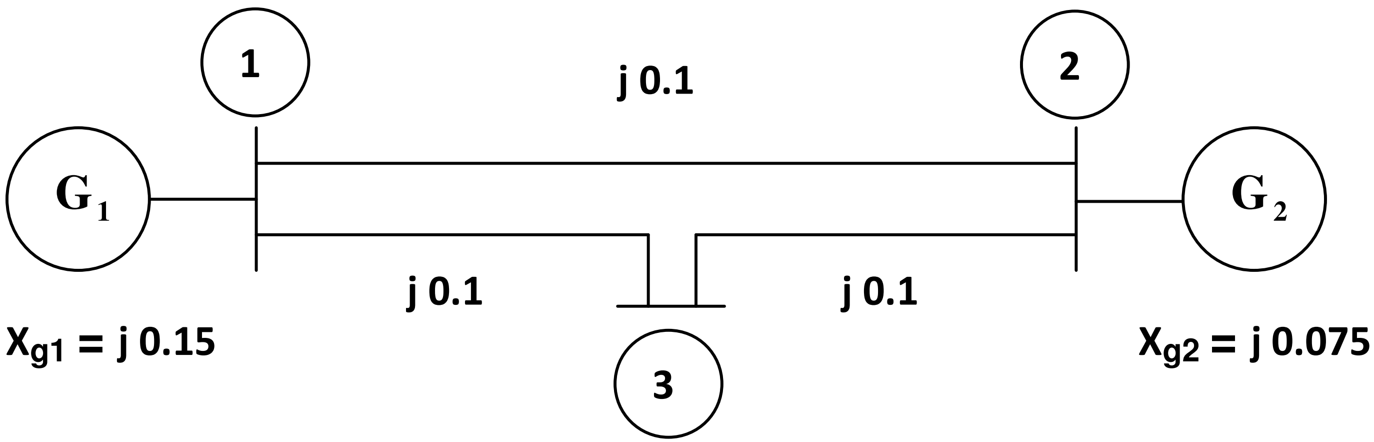


Fig. 3.10 Sample power system

The network graph of the power system is shown in Fig. 3.11.

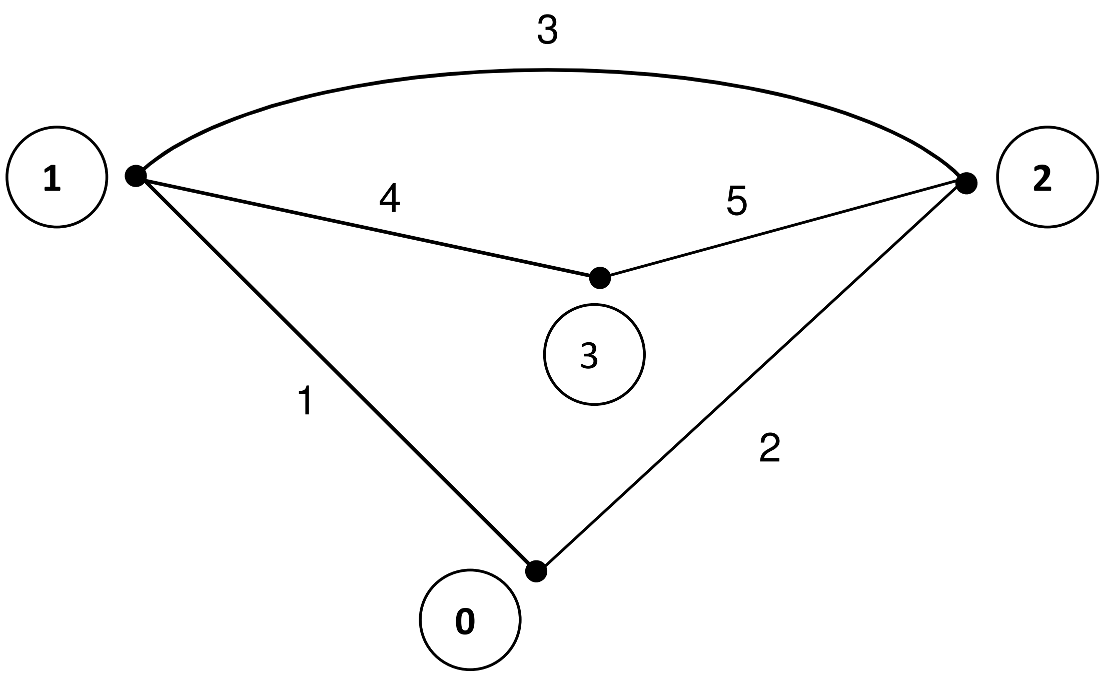


Fig. 3.11 Network graph of sample power system

The sub-graph consisting of elements 1, 2 and 3 corresponds to a partial network with buses 0, 1 and 2. In the partial network, if element 4 is added, resulting graph will be as shown in Fig. 3.12.

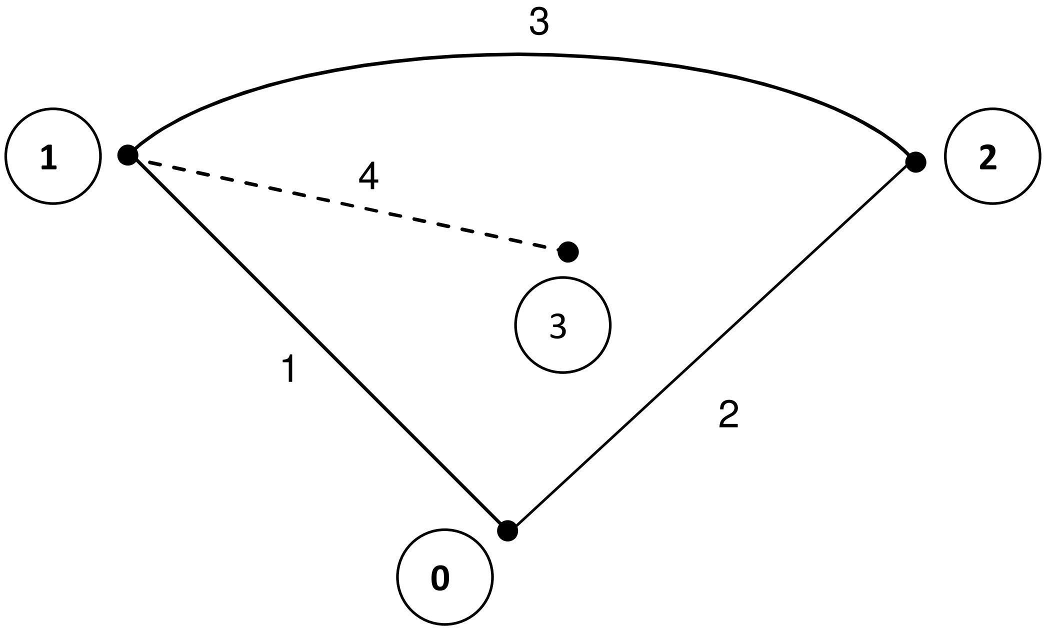


Fig. 3.12 Network graph of partial power system

Now a new bus 3 is created. The added element is a BRANCH. For the next step, network with elements 1,2,3 and 4 will be taken as partial network. This contains buses 0,1,2 and 3.

When element 5 is added to this, the network graph will be as shown in Fig. 3.13

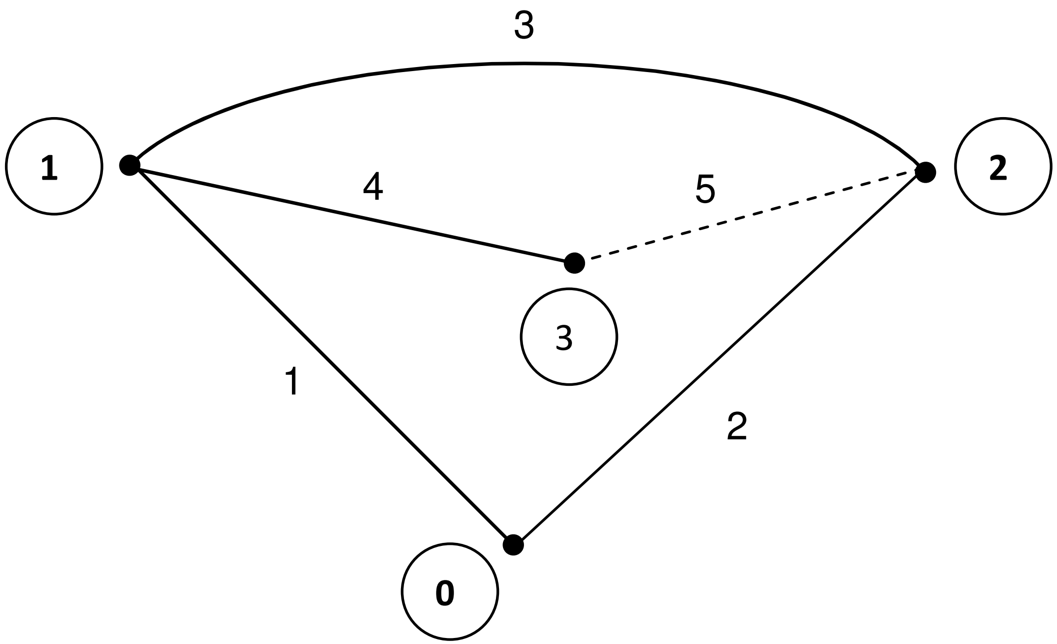


Fig. 3.13 Network graph with element 5 added

In this case, no new bus is created and the added element links buses 2 and 3 and hence it is called a LINK.

Assume that the bus impedance matrix Z_{bus}, for a partial network of m buses taking bus 0 as reference, is known.

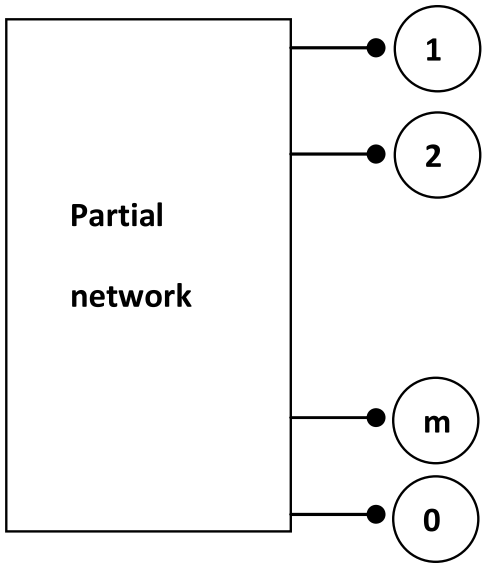


Fig. 3.14 Partial network

The performance equation of this network shown in Fig. 3.14 is

$$E_{bus} = Z_{bus}\ I_{bus} \qquad (3.7)$$

where

> E_{bus} **= an m x 1 vector of bus voltages measured with reference to the reference bus 0**
>
> I_{bus} **= an m x 1 vector of bus current**

When an element p-q is added to the partial network, it may be a branch or a link.

Addition of a branch

An element having an impedance of z_α is added from bus p, creating a new bus q as shown in Fig. 3.15.

