Chapter 4

ECONOMIC DISPATCH AND UNIT COMMITMENT

1 INTRODUCTION

A power system has several power plants. Each power plant has several generating units. At any point of time, the total load in the system is met by the generating units in different power plants. Economic dispatch control determines the <u>power output of each power plant,</u> <u>and power output of each generating unit within a power plant ,</u> which will minimize the overall cost of fuel needed to serve the system load.

- We study first the most economical distribution of the output of a power plant between the generating units in that plant. The method we develop also applies to economic scheduling of plant outputs for a given system load without considering the transmission loss.
- Next, we express the transmission loss as a function of output of the various plants.
- Then, we determine how the output of each of the plants of a system is scheduled to achieve the total cost of generation minimum, simultaneously meeting the load on the system.

2 INPUT – OUTPUT CURVE OF GENERATING UNIT

Power plants consisting of several generating units are constructed investing huge amount of money. Fuel cost, staff salary, interest and depreciation charges and maintenance cost are some of the components of operating cost. **Fuel cost is the major portion of operating cost and it can be controlled.** Therefore, we shall consider the fuel cost alone for further consideration.

To get different output power, we need to vary the fuel input. Fuel input can be measured in Tonnes / hour or Millions of Btu / hour. Knowing the cost of the fuel, in terms of Rs. / Tonne or Rs. / Millions of Btu, input to the generating unit can be expressed as Rs / hour. Let C_i Rs / h be the input cost to generate a power of P_i MW in unit i. Fig.1 shows a typical input – output curve of a generating unit. For each generating unit there shall be a minimum and a maximum power generated as $P_{i\ min}$ and $P_{i\ max}$.
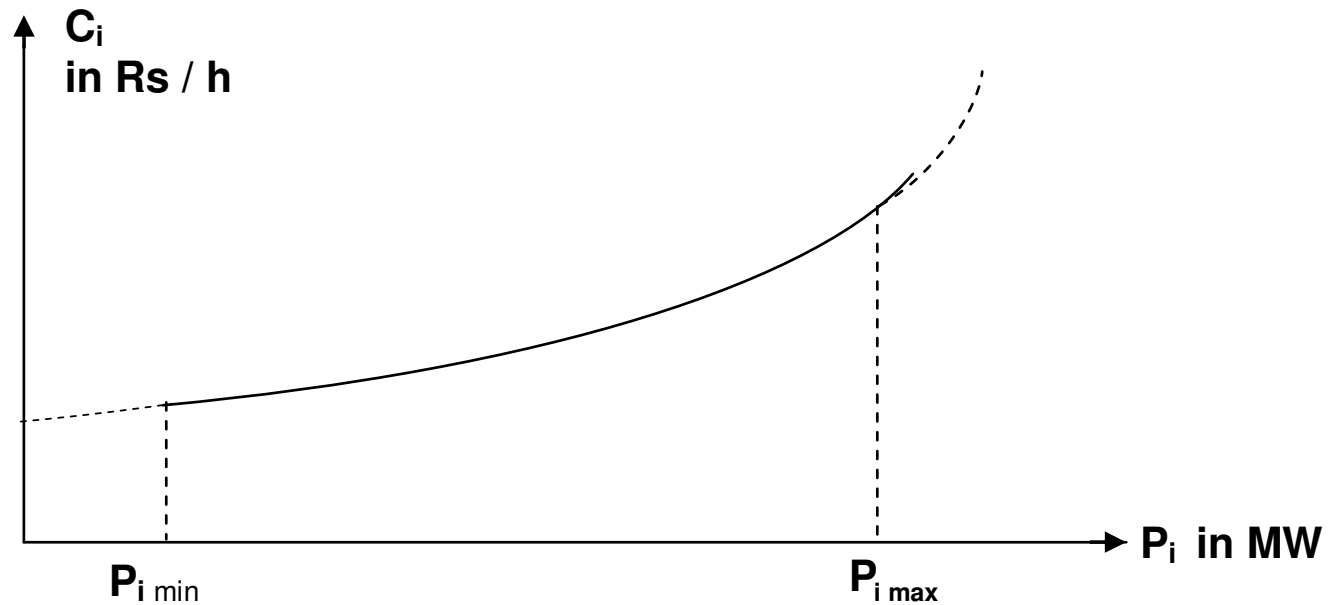

Fig.1 Input-Output curve of a generating unit

If the input-output curve of unit i is quadratic, we can write

$$C_i = \alpha_i P_i^2 + \beta_i P_i + \gamma_i \quad Rs/h \qquad (1)$$

A power plant may have several generator units. If the input-output characteristic of different generator units are identical, then the generating units can be equally loaded. But generating units will generally have different input-output characteristic. This means that, for particular input cost, the generator power P_i will be different for different generating units in a plant.

3 INCREMENTAL COST CURVE

As we shall see, the criterion for distribution of the load between any two units is based on whether increasing the generation of one unit, and decreasing the generation of the other unit by the same amount results in an increase or decrease in total cost. This can be obtained if we can calculate the change in input cost ΔC_i for a small change in power ΔP_i.

Since $\dfrac{dC_i}{dP_i} = \dfrac{\Delta C_i}{\Delta P_i}$ we can write $\Delta C_i = \dfrac{dC_i}{dP_i} \Delta P_i$

$$\Delta C_i = \frac{dC_i}{dP_i} \, \Delta P_i$$

Thus while deciding the optimal scheduling, we are concerned with $\frac{dC_i}{dP_i}$, **INCREMENTAL COST (IC)** which is determined by the slopes of the input-output curves. Thus the **incremental cost curve is the plot of** $\frac{dC_i}{dP_i}$ **versus**

P_i. The dimension of $\frac{dC_i}{dP_i}$ is **Rs / MWh.**

The unit that has the input – output relation as

$$C_i = \alpha_i P_i^2 + \beta_i P_i + \gamma_i \quad \text{Rs / h} \qquad (1)$$

has incremental cost (IC) as

$$IC_i = \frac{dC_i}{dP_i} = 2\alpha_i P_i + \beta_i \qquad (2)$$

Here α_i, β_i and γ_i are constants.

