SRM INSTITUTE OF SCIENCE AND TECHNOLOGY
FACULTY OF ENGINEERING AND TECHNOLOGY
DEPARTMENT OF MATHEMATICS
Ph.D. ENTRANCE EXAMINATION

Duration: 2 hrs. **Maximum Marks: 100**

Answer All Questions
Each Question Carries 2 Marks
PART A (MATHEMATICS)
(25×2=50 MARKS)

1. $\lim_{n\to\infty} \frac{1}{\sqrt[n]{n!}}$ is
   (A) 0 (B) ∞ (C) 1 (D) e

2. A subset in $\mathbb{R}$ is compact if and only if it is
   (A) both open and bounded
   (B) open and unbounded
   (C) closed and unbounded
   (D) both closed and bounded

3. A finite set is
   (A) Compact only
   (B) Closed only
   (C) Both Compact and Closed
   (D) Neither Compact nor Closed

4. Let X be a topological space, then union of finitely many closed sets is
   (A) open (B) open as well as closed
   (C) neither open nor closed (D) closed

5. Let X be a topological space. Let $Bd\ A = \bar{A} \cap \overline{(X - A)}$ for $A \subset X$. Then
   (A) $\bar{A} = \text{Int}\ A \cup Bd\ A$ (B) $\bar{A} = \text{Int}\ A \cap Bd\ A$
   (C) Int A and $Bd\ A$ are not disjoint (D) Int A and $Bd\ A$ are separated

6. Let X be a normed space with $||\ \ ||$ on it. For all $x, y \in X$, then $|\ \|x\| - \|y\|\ |$
   (A) $\leq \|x - y\|$ (B) $\geq \|x - y\|$
   (C) $> |x - y|$ (D) $= \|x\| - \|y\|$

7. Let $X = \mathbb{R}^4$ be the normed space with norm $||\ \ ||_P$, $1 \leq P \leq \infty$. Then the Hahn-Banach extension to X is unique if
   (A) P = ∞ (B) P = 1 (C) P = 2 (D) P = 4

8. The residue of $\frac{1}{z^2+1}$ at $z = i$ is

(A) i (B) $\frac{i}{2}$ (C) $\frac{-i}{2}$ (D) 1

9. The singularity of the function $\frac{\sin z}{z^2}$ at z = 0 is

(A) Pole of order 2 (B) A removable singularity

(C) An essential singularity (D) A simple pole

10. Which of the following is quasi group only

(A) $(\mathbb{Z}, +)$ (B) $(\mathbb{Z}, -)$

(C) $(\mathbb{Z}, \times)$ (D) $(\mathbb{N}, +)$

11. The number of group homomorphisms from $\mathbb{Z}_{10}$ to $\mathbb{Z}_{20}$ is

(A) 10 (B) 5 (C) 1 (D) 0

12. Let $A = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$. Then the smallest positive itnteger n such that $A^n = \mathrm{I}$ is

(A) 1 (B) 2 (C) 4 (D) 6

13. Let A be a $(m \times n)$ matrix and B be a $(n \times m)$ matrix over real numbers with $m < n$. Then

(A) AB is always non-singular

(B) AB is always singular

(C) BA is always singular

(D) BA is always non-singular

14. The trace of the matrix $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}^{20}$ is

(A) $2 \cdot 2^{20} + 3^{20}$ (B) $2^{20} + 3^{20}$ (C) 7^{20} (D) $2^{20} + 3^{20} + 1$

15. The initial value problem $x\frac{dy}{dx} = y, y(0) = 0, x \geq 0$ has

(A) no solution (B) uncountable number of solutions

(C) a unique solution (D) two solutions

16. Let $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ and $y = \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix}$ satisfy $\frac{dy}{dt} = Ay; t > 0; y(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Then

(A) $y_1(t) = 1, y_2(t) = 1$ (B) $y_1(t) = 1, y_2(t) = 1 + t$

(C) $y_1(t) = 1 + t, y_2(t) = 1 + t$ (D) $y_1(t) = 1 + t, y_2(t) = 1$

17. The partial differential equation of $z = xy + f(x^2 + y^2)$ is

(A) $py + x^2 = qx - y^2$ (B) $py + x^2 = qx + y^2$

(C) $qy + x^2 = px + y^2$ (D) $qy - x^2 = px + y^2$

18. The PDE $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = x$ has

(A) only one particular integral.

(B) a particular integral which is linear in *x* and *y*.

(C) a particular integral which is a quadratic polynomial in *x* and *y*.

(D) more than one particular integral.

19. For the linear programming problem

Max. $Z = 2X_1 + 4X_2$ Subject to: $X_1 + 2X_2 \leq 5$, $X_1 + X_2 \leq 3$and $X_1, X_2 \geq 0$. The optimum solution is

(A) (1,2) (B) (1,3) (C) (0,0.25) (D) (2,1)

20. A graph G that has a circuit that contains all the vertices of G is called

(A) an Eulerian graph (B) an Euclidean graph

(C) a Hamiltonian graph (D) a lagrangian graph

21. Under the conditions of incompressibility, the velocity in terms of the potential continuity equation $\nabla . v = 0$ implies

(A) $\nabla \times \nabla \varphi = 0$ (B) $\nabla \varphi = 0$ (C) $\nabla . (\nabla \times \varphi) = 0$ (D) $\nabla^2 \varphi = 0$

22. The value of $(1 + \Delta)(1 - \nabla)$ is

(A) 1 (B) 0 (C) μ (D) Δ

23. Given $\frac{dy}{dx} = x + y, y(1) = 0$, by using Taylor's method, $y(1.1)$ is approximately equals to

1.1103 (B) 0.1103 (C) 0.01103 (D) 0.0113

24. If λ is the angle of friction then the coefficient of friction μ =

$\sin \lambda$ (B) $\sec \lambda$ (C) $\tan \lambda$ (D) $cosec\, \lambda$

25. The height $h(A)$ of a fuzzy set A is defined as $h(A) = sup\, A(x)$ where x belongs to A. Then the fuzzy set A is called normal when

(A) $h(A) = 0$ (B) $h(A) < 0$ (C) $h(A) = 1$ (D) $h(A) < 1$

PART B (Research Methodology)

(50× 1 = 50 MARKS)

ANSWER KEY

Q.No.	Ans	Q.No.	Ans	Q.No.	Ans	Q.No.	Ans	Q.No.	Ans
1	A	6	A	11	A	16	D	21	D
2	D	7	C	12	D	17	B	22	A
3	C	8	C	13	C	18	D	23	B
4	D	9	D	14	A	19	B	24	C
5	A	10	B	15	B	20	C	25	C