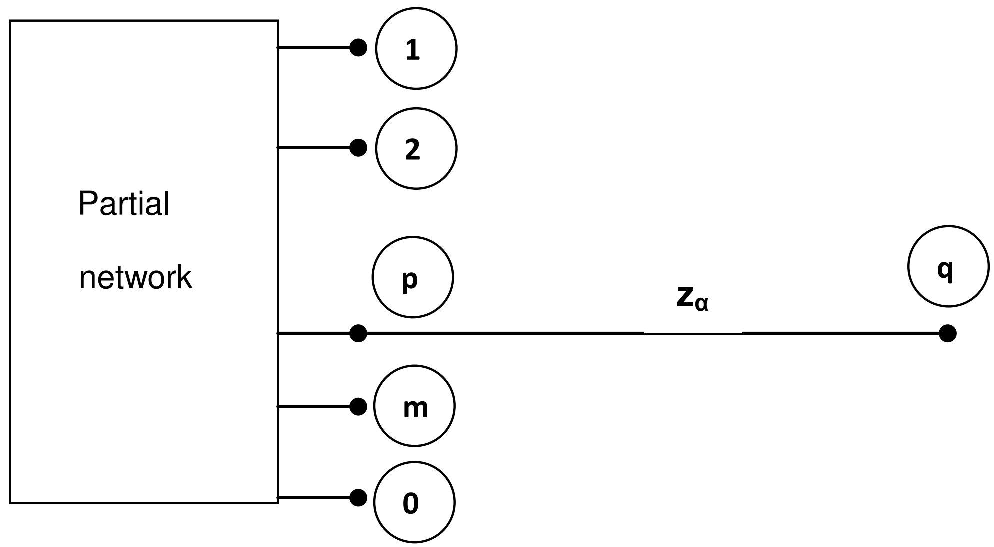


Fig. 3.15 Partial network with added branch

The performance equation of the new network will be

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_p \\ \vdots \\ E_m \\ \cdots \\ E_q \end{bmatrix} = \left[\begin{array}{cccccccc|c} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & & & Z_{1q} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & & & Z_{2q} \\ \vdots & \vdots & & \vdots & & \vdots & & & \vdots \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & & & Z_{pq} \\ \vdots & \vdots & & \vdots & & \vdots & & & \vdots \\ Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & & & Z_{mq} \\ \hline Z_{q1} & Z_{q2} & \cdots & Z_{qp} & \cdots & Z_{qm} & & & Z_{qq} \end{array}\right] \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ \cdots \\ I_q \end{bmatrix} \quad (3.8)$$

$$
\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_p \\ \vdots \\ E_m \\ \cdots \\ E_q \end{bmatrix} = \left[\begin{array}{cccccc:c} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1q} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2q} \\ \vdots & \vdots & & \vdots & & \vdots & \vdots \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pq} \\ \vdots & \vdots & & \vdots & & \vdots & \vdots \\ Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{mq} \\ \hdashline Z_{q1} & Z_{q2} & \cdots & Z_{qp} & \cdots & Z_{qm} & Z_{qq} \end{array}\right] \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ \cdots \\ I_q \end{bmatrix} \qquad (3.8)
$$

To obtain the elements in k^{th} column of Z_{bus} , $k = 1,....., m$; $k \neq q$, 1 p.u. current is injected at bus k , other bus currents are kept zero, the voltages are measured at buses 1,…., m. These values remain same as those in partial network. Hence it can be concluded that while finding the new bus impedance matrix, it is required to compute only the elements in the new row and column alone, other elements remain unaltered.

It is assumed that the network consists of bilateral passive elements. Then $Z_{iq} = Z_{qi}$ for $i = 1,2,….,m$.

To compute Z_{qi}

The element Z_{qi} can be determined by calculating the voltage at the q^{th} bus w.r.t. reference bus, by injecting 1 p.u. current at the i^{th} bus and keeping all other buses open circuited as shown in Fig. 3.16.

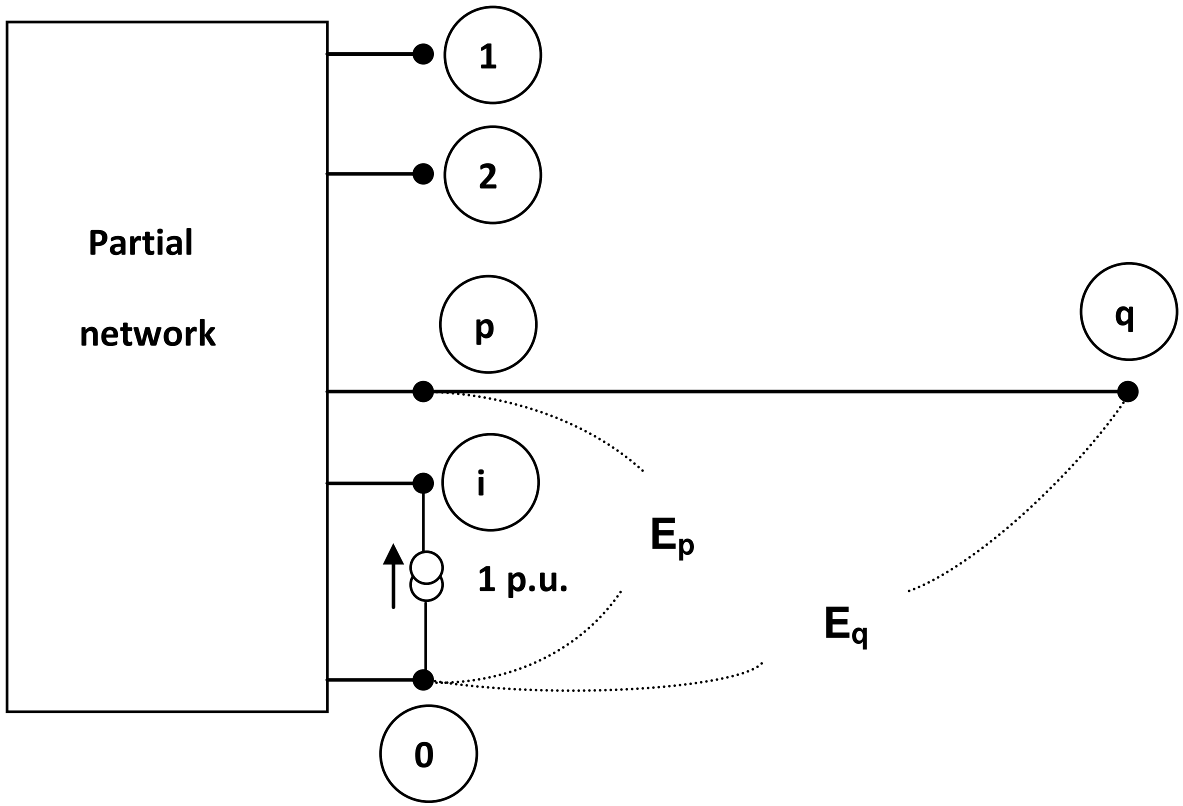


Fig. 3.16 Determining Z_{qi}

Now the bus currents are

$$\mathbf{I}_{bus} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1_i \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{3.9}$$

Corresponding to this bus current, from eqn. (3.8) we get

$$\begin{aligned} E_1 &= Z_{1i} \\ E_2 &= Z_{2i} \\ &\vdots \\ E_p &= Z_{pi} \\ &\vdots \\ E_m &= Z_{mi} \\ E_q &= Z_{qi} \end{aligned} \tag{3.10}$$

There is no current flow in the added element. It is also assumed that there is no mutual coupling between the added element and the elements in the partial network. Then

$$E_q = E_p \tag{3.11}$$

Using eqn. (3.11) in eqn. (3.10) we get $Z_{qi} = Z_{pi}$

Thus

$$Z_{qi} = Z_{pi} \quad \text{for } i = 1, 2, \ldots\ldots, m; \; i \neq q \tag{3.12}$$

If p happens to be reference bus, then $E_p = 0$ **and hence** $Z_{pi} = 0$ **.**

Thus

$$Z_{pi} = 0 \tag{3.13}$$

To compute Z_{qq}

The element Z_{qq} can be calculated by measuring the voltage at bus q due to current injection of 1 p.u. at bus q and keeping the other buses open circuited, as shown in Fig. 3.17.

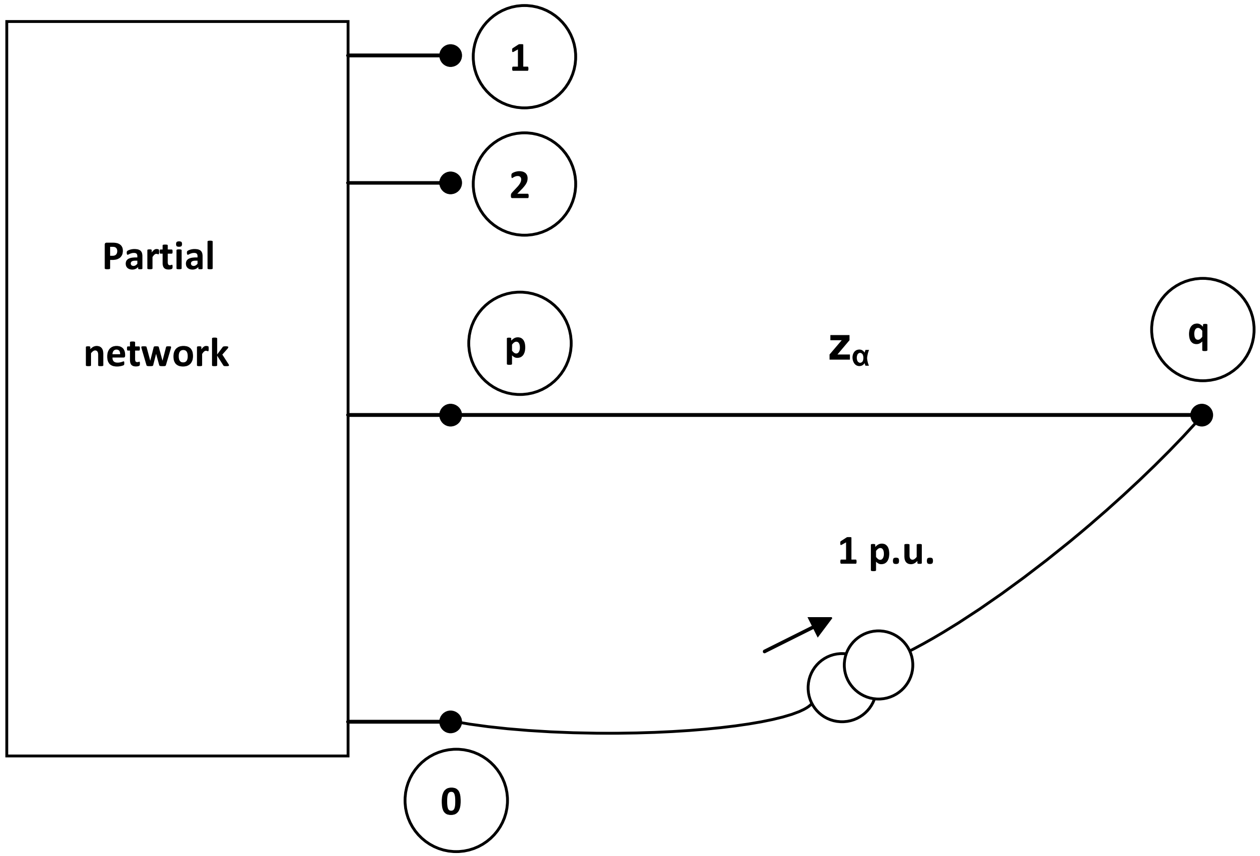


Fig. 3.17 Determining Z_{qq}

Now the bus currents are

$$\mathbf{I}_{bus} = \begin{bmatrix} 0 \\ \vdots \\ \vdots \\ 0 \\ 1_q \end{bmatrix} \qquad \textbf{(3.14)}$$

Using the above in the performance eqn. (3.8), we get

$$\begin{aligned} E_1 &= Z_{1q} \\ E_2 &= Z_{2q} \\ &\vdots \\ E_p &= Z_{pq} \\ &\vdots \\ E_m &= Z_{mq} \\ E_q &= Z_{qq} \end{aligned} \qquad \textbf{(3.15)}$$

The added element p-q has an impedance of z_α and there is no mutual coupling between the added element and the elements in the partial network. Then, from Fig. 3.17 it is clear that

$$E_q - z_\alpha = E_p \qquad \text{i.e.} \qquad E_q = E_p + z_\alpha$$

$E_q - z_\alpha = E_p$ i.e. $E_q = E_p + z_\alpha$

Using eqn. (3.15) in the above equation, we get

$$Z_{qq} = Z_{pq} + z_\alpha \tag{3.16}$$

If p happens to be the reference bus, then $E_p = 0$ and hence $Z_{pq} = 0$. Thus

$$Z_{qq} = z_\alpha \tag{3.17}$$

Addition of a link

If the added element p-q is a link, the procedure for recalculating the elements of Z_{bus} is to connect in series with the added element a voltage source e_ℓ as shown in Fig. 3.18.