A typical plot of IC versus power output is shown in Fig.2.
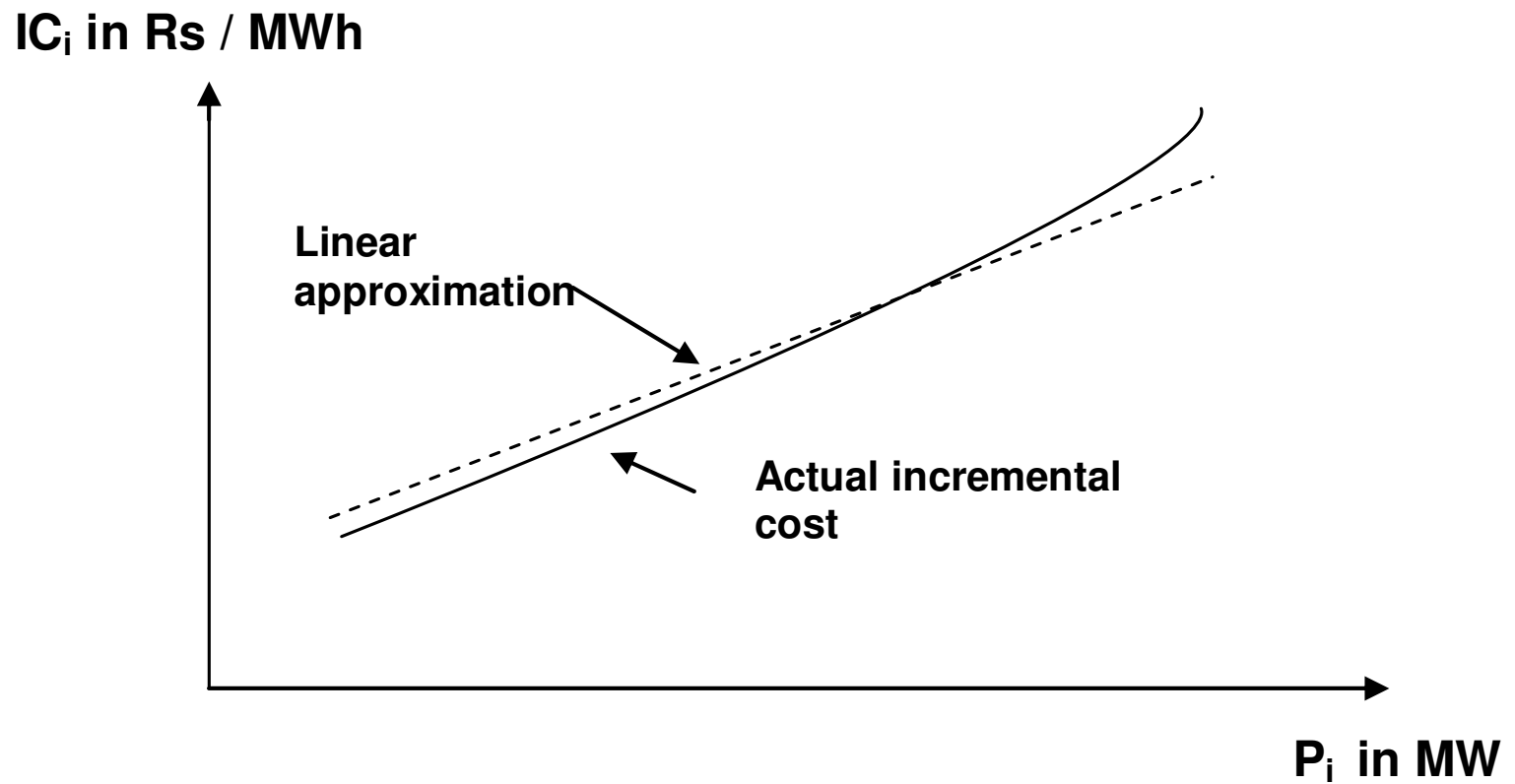

Fig.2 Incremental cost curve

This figure shows that incremental cost is quite linear with respect to power output over an appreciable range. In analytical work, the curve is usually approximated by one or two straight lines. The dashed line in the figure is a good representation of the curve.

We now have the background to understand the principle of economic dispatch which guides distribution of load among the generating units within a plant.

4 ECONOMICAL DIVISION OF PLANT LOAD BETWEEN GENERATING UNITS IN A PLANT

Various generating units in a plant generally have different input-output characteristics. Suppose that the total load in a plant is supplied by two units and that the division of load between these units is such that the incremental cost of one unit is higher than that of the other unit.

Now suppose some of the load is transferred from the unit with higher incremental cost to the unit with lower incremental cost. Reducing the load on the unit with higher incremental cost will result in greater reduction of cost than the increase in cost for adding the same amount of load to the unit with lower incremental cost. The transfer of load from one to other can be continued with a reduction of total cost until the incremental costs of the two units are equal.

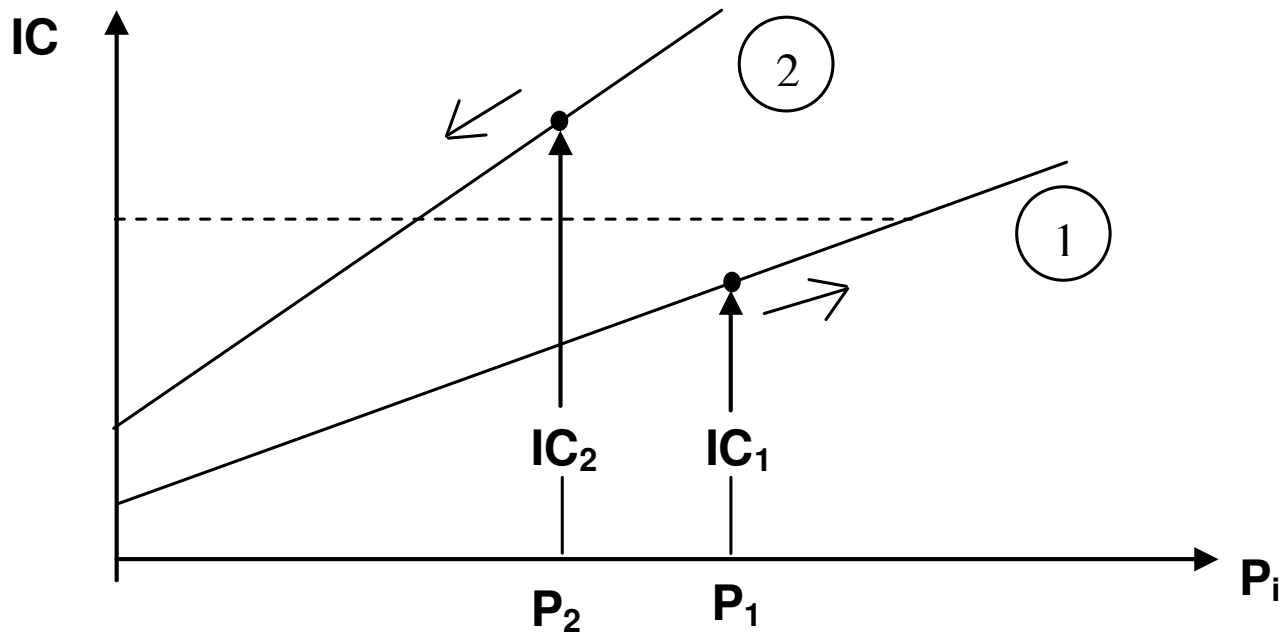This is illustrated through the characteristics shown in Fig. 3

Fig. 3 Two units case

Initially, $IC_2 > IC_1$. Decrease the output power in unit 2 by ΔP and increase output power in unit 1 by ΔP. Now $IC_2 \, \Delta P > IC_1 \, \Delta P$. Thus there will be more decrease in cost and less increase in cost bringing the total cost less. This change can be continued until $IC_1 = IC_2$ at which the total cost will be minimum.

The same reasoning can be extended to a plant with more than two generating units also. In this case, if any two units have different incremental costs, then decrease the output power in unit having higher IC and increase the output power in unit having lower IC. When this process is continued, a stage will reach wherein incremental costs of all the units will be equal. Now the total cost of generation will be minimum.

Thus the economical division of load between units within a plant is that all units must operate at the same incremental cost. Now we shall get the same result mathematically.