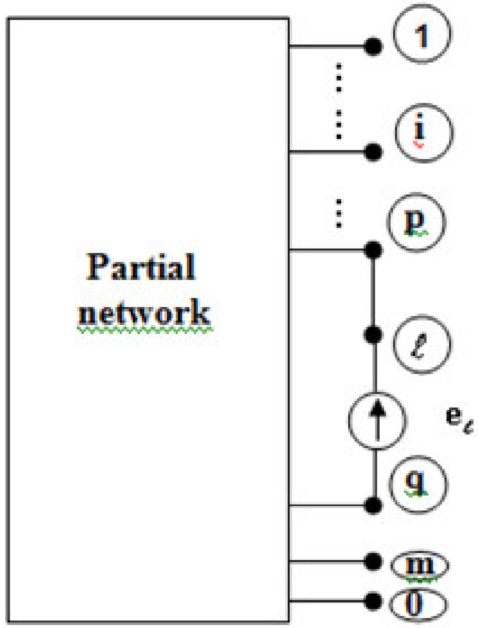


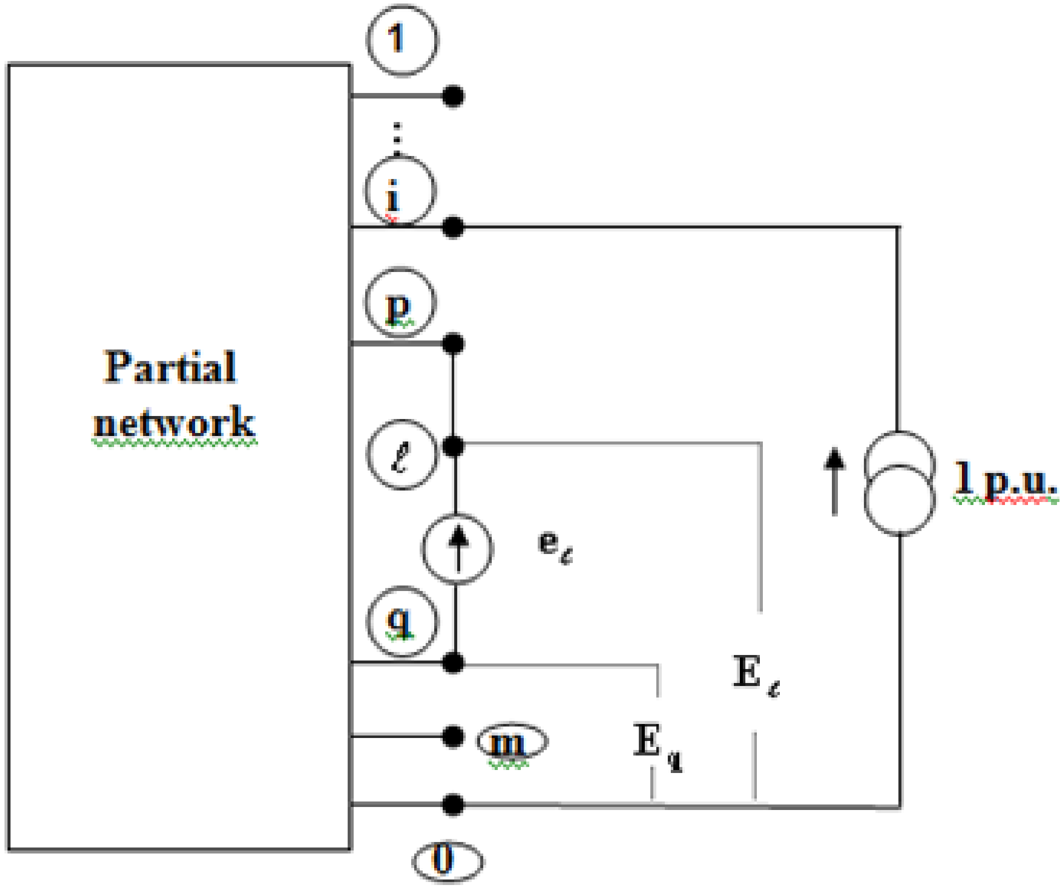
Fig. 3.18 Addition of a link

This creates a fictitious bus ℓ which will be eliminated later. The voltage source e_ℓ is selected such that the current through the added link is zero. Now the performance equation is

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_p \\ \vdots \\ E_m \\ e_\ell \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1\ell} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2\ell} \\ \vdots & \vdots & & \vdots & & \vdots & \vdots \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{p\ell} \\ \vdots & \vdots & & \vdots & & \vdots & \vdots \\ Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{m\ell} \\ Z_{\ell 1} & Z_{\ell 2} & \cdots & Z_{\ell p} & \cdots & Z_{\ell m} & Z_{\ell\ell} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ I_\ell \end{bmatrix} \tag{3.18}$$

To determine $Z_{\ell i}$

The element $Z_{\ell i}$ can be determined by calculating the voltage at the ℓ^{th} bus w.r.t. to bus q when 1 p.u. current is injected into bus i and other buses are open circuited as shown in Fig. 3.19. For this condition, the bus current vector is



$$I_{bus} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1_i \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad (3.19)$$

Fig. 3.19 Addition of a link – Determine $Z_{\ell i}$

Substituting eq. (3.19) in eq. (3.18) performance equation reduces to

$$E_1 = Z_{1i}$$

$$E_2 = Z_{2i}$$

$$\vdots$$

$$E_p = Z_{pi} \qquad (3.20)$$

$$\vdots$$

$$E_m = Z_{mi}$$

$$e_\ell = Z_{\ell i}$$

As there is no current in p- ℓ, $E_q + e_\ell = E_p$ i.e.

$$e_\ell = E_p - E_q \qquad (3.21)$$

Using eqn. (3.20) in the above equation

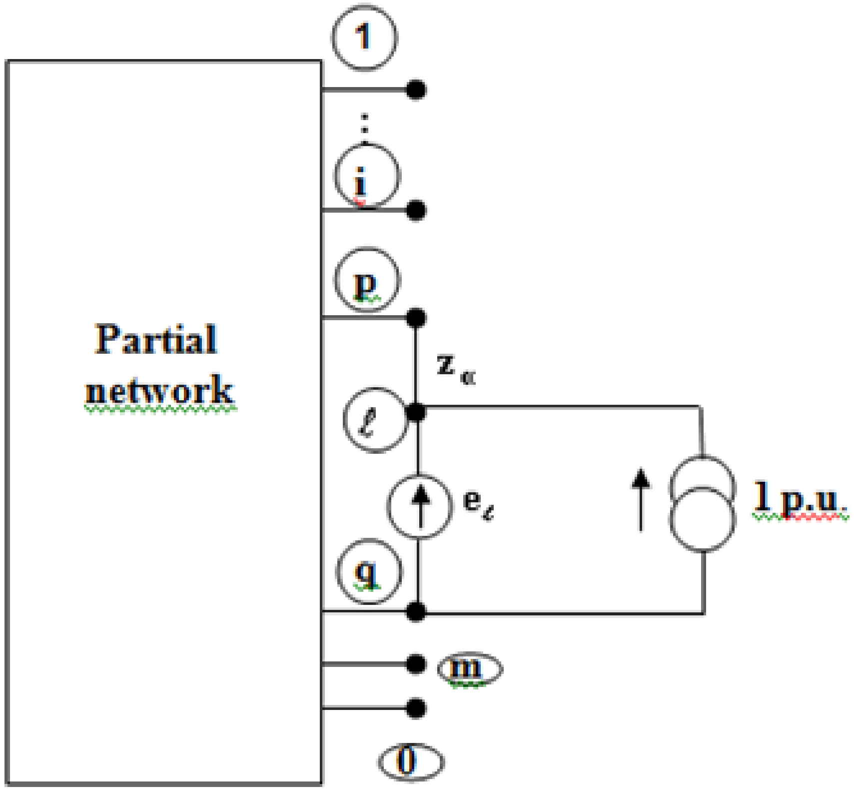
$$Z_{\ell i} = Z_{pi} - Z_{qi} \qquad i = 1, 2, \ldots\ldots., m \qquad (3.22)$$

If p happens to be reference bus $E_p = 0$ and hence $Z_{p\,i} = 0$. Thus

$$Z_{\ell i} = -Z_{qi} \qquad i = 1, 2, \ldots\ldots., m \qquad (3.23)$$

To determine $Z_{\ell\ell}$

The element $Z_{\ell\ell}$ can be calculated by measuring the voltage at the ℓ^{th} bus w.r.t. bus q after injecting 1 p.u. current at the ℓ^{th} bus and keeping other buses open circuited as shown in Fig. 3.20. Now the bus current is



$$I_{bus} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 1_{\ell} \end{bmatrix} \quad (3.24)$$

Fig. 3.20 Addition of a link – Determine Z_{ℓ}

Substituting eqn.(3.24) in eqn. (3.18), performance equation reduces to

$$\begin{aligned} E_1 &= Z_{1\ell} \\ E_2 &= Z_{2\ell} \\ &\vdots \\ E_p &= Z_{p\ell} \\ &\vdots \\ E_q &= Z_{q\ell} \\ &\vdots \\ E_m &= Z_{m\ell} \\ e_\ell &= Z_{\ell\ell} \end{aligned} \tag{3.25}$$

From the Fig. 3.20 it is clear that $E_q + e_\ell - z_\alpha = E_p$ i.e.

$$e_\ell = E_p - E_q + z_\alpha \tag{3.26}$$

Substituting eqn. (3.25) in the above equation we get

$$Z_{\ell\ell} = Z_{p\ell} - Z_{q\ell} + z_\alpha \tag{3.27}$$

If p happens to be reference bus, $E_p = 0$ and hence $Z_{p\ell} = 0$. Thus

$$Z_{\ell\ell} = - Z_{q\ell} + z_\alpha \tag{3.28}$$

Elimination of bus ℓ

Once we have calculated $Z_{\ell i}$ i = 1,2,……..,m and $Z_{\ell\ell}$ we have an (m+1) x (m+1) impedance matrix. However, we know that eventually we must connect ℓ and q together. This will make the resultant bus impedance matrix of dimension m x m . Equation (3.18) can be written as

$$\begin{bmatrix} \mathbf{E}_{bus} \\ e_\ell \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{bus} & \bar{Z}_{i\ell} \\ \bar{Z}_{\ell j} & Z_{\ell\ell} \end{bmatrix} \begin{bmatrix} \mathbf{I}_{bus} \\ I_\ell \end{bmatrix} \qquad \begin{matrix} i = 1,2,\ldots,m \\ j = 1,2,\ldots,m \end{matrix} \tag{3.29}$$

Therefore $\mathbf{E}_{bus} = \mathbf{Z}_{bus}\ \mathbf{I}_{bus} + \bar{Z}_{i\ell}\ I_\ell$ (3.30)

$$e_\ell = \bar{Z}_{\ell j}\ \mathbf{I}_{bus} + Z_{\ell\ell}\ I_\ell \tag{3.31}$$

If we join ℓ and q, e_ℓ = 0. Then we can solve for I_ℓ from eqn. (3.31). Thus

$I_\ell = -\dfrac{\bar{Z}_{\ell j}\ \mathbf{I}_{bus}}{Z_{\ell\ell}}$. Substituting this in equation (3.30) we get

$$\mathbf{E}_{bus} = \mathbf{Z}_{bus}\ \mathbf{I}_{bus} - \frac{\bar{Z}_{i\ell}\ \bar{Z}_{\ell j}}{Z_{\ell\ell}}\ \mathbf{I}_{bus} = [\mathbf{Z}_{bus} - \frac{\bar{Z}_{i\ell}\ \bar{Z}_{\ell j}}{Z_{\ell\ell}}]\ \mathbf{I}_{bus}$$

$$E_{bus} = Z_{bus}\, I_{bus} - \frac{\bar{Z}_{i\ell}\, \bar{Z}_{\ell j}}{Z_{\ell\ell}}\, I_{bus} = \left[Z_{bus} - \frac{\bar{Z}_{i\ell}\, \bar{Z}_{\ell j}}{Z_{\ell\ell}} \right] I_{bus}$$

Thus $$Z_{bus\ (modified)} = Z_{bus\ (before\ modification)} - \frac{\bar{Z}_{i\ell}\, \bar{Z}_{\ell j}}{Z_{\ell\ell}} \qquad (3.32)$$

Note that $\bar{Z}_{i\ell}\ \bar{Z}_{\ell j}$ will be a m x m matrix. Elements of modified bus impedance matrix can be obtained as

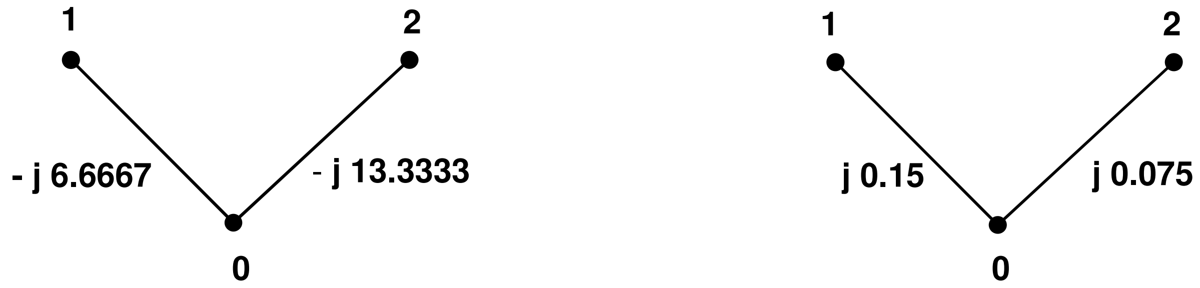
$$Z_{ij\ (modified)} = Z_{ij\ (before\ modification)} - \frac{Z_{i\ell}\, Z_{\ell j}}{Z_{\ell\ell}} \qquad \begin{matrix} i = 1,2,\ldots,m \\ j = 1,2,\ldots,m \end{matrix} \qquad (3.33)$$

Summary of formulas

Added element	p is not reference bus	p is the reference bus
p − q is a Branch	$Z_{qi} = Z_{pi}$ $i = 1,2,\ldots\ldots\ldots,m$ $i \neq q$ $Z_{qq} = Z_{pq} + z_{\alpha}$	$Z_{qi} = 0$ $i = 1,2,\ldots\ldots\ldots,m$ $i \neq q$ $Z_{qq} = z_{\alpha}$
Added element p − q is a Link	$Z_{\ell i} = Z_{pi} - Z_{qi}$ $i = 1,2,\ldots\ldots\ldots,m$ $i \neq \ell$ $Z_{\ell\ell} = Z_{p\ell} - Z_{q\ell} + z_{\alpha}$	$Z_{\ell i} = -Z_{qi}$ $i = 1,2,\ldots\ldots\ldots,m$ $i \neq \ell$ $Z_{\ell\ell} = -Z_{q\ell} + z_{\alpha}$
	$Z_{ij\,(\text{modified})} = Z_{ij\,(\text{before modification})} - \dfrac{Z_{i\ell}\,Z_{\ell j}}{Z_{\ell\ell}}$ $i = 1,2,\ldots\ldots\ldots\ldots,m$ and $j = 1,2,\ldots\ldots\ldots\ldots,m$	

How to start the building algorithm?

Consider a network having a graph as shown in Fig. 3.21.



Marked values are admittances **Marked values are impedances**

Fig. 3.21 Sample network graphs

For this network

$$Y_{Bus} = -j \begin{matrix} & 1 & 2 \\ 1 & 6.6667 & 0 \\ 2 & 0 & 13.3333 \end{matrix}$$

and hence its Z_{Bus} is given by

$$Z_{Bus} = j \begin{matrix} & 1 & 2 \\ 1 & 0.15 & 0 \\ 2 & 0 & 0.075 \end{matrix}$$

Note that these element values are the values of the shunt impedances at buses 1 and 2. This result can be extended to more number of buses also. Generally power system network will have a few shunt elements. Bus impedance algorithm can be started using these. While adding element p-q, it must be ensured that either bus p or bus q or both buses p and q must be present in the partial network.

Example 3.2

Consider the power system shown in Fig. 3.21. The values marked are p.u. impedances. The p.u. reactances of the generator 1 and 2 are 0.15 and 0.075 respectively. Compute the bus impedance matrix of the generator – transmission network.

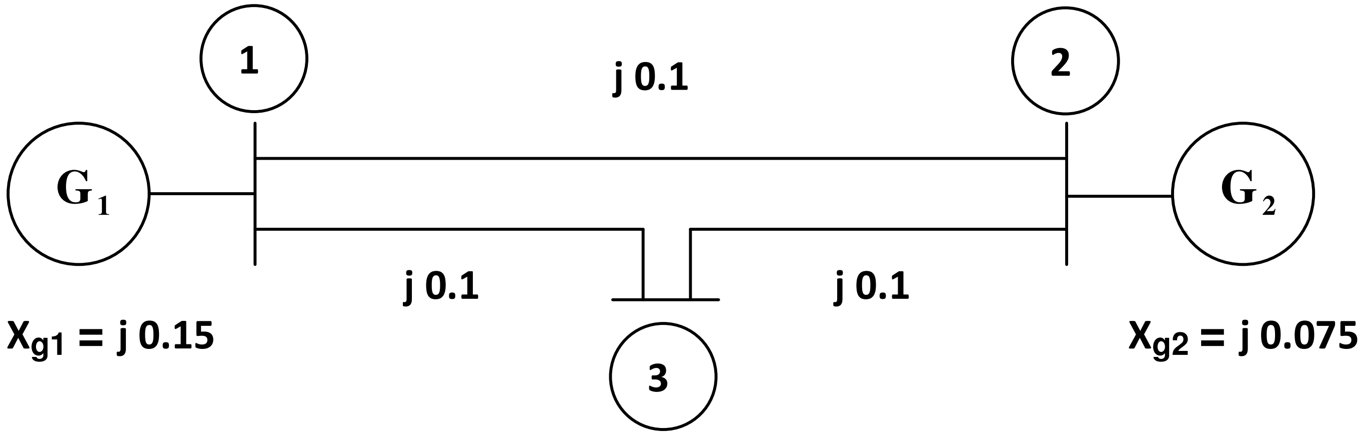


Fig. 3.21 Power system for Example 3.2

Solution

The ground bus is numbered as 0 and it is taken as reference bus. The p.u. impedance diagram is shown in Fig. 3.22.

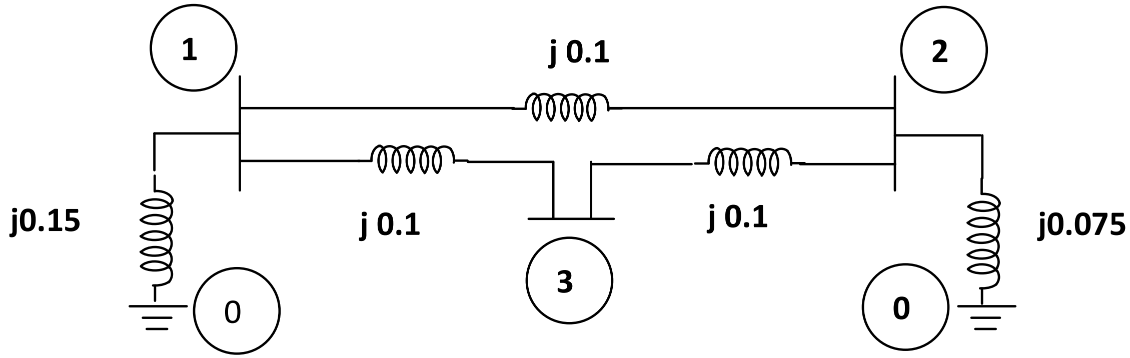


Fig. 3.22 p.u. impedance diagram for Example 3.2

When element 0 – 1 is included

$$Z_{bus} = j \begin{matrix} & 1 \\ 1 & [0.15] \end{matrix} \text{ ; When element } 0-2 \text{ is included } Z_{bus} = j \begin{matrix} & 1 & 2 \\ 1 & \\ 2 & \end{matrix} \begin{bmatrix} 0.15 & 0 \\ 0 & 0.075 \end{bmatrix}$$

Element 1 – 2 is added; it is a link between buses 1 and 2. With bus ℓ

$$Z_{bus} = j \begin{matrix} & 1 & 2 & \ell \\ 1 & & & \\ 2 & & & \\ \ell & & & \end{matrix} \left[\begin{array}{cc|c} 0.15 & 0 & 0.15 \\ 0 & 0.075 & -0.075 \\ \hline 0.15 & -0.075 & 0.325 \end{array}\right];$$

Eliminating the ℓ^{th} bus

$$Z_{bus} = j \begin{matrix} & 1 & 2 \\ 1 & & \\ 2 & & \end{matrix} \begin{bmatrix} 0.08077 & 0.034615 \\ 0.034615 & 0.05769 \end{bmatrix}$$

Add element 1 – 3. It is a branch from bus 1 and it creates bus 3.

$$Z_{bus} = j \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.08077 & 0.034615 & 0.08077 \\ 0.034615 & 0.05769 & 0.034615 \\ 0.08077 & 0.034615 & 0.18077 \end{bmatrix} \end{array}$$

Finally add element 2 – 3. It is a link between buses 2 and 3. With bus ℓ

$$Z_{bus} = j \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ \ell \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & \ell \end{array} \\ \left[\begin{array}{ccc|c} 0.08077 & 0.034615 & 0.08077 & -0.046155 \\ 0.034615 & 0.05769 & 0.034615 & 0.023075 \\ 0.08077 & 0.034615 & 0.18077 & -0.146155 \\ \hline -0.046155 & 0.023075 & -0.146155 & 0.26923 \end{array}\right] \end{array}$$

Eliminating the ℓ^{th} bus, final bus impedance matrix is obtained as

$$Z_{bus} = j \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.07286 & 0.03857 & 0.05571 \\ 0.03857 & 0.05571 & 0.04714 \\ 0.05571 & 0.04714 & 0.10143 \end{bmatrix} \end{array}$$

Symmetrical fault analysis through bus impedance matrix

Once the bus impedance matrix is constructed, symmetrical fault analysis can be carried out with a very few calculations. Bus voltages and currents in various elements can be computed quickly. When faults are to be simulated at different buses, this method is proved to be good.