Consider a plant having N number of generating units. Input-output curve of the units are denoted as $C_1(P_1), C_2(P_2)\ldots\ldots\ldots, C_N(P_N)$. Our problem is, for a given load demand P_D, find the set of P_i s which minimizes the cost function

$$C_T = C_1(P_1) + C_2(P_2) + \ldots\ldots\ldots + C_N(P_N). \tag{3}$$

subject to the constraints

$$P_D - (P_1 + P_2 + \ldots\ldots\ldots + P_N) = 0 \tag{4}$$

and $\quad P_{i\,min} \leq P_i \leq P_{i\,max} \quad i = 1,2,\ldots\ldots, \ N \tag{5}$

Omitting the inequality constraints for the time being, the problem to be solved becomes

$$\text{Minimize } \ C_T = \sum_{i=1}^{N} C_i(P_i) \tag{6}$$

$$\text{subject to } P_D - \sum_{i=1}^{N} P_i = 0 \tag{7}$$

This optimizing problem with equality constraint can be solved by the method of Lagrangian multipliers. In this method, the Lagrangian function is formed by augmenting the equality constraints to the objective function using proper Lagrangian multipliers. For this case, Lagrangian function is

$$L\ (P_1, P_2, \ldots\ldots, P_N, \lambda) = \sum_{i=1}^{N} C_i\ (P_i) + \lambda\ (P_D - \sum_{i=1}^{N} P_i) \tag{8}$$

where λ is the Lagrangian multiplier. Now this Lagrangian function has to be minimized with no constraints on it. The necessary conditions for a minimum are

$$\frac{\partial L}{\partial P_i} = 0 \quad i = 1, 2, \ldots\ldots, N \text{ and } \frac{\partial L}{\partial \lambda} = 0.$$

For a plant with 3 units

$$L\ (P_1, P_2, P_3, \lambda) = C_1(P_1) + C_2(P_2) + C_3(P_3) + \lambda(P_D - P_1 - P_2 - P_3)$$

Necessary conditions for a minimum are

$$\frac{\partial L}{\partial P_1} = \frac{\partial C_1}{\partial P_1} + \lambda(-1) = 0$$

$$\frac{\partial L}{\partial P_2} = \frac{\partial C_2}{\partial P_2} + \lambda(-1) = 0$$

$$\frac{\partial L}{\partial P_3} = \frac{\partial C_3}{\partial P_3} + \lambda(-1) = 0$$

$$\frac{\partial L}{\partial \lambda} = P_D - P_1 - P_2 - P_3 = 0$$

Generalizing the above, the necessary conditions are

$$\frac{\partial C_i}{\partial P_i} - \lambda = 0 \qquad i = 1,2,\ldots\ldots,N \tag{9}$$

and

$$P_D - \sum_{i=1}^{N} P_i = 0 \tag{10}$$

Here $\dfrac{\partial C_i}{\partial P_i}$ **is the change in production cost in unit i for a small change in generation in unit i. Since change in generation in unit i will affect the production cost of this unit ALONE, we can write**

$$\frac{\partial C_i}{\partial P_i} = \frac{dC_i}{dP_i} \tag{11}$$

Using eqn.(11) in eqn. (9) we have

$$\frac{dC_i}{dP_i} = \lambda \qquad I = 1,2,\ldots\ldots,N \tag{12}$$

Thus the solution for the problem

$$\text{Minimize } C_T = \sum_{i=1}^{N} C_i(P_i) \qquad \text{subject to } P_D - \sum_{i=1}^{N} P_i = 0$$

is obtained when the following equations are satisfied.

$$\frac{dC_i}{dP_i} = \lambda \qquad i = 1,2,\dots\dots,N \tag{13}$$

$$\text{and } P_D - \sum_{i=1}^{N} P_i = 0 \tag{14}$$

The above two conditions give N+1 number of equations which are to be solved for the N+1 number of variables $\lambda, P_1, P_2, \dots\dots\dots, P_N$. Equation (13) simply says that at the minimum cost operating point, the incremental cost for all the generating units must be equal. This condition is commonly known as EQUAL INCREMENTAL COST RULE. Equation (14) is known as POWER BALANCE EQUATION.

It is to be remembered that we have not yet considered the inequality constraints given in eqn. (5). Fortunately, if the solution obtained without considering the inequality constraints satisfies the inequality constraints also, then the obtained solution will be optimum. If for one or more generator units, the inequality constraints are not satisfied, the optimum strategy is obtained by keeping these generator units in their nearest limits and making the other generator units to supply the remaining power as per equal incremental cost rule.

EXAMPLE 1

The cost characteristic of two units in a plant are:

$C_1 = 0.4\,P_1^2 + 160\,P_1 + K_1$ Rs./h

$C_2 = 0.45\,P_2^2 + 120\,P_2 + K_2$ Rs. / h

where P_1 and P_2 are power output in MW. Find the optimum load allocation between the two units, when the total load is 162.5 MW. What will be the daily loss if the units are loaded equally?

SOLUTION
Incremental costs are:

$IC_1 = 0.8\,P_1 + 160$ Rs. / MW h

$IC_2 = 0.9\,P_2 + 120$ Rs. / MW h

Using the equal incremental cost rule

$0.8\,P_1 + 160 = \lambda$ and $0.9\,P_2 + 120 = \lambda$

Since $P_1 + P_2 = 162.5$ we get $\dfrac{\lambda - 160}{0.8} + \dfrac{\lambda - 120}{0.9} = 162.5$

i.e. $\lambda\left[\dfrac{1}{0.8} + \dfrac{1}{0.9}\right] = \dfrac{160}{0.8} + \dfrac{120}{0.9} + 162.5$ i.e. $2.3611\,\lambda = 495.8333$

2.3611 λ = 495.8333

This gives λ = 210 Rs / MWh

Knowing $0.8\ P_1 + 160 = \lambda$ and $0.9\ P_2 + 120 = \lambda$

Optimum load allocation is

$$P_1 = \frac{210 - 160}{0.8} = 62.5\,\text{MW} \quad \text{and} \quad P_2 = \frac{210 - 120}{0.9} = 100\,\text{MW}$$

When the units are equally loaded, $P_1 = P_2 = 81.25$ MW and we have

deviated from the optimum value of P_1 = 62.5 MW and P_2 = 100 MW.

Knowing $C_1 = 0.4\ P_1^2 + 160\ P_1 + K_1$ Rs./h

$\quad\quad\quad C_2 = 0.45\ P_2^2 + 120\ P_2 + K_2$ Rs. / h

Daily loss can also be computed by calculating the total cost C_T, which is

$C_1 + C_2$, for the two schedules.