Symmetrical short circuit analysis essentially consists of determining the steady state solution of linear network with balanced sources.

Since the short circuit currents are much larger compared to prefault currents the following assumptions are made while conducting short circuit study.

1. all the shunt parameters like loads, line charging admittances etc. are neglected.
2. all the transformer taps are at nominal position.
3. prior to the fault, all the generators are assumed to operate at rated voltage of 1.0 p.u. with their emf's in phase.

With these assumptions, in the prefault condition, there will not be any current flow in the network and all the bus voltages will be equal to 1.0 p.u.

The linear network that has to be solved comprises of

i) Transmission network ii) Generation system and iii) Fault

By properly combining the representations of the above three components, we can solve the short circuit problem.

Consider the transmission network shown in Fig. 3.23.

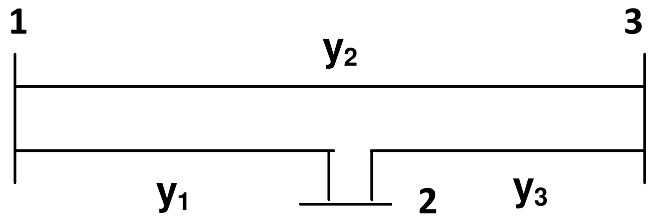


Fig. 3.23 Sample transmission system

Taking the ground as the reference bus, the bus admittance matrix is obtained as

$$Y_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} y_1 + y_2 & -y_1 & -y_2 \\ -y_1 & y_1 + y_3 & -y_3 \\ -y_2 & -y_3 & y_2 + y_3 \end{bmatrix} \end{array} \qquad (3.34)$$

If we add all the columns (or rows) we get a column (or row) of all zero elements. Hence this Y_{bus} matrix is singular and hence corresponding Z_{bus} matrix of this transmission network does not exist. Thus, when all the shunt parameters are neglected, Z_{bus} matrix will not exist for the transmission network.

However, connection to ground is established at the generator buses, representing the generator as a constant voltage source behind appropriate reactance as shown in Fig. 3.24.

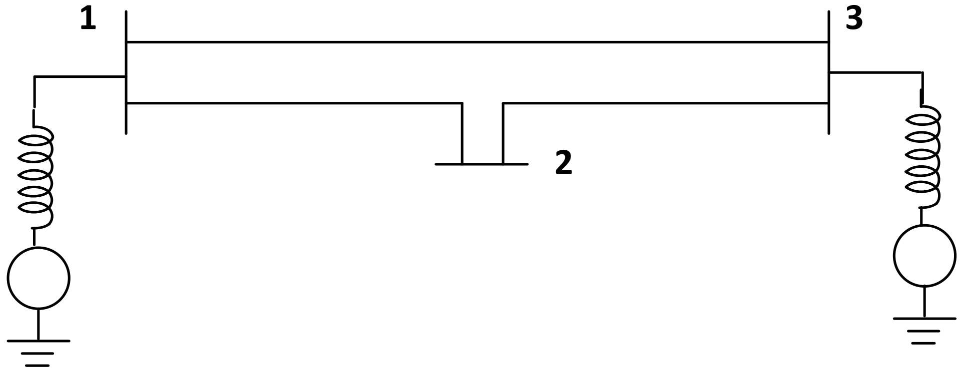


Fig. 3.24 Sample transmission system with generators

If the generator reactances are included with the transmission network, Z_{bus} matrix of the combined network can be obtained. As stated earlier, there is no current flow in the network in the pre-fault condition and all the bus voltages will be 1.0 p.u.

Consider the network shown in Fig. 3.25. Symmetrical fault occurring at bus 2 can be simulated by closing the switch shown in Fig. 3.25. Here Z_f is the fault impedance.

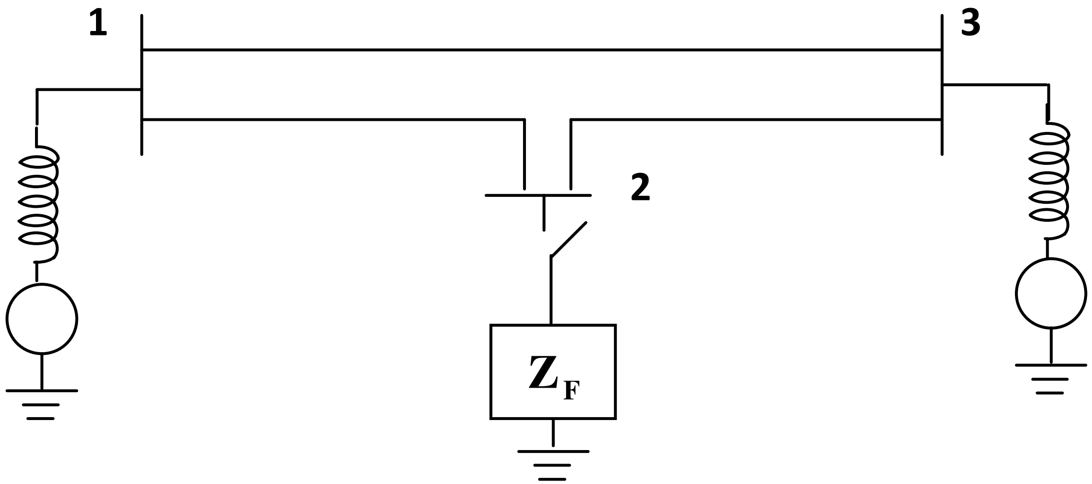


Fig. 3.25 Sample transmission system with generators and fault

When the fault is simulated, there will be currents in different elements and the bus voltages will be different from 1.0 p.u. These changes have occurred because of i) Generator voltages and ii) Fault current

Any general power system with a number of generators and N number of buses subjected to symmetrical fault at p^{th} bus will be represented as shown in Fig. 3.26.

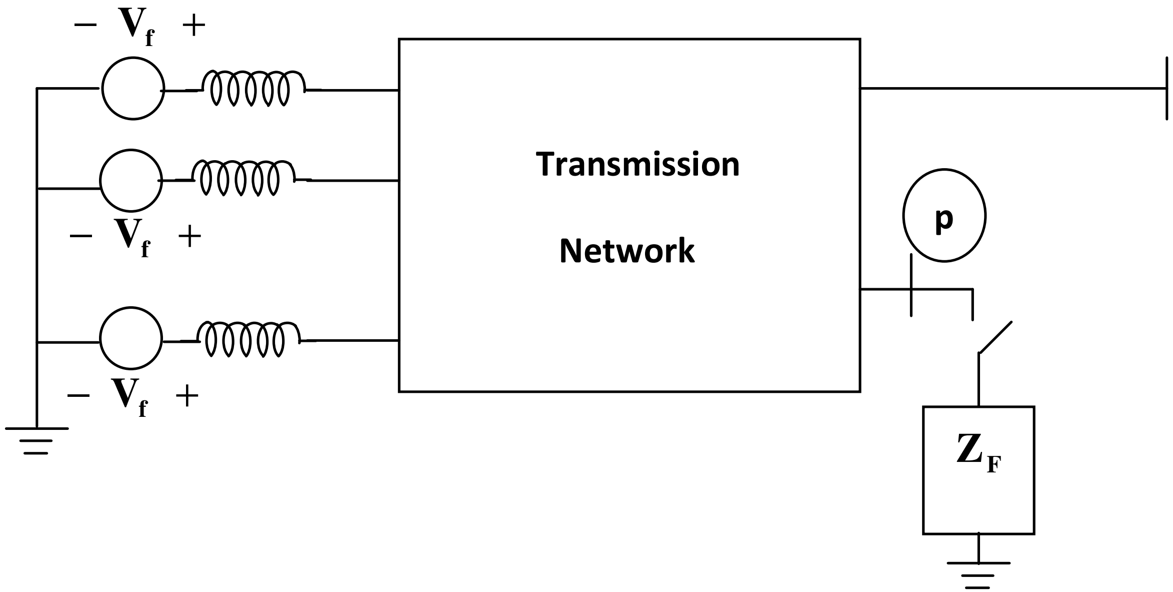


Fig. 3.26 Symmetrical fault at bus p in a power system

In the faulted system there are two types of sources:

1. Current injection at the faulted bus
2. Generated voltage sources.

The bus voltages in the faulted system can be obtained using Superposition theorem.

Bus voltages due to current injection:

Make all the generator voltages to zero. Then we have Generator-Transmission system without voltage sources. Such network has transmission parameters and generator reactances between generator buses and the ground. Let Z_{bus} be the bus impedance matrix of such Generator-Transmission network. Then the bus voltages due to the current injection will be given by

$$V_{bus} = Z_{bus} \; I_{bus\,(F)} \tag{3.35}$$

where $I_{bus\,(F)}$ is the bus current vector having only one non-zero element. Thus when the fault is at the p^{th} bus

$$I_{bus\,(F)} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ I_{p\,(F)} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Here $I_{p\,(F)}$ is the faulted bus current. (3.36)

$$\mathbf{I}_{bus\,(F)} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \mathbf{I}_{p\,(F)} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Here $\mathbf{I}_{p\,(F)}$ is the faulted bus current. **(3.36)**

Thus, bus voltages due to current injection will be

$$\mathbf{V}_{bus} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2N} \\ \vdots & \vdots & & \vdots & & \vdots \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pN} \\ \vdots & \vdots & & \vdots & & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{Np} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \mathbf{I}_{p\,(F)} \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{1p} \\ Z_{2p} \\ \vdots \\ Z_{pp} \\ \vdots \\ Z_{Np} \end{bmatrix} \mathbf{I}_{p\,(F)} \qquad \textbf{(3.37)}$$

Bus voltages due to generator voltages

Make the fault current to be zero. Since there is no shunt element, there will be no current flow and all the bus voltages are equal to V_0, the pre-fault voltage which will be normally equal to 1.0 p.u. Thus, bus voltages due to generator voltages will be

$$V_{bus} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ \vdots \\ 1 \end{bmatrix} V_0 \qquad (3.38)$$

Thus for the faulted system, wherein both the current injection and generator sources are simultaneously present, the bus voltages can be obtained by adding the voltages given by eqns. (3.37) and (3.38). Therefore, for the faulted system the bus voltages are

$$V_{bus\,(F)} = \begin{bmatrix} V_{1(F)} \\ V_{2(F)} \\ \vdots \\ V_{p(F)} \\ \vdots \\ V_{N(F)} \end{bmatrix} = \begin{bmatrix} Z_{1p} \\ Z_{2p} \\ \vdots \\ Z_{pp} \\ \vdots \\ Z_{Np} \end{bmatrix} I_{p\,(F)} + \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ \vdots \\ 1 \end{bmatrix} V_0 \qquad (3.39)$$

To calculate $V_{bus\,(F)}$ we need the faulted bus current $I_{p\,(F)}$ which can be determined as discussed below.