Thus, Daily loss = 24 x $[\,C_T(81.25, 81.25) - C_T(62.5, 100)\,]$

$\quad\quad\quad\quad$ = 24 x [28361.328 + K_1 + K_2 − (28062.5 + K_1 + K_2)] = Rs. 7171.87

EXAMPLE 2

A power plant has three units with the following cost characteristics:

$$C_1 = 0.5P_1^2 + 215P_1 + 5000 \quad \text{Rs} / \text{h}$$

$$C_2 = 1.0P_2^2 + 270P_2 + 5000 \quad \text{Rs} / \text{h}$$

$$C_3 = 0.7P_3^2 + 160P_3 + 9000 \quad \text{Rs/h}$$

where P_i^s are the generating powers in MW. The maximum and minimum

loads allowable on each unit are 150 and 39 MW. Find the economic

scheduling for a total load of   i) 320 MW   ii) 200 MW

SOLUTION

Knowing the cost characteristics, incremental cost characteristics are
obtained as

$$IC_1 = 1.0P_1 + 215 \quad \text{Rs} / \text{MWh}$$

$$IC_2 = 2.0P_2 + 270 \quad \text{Rs} / \text{MWh}$$

$$IC_3 = 1.4P_3 + 160 \quad \text{Rs} / \text{MWh}$$

Using the equal incremental cost rule

$$1.0\ P_1 + 215 \ = \ \lambda; \qquad 2.0\ P_2 + 270 \ = \ \lambda; \qquad 1.4\ P_3 + 160 \ = \ \lambda$$

$$1.0\ P_1 + 215 = \lambda; \qquad 2.0\ P_2 + 270 = \lambda; \qquad 1.4\ P_3 + 160 = \lambda$$

Case i) Total load = 320 MW   Since $P_1 + P_2 + P_3 = 320$ we have

$$\frac{\lambda - 215}{1.0} + \frac{\lambda - 270}{2.0} + \frac{\lambda - 160}{1.4} = 320$$

i.e. $\lambda\ [\frac{1}{1.0} + \frac{1}{2.0} + \frac{1}{1.4}] = \frac{215}{1.0} + \frac{270}{2.0} + \frac{160}{1.4} + 320$

i.e. $2.2143\ \lambda = 784.2857$ This gives $\lambda = 354.193$ RM / MWh

Thus $P_1 = (354.193 - 215) / 1.0 = 139.193$ MW

$P_2 = (354.193 - 270) / 2.0 = 42.0965$ MW

$P_3 = (354.193 - 16.0) / 1.4 = 138.7093$ MW

All P_i' s lie within maximum and minimum limits. Therefore, economic scheduling is

$P_1 = 139.193$ MW; $P_2 = 42.0965$ MW; $P_3 = 138.7093$ MW

Case ii) Total load = 200 MW   Since $P_1 + P_2 + P_3 = 200$ we have

$$\lambda \left[\frac{1}{1.0} + \frac{1}{2.0} + \frac{1}{1.4} \right] = \frac{215}{1.0} + \frac{270}{20} + \frac{160}{1.4} + 200 \quad \text{i.e.} \quad 2.21429 \lambda = 664.2857$$

This gives $\lambda = 300$ Rs / MWh

Thus $P_1 = (300 - 215) / 1.0 = 85$ MW

$P_2 = (300 - 270) / 2.0 = 15$ MW

$P_3 = (300 - 160) / 1.4 = 100$ MW

It is noted that $P_2 < P_{2\,min}$. Therefore P_2 is set at the min. value of 39 MW.

Then $P_1 + P_3 = 200 - 39 = 161$ MW. This power has to be scheduled

between units 1 and 3. Therefore

$$\lambda \left[\frac{1}{1.0} + \frac{1}{1.4} \right] = \frac{215}{1.0} + \frac{160}{1.4} + 161 \quad \text{i.e.} \quad 1.71429 \lambda = 490.2857$$

This gives $\lambda = 286$ Rs / MWh

Thus $P_1 = (286 - 215) / 1.0 = 71$ MW

$P_3 = (286 - 160) / 1.4 = 90$ MW

P_1 and P_3 are within the limits. Therefore economic scheduling is

$P_1 = 71$ MW; $P_2 = 39$ MW; $P_3 = 90$ MW

EXAMPLE 3

Incremental cost of two units in a plant are:

$$IC_1 = 0.8 \ P_1 + 160 \quad \text{Rs / MWh} \qquad IC_2 = 0.9 \ P_2 + 120 \quad \text{Rs / MWh}$$

where P_1 and P_2 are power output in MW. Assume that both the units are operating at all times. Total load varies from 50 to 250 MW and the minimum and maximum loads on each unit are 20 and 125 MW respectively. Find the incremental cost and optimal allocation of loads between the units for various total loads and furnish the results in a graphical form.

SOLUTION

For lower loads, IC of unit 1 is higher and hence it is loaded to minimum value i.e. P_1 = 20 MW. Total minimum load being 50 MW, when P_1 = 20 MW, P_2 must be equal to 30 MW. Thus initially P_1 = 20 MW , IC_1 = 176 Rs / MWh, P_2 = 30 MW and IC_2 = 147 Rs / MWh. As the load increased from 50 MW, load on unit 2 will be increased until its IC i.e. IC_2 reaches a value of 176 Rs / MWh. When IC_2 = 176 Rs / MWh load on unit 2 is P_2 = (176 − 120) / 0.9 = 62.2 MW. Until that point is reached, P_1 shall remain at 20 MW and the plant IC, i.e. λ is determined by unit 2.

When the plant IC, λ is increased beyond 176 Rs / MWh, unit loads are calculated as

P_1 = (λ - 160) / 0.8 MW

P_2 = (λ - 120) / 0.9 MW

Then the load allocation will be as shown below.

Plant IC λ Rs/MWh	Load on unit 1 P_1 MW	Load on unit 2 P_2 MW	Total load P_1 + P_2 MW
147	20	30	50
176	20	62.2	82.2
180	25	66.6	91.6
190	37.5	77.7	115.2
200	50	88.8	138.8
210	62.5	100.0	162.5
220	75	111.1	186.1
230	87.5	122.2	209.7

Load on unit 2 reaches the maximum value of 125 MW, when λ = (0.9 x 125) + 120 = 232.5 Rs / Mwh. When the plant IC, λ increases further, P_2 shall remain at 125 MW and the load on unit 1 alone increases and its value is computed as P_1 = (λ - 120) / 0.9 MW. Such load allocations are shown below.

Plant IC λ Rs / MWh	Load on unit 1 P_1 MW	Load on unit 2 P_2 MW	Total load $P_1 + P_2$ MW
232.5	90.62	125	215.62
240	100	125	225
250	112.5	125	237.5
260	125	125	250