The fault can be described as shown in Fig. 3.27.

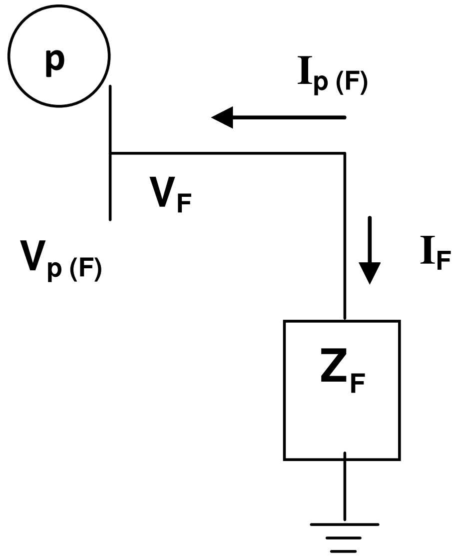


Fig. 3.27 Description of fault at bus p

It is clear that $V_F = Z_F\ I_F$, $V_{p\,(F)} = V_F$ **and** $I_{p\,(F)} = -\,I_F$ **(3.40)**

Therefore

$$V_{p\,(F)} = -\,Z_F\ I_{p\,(F)} \qquad (3.41)$$

The p^{th} equation extracted from eqn. (3.39) gives

$$V_{p\,(F)} = Z_{pp}\ I_{p\,(F)} + V_0 \qquad (3.42)$$

$$V_{p(F)} = -Z_F\ I_{p(F)} \tag{3.41}$$

The p^{th} equation extracted from eqn. (3.39) gives

$$V_{p(F)} = Z_{pp}\ I_{p(F)} + V_0 \tag{3.42}$$

Substituting eqn. (3.41) in the above, we get

$$-Z_F\ I_{p(F)} = Z_{pp}\ I_{p(F)} + V_0$$

Thus the faulted bus current $I_{p(F)}$ is given by

$$I_{p(F)} = -\frac{V_0}{Z_{pp} + Z_F} \tag{3.43}$$

Substituting the above in eqn. (3.41), the faulted bus voltage $V_{p(F)}$ is

$$V_{p(F)} = \frac{Z_F}{Z_{pp} + Z_F}\ V_0 \tag{3.44}$$

Finally voltages at other buses at faulted condition are to be obtained. The i^{th} equation extracted from eqn. (3.39) gives

$$V_{i\,(F)} = Z_{ip}\ I_{p\,(F)} + V_0$$

Substituting eqn. (3.43) in the above, we get

$$V_{i\,(F)} = V_0 - \frac{Z_{ip}}{Z_{pp} + Z_F}\ V_0 \qquad i = 1,2,\ldots\ldots\ldots,N \quad i \neq p \qquad (3.45)$$

Knowing all the bus voltages, current flowing through the various network elements can be computed as

$i_{km(F)} = (V_{k(F)} - V_{m(F)})\ y_{km}$ where y_{km} is the admittance of element k-m. (3.46)

When the fault is direct, $Z_F = 0$ and hence

$$\left.\begin{aligned} I_{p(F)} &= -\frac{V_0}{Z_{pp}} \\ V_{p(F)} &= 0 \text{ and} \\ V_{i(F)} &= V_0 - \frac{Z_{ip}}{Z_{pp}} V_0 \qquad i = 1,2,\ldots\ldots\ldots,N \quad i \neq p \end{aligned}\right\} \qquad (3.47)$$

<u>It is to be noted that when the fault occurs at the</u> p^{th} <u>bus, only the</u> p^{th} <u>column of</u> Z_{bus} <u>matrix</u> (<u>and not the entire</u> Z_{bus} <u>matrix</u>) <u>is required for further calculations.</u>

The following are the various steps for conducting symmetrical short circuit analysis.

Step 1 Read
i) Transmission line data
ii) Generator reactances data
iii) Faulted bus number p and
iv) Fault impedance Z_F.

Step 2 Construct the bus impedance matrix of the transmission network including the generator reactances.

Step 3 Compute $I_{p(F)} = -\dfrac{V_0}{Z_{pp} + Z_F}$

Step 4 Compute $V_{p(F)} = \dfrac{Z_F}{Z_{pp} + Z_F} V_0$

Step 5 Compute $V_{i(F)} = V_0 - \dfrac{Z_{ip}}{Z_{pp} + Z_F} V_0$ $i = 1,2,\ldots\ldots\ldots,N$; $i \neq p$

Step 6 Calculate the element currents from $i_{km(F)} = (V_{k(F)} - V_{m(F)})\, y_{km}$

Example 3.3

Consider the power system discussed in Example 3.2. The p.u. impedances are on a base of 50 MVA and 12 kV. Symmetrical short circuit occurs at bus 3 with zero fault impedance. Using Z_{bus} matrix determine the fault current, bus voltages and also the currents contributed by the generators.

Solution

As seen in example 3.2, Z_{bus} matrix of the transmission-generator network is

$$Z_{bus} = j \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.07286 & 0.03857 & 0.05571 \\ 0.03857 & 0.05571 & 0.04714 \\ 0.05571 & 0.04714 & 0.10143 \end{bmatrix} \end{array}$$

Faulted system is shown in Fig. 3.28

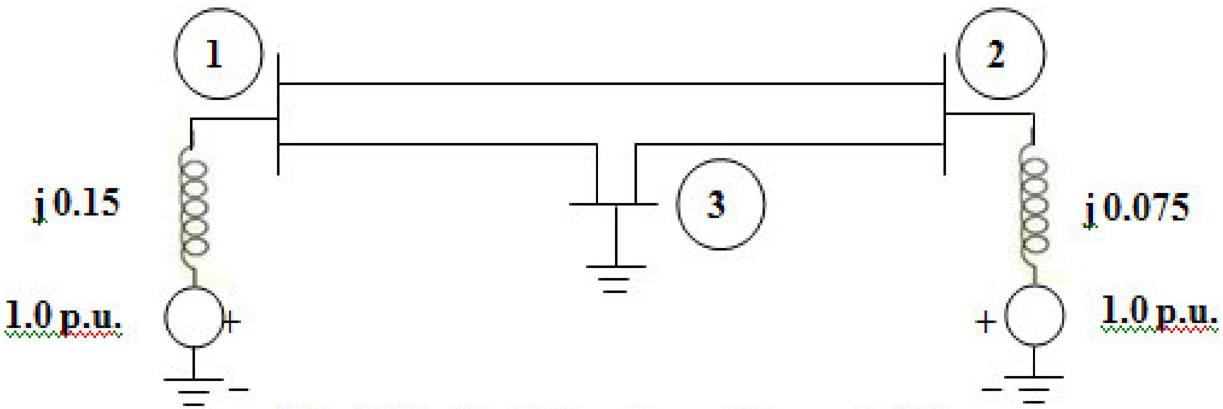


Fig. 3.28 Faulted system – Example 3.3

Faulted bus current $I_{3(F)} = -\dfrac{1}{j0.10143} = j9.8590$ p.u.

Fault current $I_F = -I_{3(F)} = -j9.8590$ p.u.

$$\text{Base current} = \frac{50 \times 1000}{\sqrt{3} \times 12} = 2405.6 \text{ amp.}$$

Fault current I_F = - j 9.8590 x 2405.6 = - j 23717 amp.

Since fault impedance is zero, $V_{3(F)} = V_F = 0$

Taking the pre-fault voltage $V_0 = 1.0$ p.u.

$$V_{1(F)} = 1.0 - \frac{Z_{13}}{Z_{33}} = 1.0 - \frac{0.05571}{0.10143} = 0.45075 \text{ p.u.} = \frac{12}{\sqrt{3}} \times 0.45075 = 3.1229 \text{ kV}$$

$$V_{2(F)} = 1.0 - \frac{Z_{23}}{Z_{33}} = 1.0 - \frac{0.04714}{0.10143} = 0.53525 \text{ p.u.} = \frac{12}{\sqrt{3}} \times 0.53525 = 3.7083 \text{ kV}$$

Current supplied by gen. 1, $I_{G1} = \dfrac{1.0 - 0.45075}{j0.15} = -j3.6617$ p.u. = - j8808.6 amp.

Current supplied by gen. 2, $I_{G2} = \dfrac{1.0 - 0.53525}{j0.075} = -j6.1967$ p.u. = -j14906.8 amp.

Note that in this example as the fault occurs at bus 3 we used only the column 3 of Z_{bus} matrix and not the entire Z_{bus} matrix.

Example 3.4

For the transmission-generator system shown in Fig.3.29, the bus impedance matrix is obtained as

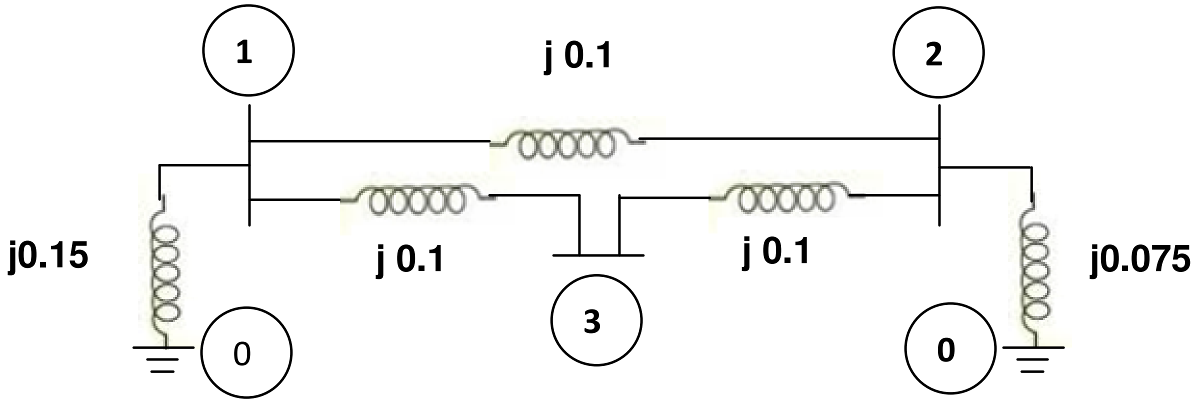


Fig. 3.29 Trans. Gen. system – Example 3.4

$$Z_{bus} = j \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.072857 & 0.038571 & 0.055714 \\ 0.038571 & 0.055714 & 0.047143 \\ 0.055714 & 0.047143 & 0.101429 \end{bmatrix} \end{array}$$

Symmetrical three phase fault with fault impedance j 0.052143 p.u. occurs at bus 1. Find the p.u. currents in all the elements and mark them on the single line diagram.

Solution

Fault occurs at bus 1 and we need the first column of Z_{Bus}, which is

$$
j \quad \begin{matrix} \\ 1 \\ 2 \\ 3 \end{matrix} \begin{matrix} 1 \\ \begin{bmatrix} 0.072857 \\ 0.038571 \\ 0.055714 \end{bmatrix} \end{matrix} \quad \text{and} \quad Z_F = j\,0.052143
$$

Faulted bus current $I_{1(F)} = -\dfrac{1}{j0.072857 + j0.052143} = -\dfrac{1}{j0.125} = j\,8$

Fault current $I_F = -I_{1(F)} = -j\,8$ **p.u.**

$V_1(F) = V_F = Z_F\,I_F = (j\,0.052143)(-j\,8) = 0.41714$ **p.u.**

$V_2(F) = 1 - \dfrac{0.038571}{0.125} = 0.69143$ **p..u.** $V_3(F) = 1 - \dfrac{0.055714}{0.125} = 0.55429$ **p..u.**

$i_{2\text{-}1}$ = (0.69143 – 0.41714) / (j 0.1) = - j 2.7429 p.u.

$i_{3\text{-}1}$ = (0.55429 – 0.41714) / (j 0.1) = - j 1.3715 p.u.

$i_{2\text{-}3}$ = (0.69143 – 0.55429) / (j 0.1) = - j 1.3714 p.u.

i_{G1} = (1 – 0.41714) / (j 0.15) = - j 3.8857 p.u.

i_{G2} = (1 – 0.69143) / (j 0.075) = - j 4.1143 p.u.

$V_1(F)$ = 0.41714 p.u.

$V_2(F)$ = 0.69143 p.u.

$V_3(F)$ = 0.55429 p.u.

Currents are marked in Fig. 3.30.

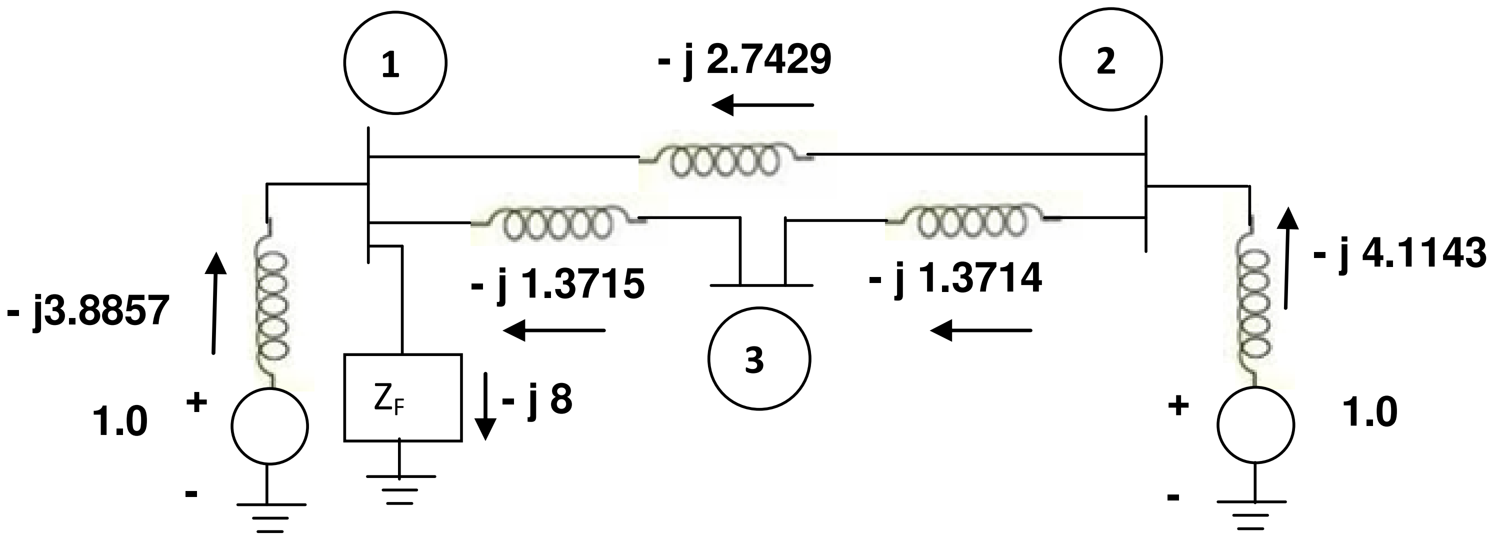


Fig. 3.30 Network with element currents

Short circuit MVA or Fault level

When a fault very large currents used to flow, In order to minimize the damage caused by the short circuit, the faulty section need to be disconnected. Circuit breakers are used to disconnect the faulty section. The rating of the circuit breaker depends on the Short Circuit MVA which is also known as FAULT LEVEL or FAULT MVA. The circuit breaker breaking capacity must be equal to or greater than the short circuit MVA. The estimation of circuit breaker capacity is made on the basis that it must clear a three phase fault with zero fault impedance as that is generally the worst case. By simulating three phase fault at a point and using subtransient reactances of the machines short circuit level at that point can be computed as

Short Circuit MVA = prefault voltage in p.u. x fault current in p.u. x Base MVA (3.48)

Unless it is given otherwise, prefault voltage shall be taken as 1.0 p.u.

Example 3.5

Fig. 3.31 shows four identical alternators in parallel. Each machine is rated for 25 MVA, 11 kV and has a subtransient reactance of 16 % on its rating. Compute the short circuit MVA when a three phase fault occurs at one of the outgoing feeders.

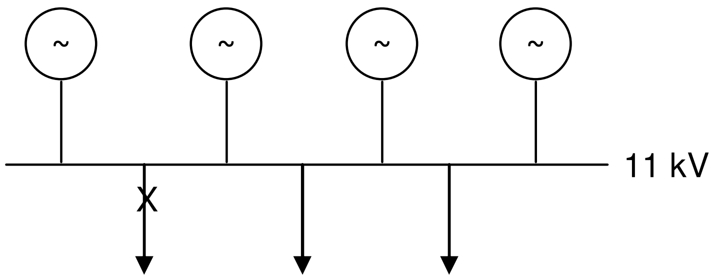


Fig. 3.31 Four alternators – Example 3.5

Solution

Fault is simulated by closing the switch shown in the p.u. reactance diagram shown in Fig. 3.32 (a). Its Thevenin's equivalent is shown in Fig. 3.32 (b).

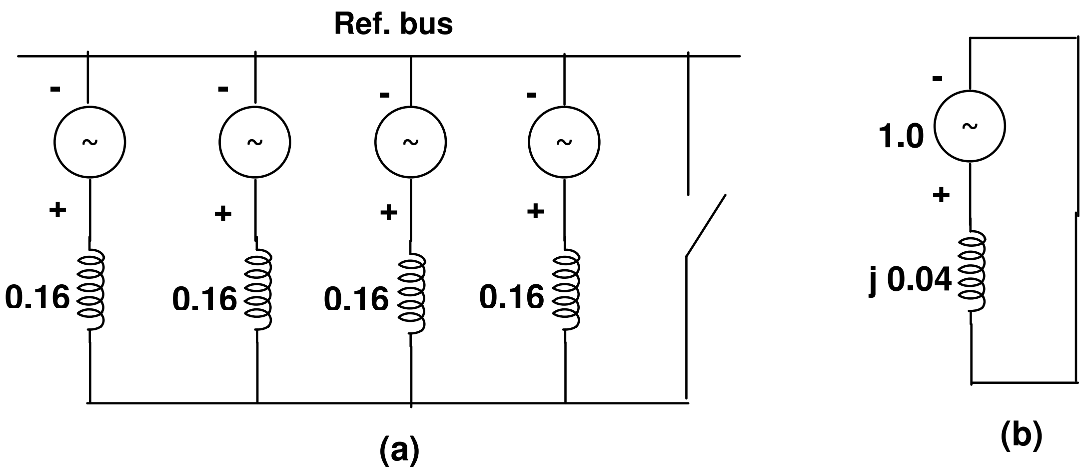


Fig. 3.32 Reactance diagram and its Thevenin's equivalent circuit

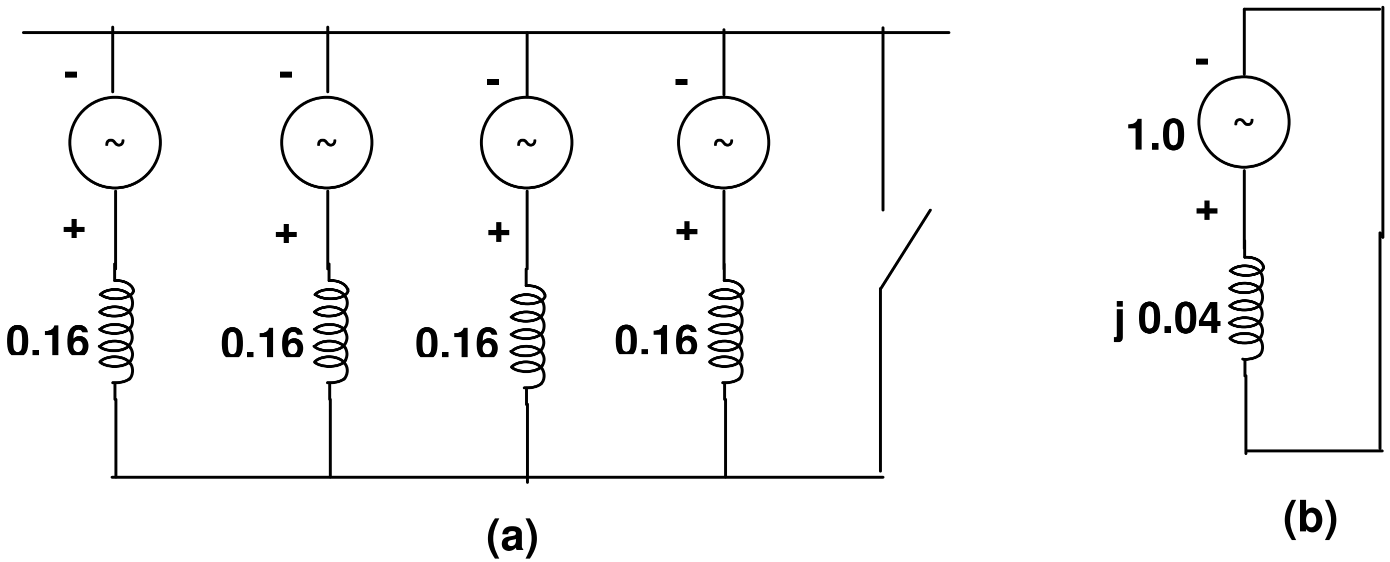


Fig. 3.32 Reactance diagram and its Thevenin's equivalent circuit

Fault current $|I_F| = \frac{1}{0.04} = 25$ p.u.

Short circuit MVA = prefault voltage in p.u. x fault current in p.u. x Base MVA

= 1.0 x 25 x 25

= 625

Selection of circuit breakers

Much study has been made to circuit-breaker ratings and applications. From the *circuit breaker current* view point two factors that are to be considered are:

* **Maximum instantaneous current which the breaker must withstand**

* **The total current when the breaker contacts part to interrupt the circuit.**

Up to this point we have devoted most of our attention to the subtransient current called the *initial symmetrical current*, which does not include the dc component of current. Inclusion of dc component of current results in a rms value of current immediately after the fault, which will be higher than the subtransient current. For the circuit breakers above 5 kV subtransient current multiplied by 1.6 is considered to be the rms value of the current the circuit breaker must withstand during the first half cycle after the fault occurs. This current is called the *momentary current*.

The *interrupting rating* of a circuit breaker is specified in kVA or MVA. Interrupting kVA = $\sqrt{3}$ x kV of the bus to which the breaker is connected

x interrupting current when its contacts part

This interrupting current is , of course, lower than the momentary current and depends on the speed of the circuit breaker, such as 8, 5, 3, or 2 cycles, which is a measure of the time from the occurrence of the fault to the extinction of the arc.

The current which a breaker must interrupt is usually asymmetrical since it still contains some of the decaying dc component of current. We shall limit our discussion to a brief treatment symmetrical basis of breaker selection.