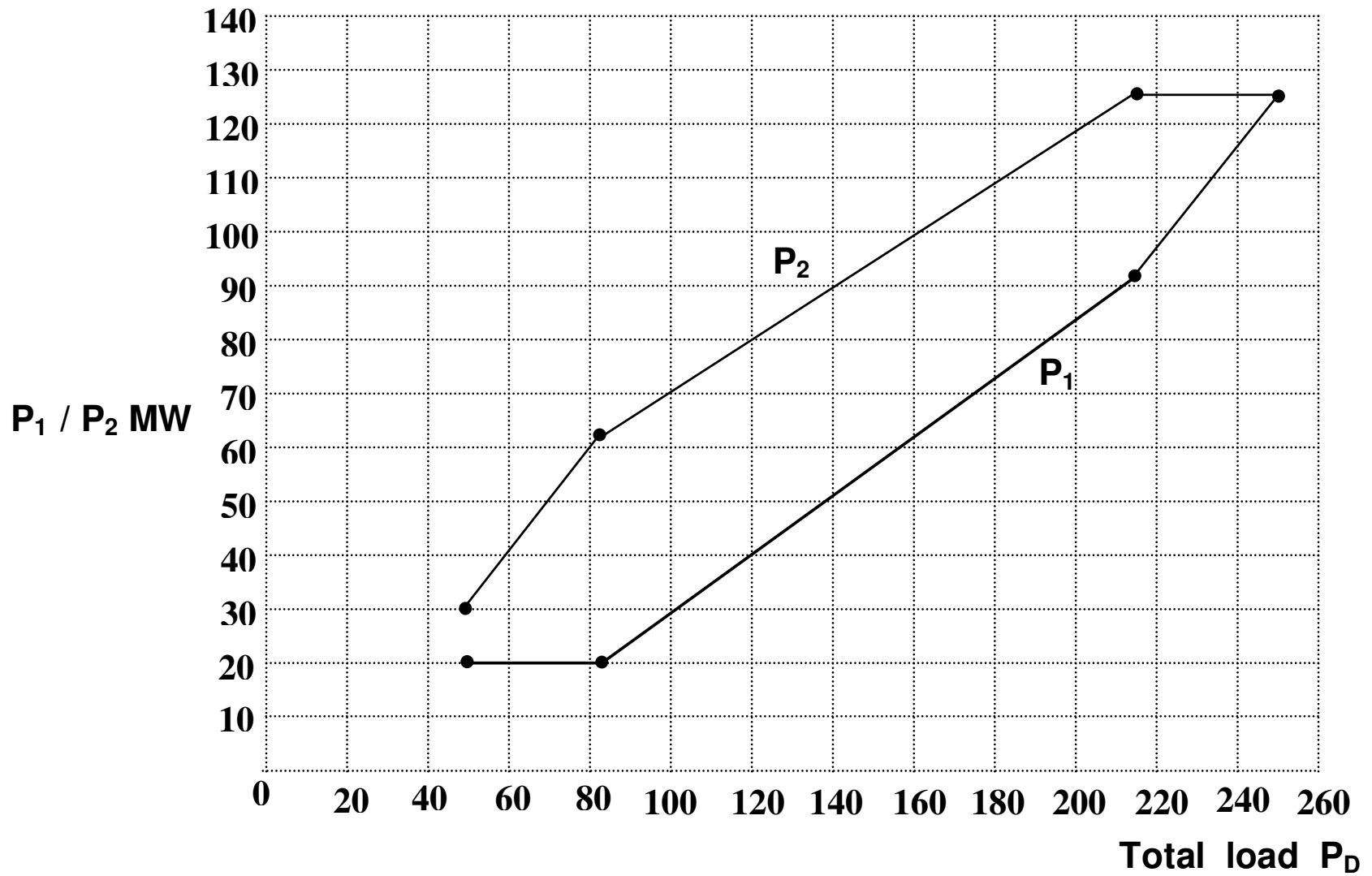The results are shown in graphical form in Fig. 4

Fig. 4 Load allocation for various plant load

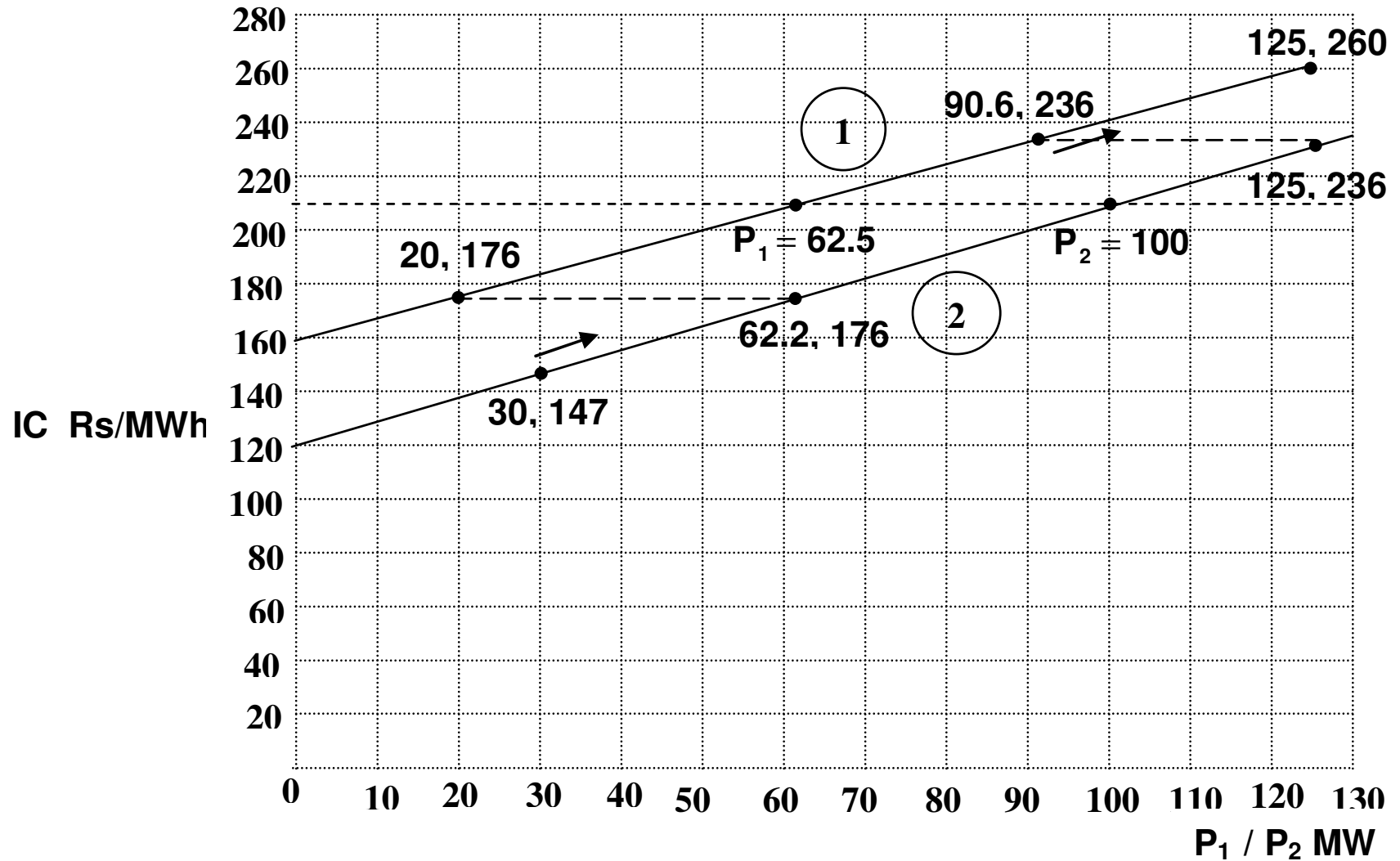**Alternative way of explaination for this problem is shown in Fig. 5**

Fig. 5 Load allocation between two units

5 TRANSMISSION LOSS

Generally, in a power system, several plants are situated at different places. They are interconnected by long transmission lines. The entire system load along with transmission loss shall be met by the power plants in the system. Transmission loss depends on i) line parameters ii) bus voltages and iii) power flow. Determination of transmission loss requires complex computations. However, with reasonable approximations, for a power system with N number of power plants, transmission loss can be represented as

$$P_L = \begin{bmatrix} P_1 & P_2 & \cdots & P_N \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1N} \\ B_{21} & B_{22} & \cdots & B_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ B_{N1} & B_{N2} & \cdots & B_{NN} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_N \end{bmatrix} \qquad (15)$$

where $P_1, P_2, ..., P_N$ are the powers supplied by the plants 1,2,.......,N respectively.

The coefficient matrix **[B]** is called as **LOSS COEFFICIENT MATRIX** or B_{mn} **COEFFICIENT MATRIX** or **B – COEFFICIENT MATRIX. This matrix is symmetric.**

From eq.(15)

$$P_L = \begin{bmatrix} P_1 & P_2 & \ldots & \ldots & P_N \end{bmatrix} \begin{bmatrix} \sum\limits_{n=1}^{N} B_{1n} \, P_n \\ \sum\limits_{n=1}^{N} B_{2n} \, P_n \\ \vdots \\ \sum\limits_{n=1}^{N} B_{Nn} \, P_n \end{bmatrix}$$

$$= P_1 \sum\limits_{n=1}^{N} B_{1n} \, P_n + P_2 \sum\limits_{n=1}^{N} B_{2n} \, P_n + \ldots\ldots + P_N \sum\limits_{n=1}^{N} B_{Nn} \, P_n$$

$$= \sum\limits_{n=1}^{N} P_1 B_{1n} \, P_n + \sum\limits_{n=1}^{N} P_2 B_{2n} \, P_n + \ldots\ldots + \sum\limits_{n=1}^{N} P_N B_{Nn} \, P_n$$

$$= \sum\limits_{m=1}^{N} \sum\limits_{n=1}^{N} P_m \, B_{mn} \, P_n$$

Thus P_L can be written as

$$P_L = \sum\limits_{m=1}^{N} \sum\limits_{n=1}^{N} P_m \, B_{mn} \, P_n \tag{16}$$

$$P_L = \sum_{m=1}^{N} \sum_{n=1}^{N} P_m \, B_{mn} \, P_n \qquad\qquad (16)$$

When the powers are in MW, the B_{mn} coefficients are of dimension 1/ MW. If powers are in per-unit, then B_{mn} coefficients are also in per-unit. Loss coefficient matrix of a power system shall be determined before hand and made available for economic dispatch.

For a two plant system, the expression for the transmission loss is

$$P_L = \begin{bmatrix} P_1 & P_2 \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

$$= \begin{bmatrix} P_1 & P_2 \end{bmatrix} \begin{bmatrix} B_{11} \, P_1 + B_{12} \, P_2 \\ B_{21} \, P_1 + B_{22} \, P_2 \end{bmatrix} = P_1^2 \, B_{11} + P_1 \, P_2 \, B_{12} + P_1 \, P_2 \, B_{21} + P_2^2 \, B_{22}$$

Since B_{mn} coefficient matrix is symmetric, for two plant system

$$P_L = P_1^2 B_{11} + 2 P_1 P_2 B_{12} + P_2^2 B_{22} \qquad (17)$$

In later calculations we need the **Incremental Transmission Loss** (ITL),

$$\frac{\partial P_L}{\partial P_i}.$$

For two plant system

$$\frac{\partial P_L}{\partial P_1} = 2B_{11} P_1 + 2B_{12} P_2 \qquad (18)$$

$$\frac{\partial P_L}{\partial P_2} = 2B_{12} P_1 + 2B_{22} P_2 \qquad (19)$$

This can be generalized as

$$\frac{\partial P_L}{\partial P_m} = \sum_{n=1}^{N} 2 B_{mn} P_n \qquad m = 1,2,\ldots\ldots\ldots, N \qquad (20)$$

6 ECONOMIC DIVISION OF SYSTEM LOAD BETWEEN VARIOUS PLANTS IN THE POWER SYSTEM

It is to be noted that different plants in a power system will have different cost characteristics.

Consider a power system having N number of plants. Input-output characteristics of the plants are denoted as $C_1(P_1), C_2(P_2), \ldots\ldots\ldots, C_N(P_N)$.

Our problem is for a given system load demand P_D, find the set of plant generation $P_1, P_2, \ldots\ldots\ldots, P_N$ which minimizes the cost function

$$C_T = C_1(P_1) + C_2(P_2) + \ldots\ldots\ldots + C_N(P_N) \tag{21}$$

subject to the constraints

$$P_D + P_L - (P_1 + P_2 + \ldots\ldots\ldots + P_N) = 0 \tag{22}$$

and $P_{i\,min} \leq P_i \leq P_{i\,max}$ I = 1,2,………,N $\tag{23}$

Inequality constraints are omitted for the time being.

The problem to be solved becomes

$$\text{Minimize} \quad C_T = \sum_{i=1}^{N} C_i(P_i) \tag{24}$$

$$\text{subject to} \quad P_D + P_L - \sum_{i=1}^{N} P_i = 0 \tag{25}$$

For this case the Lagrangian function is

$$L(P_1, P_2, \ldots \ldots, P_N, \lambda) = \sum_{i=1}^{N} C_i(P_i) + \lambda \left(P_D + P_L - \sum_{i=1}^{N} P_i \right) \tag{26}$$

This Lagrangian function has to be minimized with no constraint on it. The necessary conditions for a minimum are

$$\frac{\partial L}{\partial P_i} = 0 \quad i = 1, 2, \ldots \ldots, N$$

and $\quad \dfrac{\partial L}{\partial \lambda} = 0.$

$$L(P_1, P_2, \ldots\ldots, P_N, \lambda) = \sum_{i=1}^{N} C_i(P_i) + \lambda \left(P_D + P_L - \sum_{i=1}^{N} P_i \right) \tag{26}$$

For a system with 2 plants

$$L(P_1, P_2, \lambda) = C_1(P_1) + C_2(P_2) + \lambda(P_D + P_L - P_1 - P_2)$$

$$\frac{\partial L}{\partial P_1} = \frac{\partial C_1}{\partial P_1} + \lambda\left(\frac{\partial P_L}{\partial P_1} - 1\right) = 0$$

$$\frac{\partial L}{\partial P_2} = \frac{\partial C_2}{\partial P_2} + \lambda\left(\frac{\partial P_L}{\partial P_2} - 1\right) = 0$$

$$\frac{\partial L}{\partial \lambda} = P_D + P_L - P_1 - P_2 = 0$$

Generalizing this, for system with N plants, the necessary conditions for a minimum are

$$\frac{\partial C_i}{\partial P_i} + \lambda \left[\frac{\partial P_L}{\partial P_i} - 1 \right] = 0 \qquad i = 1,2,\ldots\ldots,N \qquad (27)$$

and
$$P_D + P_L - \sum_{i=1}^{N} P_i = 0 \qquad (28)$$

As discussed in earlier case, we can write

$$\frac{\partial C_i}{\partial P_i} = \frac{dC_i}{dP_i} \qquad (29)$$

Using eqn. (29) in eqn. (27) we have

$$\frac{dC_i}{dP_i} + \lambda \frac{\partial P_L}{\partial P_i} = \lambda \quad i = 1,2,\ldots\ldots,N \qquad (30)$$

and
$$P_D + P_L - \sum_{i=1}^{N} P_i = 0 \qquad (31)$$

$$\frac{dC_i}{dP_i} + \lambda \frac{\partial P_L}{\partial P_i} = \lambda \quad i=1,2,\ldots\ldots,N \tag{30}$$

and $$P_D + P_L - \sum_{i=1}^{N} P_i = 0 \tag{31}$$

These equations can be solved for the plant generations $P_1, P_2, \ldots\ldots, P_N$.
As shown in the next section the value of λ in eqn. (30) is the INCREMENTAL COST OF RECEIVED POWER.