Breakers are identified by:

i) nominal voltage class, such as 69 kV

ii) rated continuous current

iii) rated maximum voltage

iv) voltage range factor, K

v) rated short circuit current at rated maximum voltage

i) nominal voltage class, such as 69 kV

ii) rated continuous current

iii) rated maximum voltage

iv) voltage range factor, K

v) rated short circuit current at rated maximum voltage

The rated maximum voltage of a circuit breaker is the highest rms voltage for which the circuit breaker is designed.

$$\text{Rated voltage range factor, } K = \frac{\text{rated maximum voltage}}{\text{lower limit of the range of operating voltage}}$$

Value of K determines the range of voltage over which the product {rated short circuit current x operating voltage} is constant.

In the application of circuit breakers it is important not to exceed the short-circuit capabilities of the breakers. ***A breaker is required to have a maximum symmetrical interrupting capability equal to K x rated short circuit current.***

Example 3.6

A 69-kV circuit breaker having a voltage range factor K of 1.21 and a continuous current rating of 1200 A has a rated short circuit current of 19000 A at the maximum rated voltage of 72.5 kV. Determine the maximum symmetrical interrupting capability of the breaker and explain its significance at lower operating voltage.

Solution

Given: **Nominal voltage = 69 kV**

Voltage range factor K = 1.21

Continuous current rating = 1200 A

Rated short circuit current = 19000 A

Maximum rated voltage = 72.5 kV

The max. symmetrical interrupting capability = K x rated short circuit current

$$= 1.21 \times 19000 = 22990 \text{ A}$$

Since voltage range factor $K = \dfrac{\text{rated maximum voltage}}{\text{lower limit of the range of operating voltage}}$

Lower limit of operating voltage $= \dfrac{72.5}{1.21} = 60$ kV

Operating voltage range is 72.5 kV to 60 kV. In this range, the symmetrical interrupting current may exceed the rated short circuit current of 19000 A, but it is limited to 22990 A.

For example at 66 kV, interrupting current $= \dfrac{72.5}{66} \times 19000 = 20871$ A

Current limiting reactors

In the example 3.5, the fault level is 625 MVA where as the nearest standard ratings of the circuit breakers are 500 MVA and 750 MVA. If for economic reason it is necessary to use 500 MVA circuit breaker, the fault current must be reduced by increasing the system reactance. This can be done by connecting current limiting reactor externally. Different ways of connecting the current limiting reactors are shown in Fig. 3. 33.

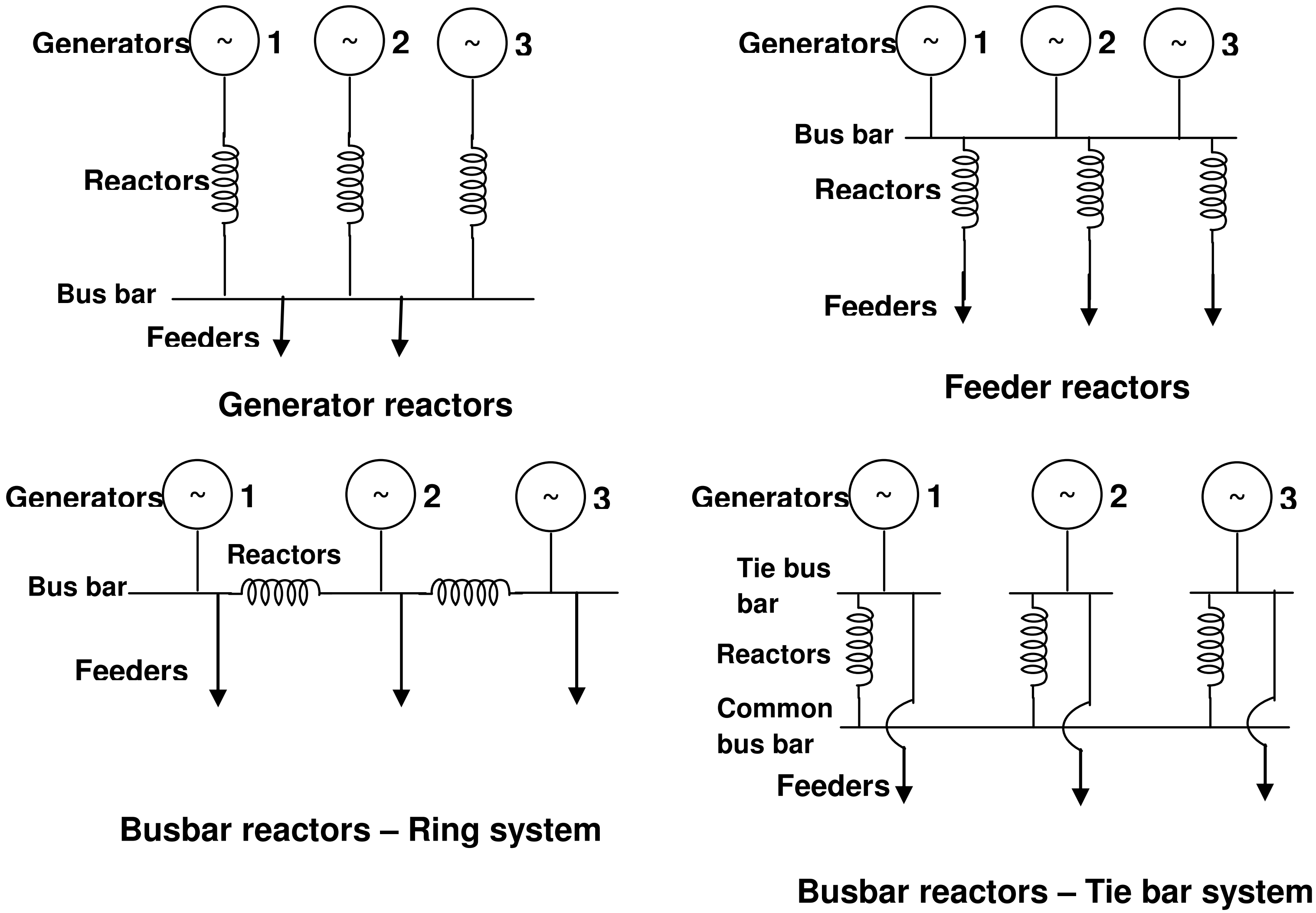


Fig. 3.33 Current limiting reactors

Example 3.7

In the generator system considered in Example 3.5, it is decided to limit the fault level to 500 MVA. Find the external generator reactor need to be connected in series with each generator.

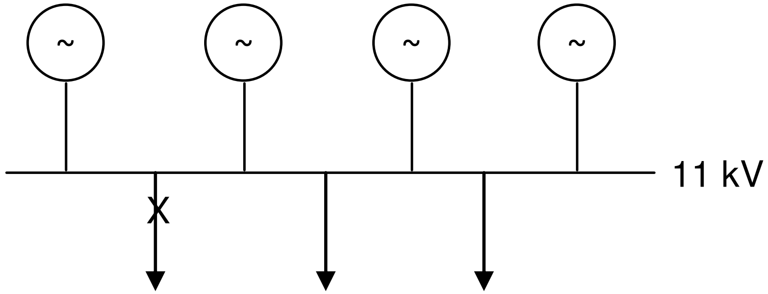


Fig. 3.31 Four alternators – Example 3.5

Solution **Knowing**

Short circuit MVA = prefault voltage in p.u. x fault current in p.u. x Base MVA

$$\text{Fault current in p.u.} = \frac{500}{1.0 \times 25} = 20$$

$$\text{Since Fault current } |I_F| = \frac{1}{|Z_{th}|}, \text{ Thevenin's impedance } |Z_{th}| = \frac{1}{20} = 0.05 \text{ p.u.}$$

Thus reactance in each generator branch $X_g + X_{ext} = 4 \times 0.05 = 0.2$

Therefore X_{ext}, generator reactance needed = 0.2 – 0.16 = 0.04 p.u.

Example 3.8

In the 4-alternator system shown in Fig. 3.32 find the reactance of the current limiting reactor X required to limit the fault level at F in a feeder to 500 MVA.

Each alternator is rated 25 MVA, 11 kV with reactance of 16 %

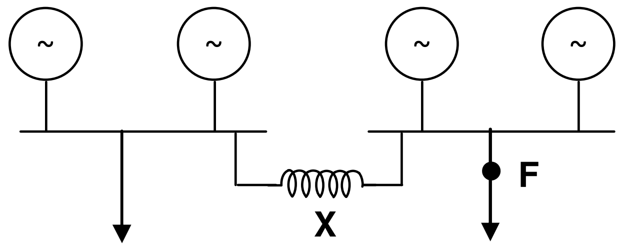


Fig. 3.32 4- alternator system – Example 3.8

Solution

Let the base MVA be 25 and base voltage 11 kV. The p.u. reactance diagram is shown in Fig. 3.33.

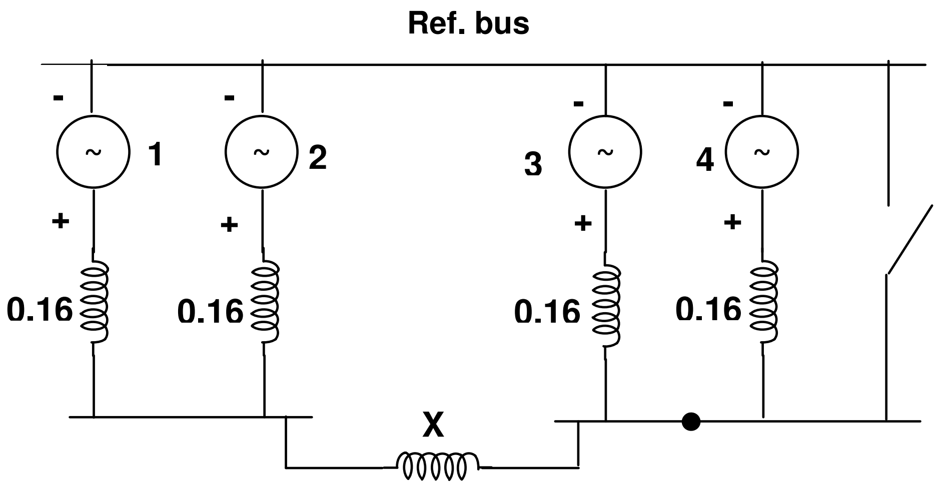


Fig. 3.33 p.u. diagram – Example 3.8

Fault level = 1.0 X $|I_F|$ x 25 = 500 Thus fault current $|I_F|$ = 20 p.u.

Also fault current $|I_F| = \dfrac{1}{|Z_{th}|}$: This gives $X_{th} = \dfrac{1}{20} = 0.05$ p.u.

From the p.u. reactance diagram: $X_{th} = \dfrac{(0.08 + X)\,(0.08)}{0.16 + X} = 0.05$

On solving the above, X = 0.05333 p.u. = $0.05333 \times \dfrac{(11)^2}{25} = 0.2581$ ohm.

<u>**ALITER**</u>

Fault MVA contributed by alternators 3 and 4 = $1.0 \times \dfrac{1}{0.08} \times 25 = 312.5$

Therefore fault MVA of the other side = 500 = 312.5 = 187.5

Equivalent reactance of alternators 1 and 2 with reactance X = 0.08 + X

Thus fault MVA of the other side = $1.0 \times \dfrac{1}{0.08 + X} \times 25 = 187.5$

On solving the above X = 0.05333 p.u. = $0.05333 \times \dfrac{(11)^2}{25} = 0.2581$ ohm.