The eqns. described in eqn. (30) are commonly known as COORDINATION EQUATIONS as they link the incremental cost of plant $\frac{dC_i}{dP_i}$, incremental cost of received power λ and the incremental transmission loss (ITL) $\frac{\partial P_L}{\partial P_i}$. Equation (31) is the POWER BALANCE EQUATION.

Coordination equations

$$\frac{dC_i}{dP_i} + \lambda \frac{\partial P_L}{\partial P_i} = \lambda \quad i=1,2,\ldots\ldots.,N \tag{30}$$

can also be written as

$$IC_i + \lambda \, ITL_i = \lambda \quad i = 1,2,\ldots\ldots.,N \tag{32}$$

The N number of coordinate equations together with the power balance equation are to be solved for the plant loads $P_1, P_2, \ldots\ldots.,P_N$ to obtain the economic schedule.

7 INCREMENTAL COST OF RECEIVED POWER

The value of λ in the coordination equations is the incremental cost of received power. This can be proved as follows:

The coordination equations can be written as

$$\frac{\partial C_i}{\partial P_i} = \lambda \left[1 - \frac{\partial P_L}{\partial P_i} \right] \qquad (33)$$

i.e. $\dfrac{\Delta C_i}{\Delta P_i} \dfrac{1}{1 - \dfrac{\Delta P_L}{\Delta P_i}} = \lambda$ i.e. $\dfrac{\Delta C_i}{\Delta P_i - \Delta P_L} = \lambda$ (34)

Since $P_D = \displaystyle\sum_{i=1}^{N} P_i - P_L$ $\dfrac{\partial P_D}{\partial P_i} = 1 - \dfrac{\partial P_L}{\partial P_i}$ i.e. $\Delta P_D = \Delta P_i - \Delta P_L$ (35)

Using eqn.(35) in eqn.(34) gives $\dfrac{\Delta C_i}{\Delta P_D} = \lambda$ (36)

Thus λ is the incremental cost of received power.

8 PENALTY FACTORS

To have a better feel about the coordination equations, let us rewrite the

same as $\quad \dfrac{dC_i}{dP_i} = \lambda \left[1 - \dfrac{\partial P_L}{\partial P_i} \right] \qquad i = 1,2,\ldots\ldots,N$ $\qquad\qquad$ (37)

Thus $\quad \left[\dfrac{1}{1 - \dfrac{\partial P_L}{\partial P_i}} \right] \dfrac{dC_i}{dP_i} = \lambda \qquad i = 1,2,\ldots\ldots,N$ $\qquad\qquad$ (38)

The above equation is often written as

$L_i \dfrac{dC_i}{dP_i} = \lambda \qquad i = 1,2,\ldots\ldots,N$ $\qquad\qquad$ (39)

where L_i, which is called the **PENALTY FACTOR** of plant i, is given by

$L_i = \dfrac{1}{1 - \dfrac{\partial P_L}{\partial P_i}} \qquad i = 1,2,\ldots\ldots,N$ $\qquad\qquad$ (40)

The results of eqn. (39) means that minimum fuel cost is obtained when the incremental cost of each plant multiplied by its penalty factor is the same for the plants in the power system.

9 OPTIMUM SCHEDULING OF SYSTEM LOAD BETWEEN PLANTS

- SOLUTION PROCEDURE

To determine the optimum scheduling of system load between plants, the data required are i) system load, ii) incremental cost characteristics of the plants and iii) loss coefficient matrix. The iterative solution procedure is:

Step 1

For the first iteration, choose suitable initial value of λ. While finding this, one way is to assume that the transmission losses are zero and the plants are loaded equally.

Step 2

Substitute the value of λ into the coordination equations

$$\frac{dC_i}{dP_i} + \lambda \frac{\partial P_L}{\partial P_i} = \lambda \qquad i = 1,2,………..,N$$

i.e. $(2\alpha_i P_i + \beta_i) + \lambda \sum_{n=1}^{N} 2B_{mn} P_n = \lambda \qquad i = 1,2,………..,N$

The above set of linear simultaneous equations are to be solved for the values $P_i's$ by some efficient means. Gauss Seidel method is recommended.

Step 3

Compute the transmission loss P_L from $P_L = [P] [B] [P^t]$

where $[P] = [P_1 \ P_2 \ ....P_N]$ and $[B]$ is the loss coefficient matrix.

Step 4

Compare $\sum_{i=1}^{N} P_i$ with $P_D + P_L$ to check the power balance. If the power balance is satisfied within a specified tolerance, then the present solution is the optimal solution; otherwise update the value of λ.

First time updating can be done judiciously. Value of λ is increased or decreased by about 5% depending on

$$\sum_{i=1}^{N} P_i < P_D + P_L \quad \text{or} \quad \sum_{i=1}^{N} P_i > P_D + P_L$$

In the subsequent iterations, using linear interpolation, value of λ can be updated as
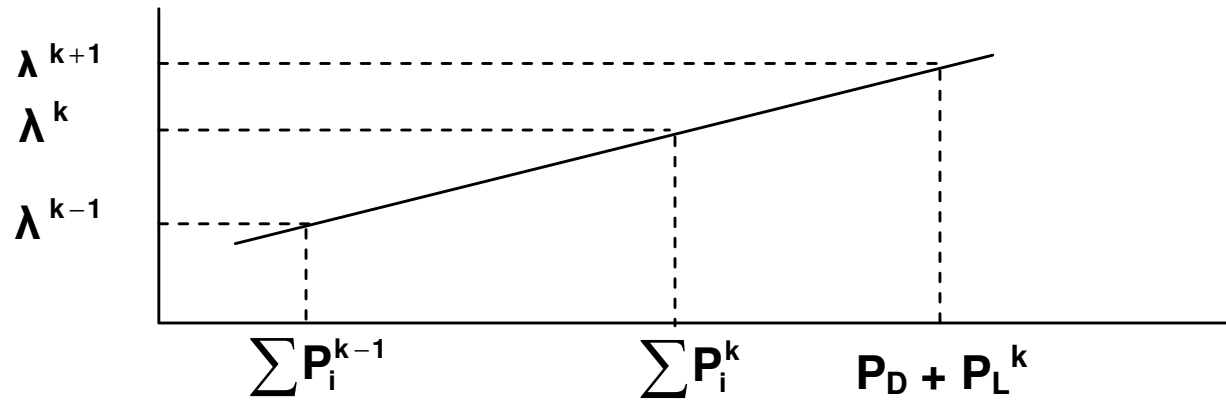
$$\lambda^{k+1} = \lambda^{k} + \frac{\lambda^{k} - \lambda^{k-1}}{\sum\limits_{i=1}^{N} P_i^{k} - \sum\limits_{i=1}^{N} P_i^{k-1}} \; [P_D + P_L^{k} - \sum\limits_{i=1}^{N} P_i^{k}] \tag{41}$$

Here **k-1, k** and **k+1** are the previous iteration count, present iterative count and the next iteration count respectively.

Step 5

Return to Step 2 and continue the calculations of Steps 2, 3 and 4 until the power balance equation is satisfied with desired accuracy.

The above procedure is now illustrated through an example.

EXAMPLE 4

Consider a power system with two plants having incremental cost as

$$IC_1 = 1.0 \, P_1 + 200 \quad Rs \, / \, MWh \qquad\qquad IC_2 = 1.0 \, P_2 + 150 \quad Rs \, / \, MWh$$

Loss coefficient matrix is given by $\quad B \; = \; \begin{bmatrix} 0.001 & -0.0005 \\ -0.0005 & 0.0024 \end{bmatrix}$

Find the optimum scheduling for a system load of 100 MW.

SOLUTION

Assume that there is no transmission loss and the plants are loaded equally. Then $P_1 = 50 \, MW$. Initial value of $\lambda = (1.0 \text{ x } 50) + 200 = 250$ Rs / MWh.

Coordination equations:

$$1.0 \, P_1 + 200 + 250 \, (\, 0.002 P_1 - 0.001 \, P_2 \,) = 250$$
$$1.0 \, P_2 + 150 + 250 \, (-0.001 P_1 + 0.0048 P_2 \,) = 250$$

i.e. $\quad 1.5 \, P_1 - 0.25 \, P_2 = 50 \qquad$ i.e. $\quad P_1 = 33.3333 + 0.1667 P_2$

$\qquad -0.25 P_1 + 2.2 \, P_2 = 100 \qquad\qquad P_2 = 45.4545 + 0.1136 P_1$

These equations are solved by Gauss Seidel method.

$$P_1 = 33.3333 + 0.1667\,P_2 \qquad\qquad P_2 = 45.4545 + 0.1136\,P_1$$

Iteration count	P_1	P_2
0	0	0
1	33.3333	49.2411
2	41.5418	50.1736
3	41.6972	50.1913
4	41.7002	50.1916

$$P_L = \begin{bmatrix} 41.7002 & 50.1916 \end{bmatrix} \begin{bmatrix} 0.001 & -0.0005 \\ -0.0005 & 0.0024 \end{bmatrix} \begin{bmatrix} 41.7002 \\ 50.1916 \end{bmatrix}$$

$$= \begin{bmatrix} 41.7002 & 50.1916 \end{bmatrix} \begin{bmatrix} 0.01660 \\ 0.09961 \end{bmatrix} = 5.6918 \text{ MW}$$

$$P_1 + P_2 = 91.8918 \text{ MW} \qquad\qquad P_D + P_L = 105.6918 \text{ MW}$$

Since $P_1 + P_2 < P_D + P_L$, λ value should be increased. It is increased by 4 %.

New value of $\lambda = 250 \times 1.04 = 260$ Rs / MWh.

Coordination equations:

$$1.0\ P_1 + 200 + 260\ (\ 0.002\,P_1 - 0.001\ P_2\ ) = 260$$

$$1.0\ P_2 + 150\ + 260\ (-0.001P_1 + 0.0048\,P_2\ ) = 260$$

i.e.$\quad 1.52\ P_1 - 0.26\ P_2\ =\ 60 \qquad$ i.e.$\quad P_1 = 39.4737 + 0.1711P_2$

$$-0.26\ P_1 + 2.248\ P_2 = 110 \qquad\qquad P_2 = 48.9324 + 0.1157\,P_1$$

These equations are solved by Gauss Seidel method.

Iteration count	P_1	P_2
0	0	0
1	39.4737	53.4995
2	48.6275	54.5586
3	48.8087	54.5796
4	48.8123	54.5800

$$P_L = \begin{bmatrix} 48.8123 & 54.5800 \end{bmatrix} \begin{bmatrix} 0.001 & -0.0005 \\ -0.0005 & 0.0024 \end{bmatrix} \begin{bmatrix} 48.8123 \\ 54.5800 \end{bmatrix}$$

$$= \begin{bmatrix} 48.8123 & 54.5800 \end{bmatrix} \begin{bmatrix} 0.02152 \\ 0.10659 \end{bmatrix}$$

$$=\ 6.8681\ MW$$

$$P_1 + P_2 = 103.3923\ MW\ ; \quad P_D + P_L = 106.8681\ MW$$

$$P_1 + P_2 \neq P_D + P_L$$

Knowing two values of λ and the corresponding total generation powers, new value of λ is computed as

$$\lambda = 260 + \frac{260 - 250}{103.3932 - 91.8918} \, (106.8681 - 103.3923) = 263 \text{ Rs / MWh}$$

With this new value of λ, coordination equations are formed and the procedure has to be repeated. The following table shows the results obtained.

λ	P_1	P_2	P_L	$P_1 + P_2$	$P_D + P_L$
250	41.7002	50.1916	5.6936	91.8918	105.6936
260	48.8123	54.5800	6.8681	103.3923	106.8681
263	50.9119	55.8769	7.2405	106.789	107.2405
263.3	51.1061	55.9878	7.2737	107.0939	107.2737
263.5	51.2636	56.0768	7.3003	107.3404	107.3003
263.467	51.2401	56.0659	7.2969	107.3060	107.2969

Optimum schedule is
$$P_1 = 51.2401 \text{ MW}$$
$$P_2 = 56.0659 \text{ MW}$$

For this transmission loss is 7.2969 MW

10 BASE POINT AND PARTICIPATION FACTORS

The system load will keep changing in a cyclic manner. It will be higher during day time and early evening when industrial loads are high. However during night and early morning the system load will be much less.

The optimal generating scheduling need to be solved for different load conditions because load demand P_D keeps changing. When load changes are small, it is possible to move from one optimal schedule to another using PARTICIPATING FACTORS.

We start with a known optimal generation schedule, P_1^0, P_2^0, …, P_N^0, for a particular load P_D. This schedule is taken as BASE POINT and the corresponding incremental cost is λ^0. Let there be a small increase in load of ΔP_D. To meet with this increased load, generations are to be increased as ΔP_1, ΔP_2, …., ΔP_N. Correspondingly incremental cost increases by $\Delta\lambda$.

Knowing that for i^{th} unit, $C_i = \alpha_i\ P_i^2 + \beta_i\ P_i + \gamma_i$, incremental cost is

$$IC_i = 2\ \alpha_i\ P_i + \beta_i = \lambda \qquad (42)$$

Small change in incremental cost and corresponding change in generation

are related as $\qquad \dfrac{\Delta\lambda}{\Delta P_i} = 2\ \alpha_i \qquad (43)$

Thus $\qquad \Delta P_i = \dfrac{\Delta\lambda}{2\alpha_i}\quad$ for $\ i = 1,2,\ldots, N \qquad (44)$

Change in load should be equal to the total change in generations.

Therefore $\qquad \Delta P_D = \displaystyle\sum_{i=1}^{N}\Delta P_i\ =\ \Delta\lambda\ \sum_{i=1}^{N}\dfrac{1}{2\alpha_i} \qquad (45)$

From the above two equations $\quad \dfrac{\Delta P_i}{\Delta P_D} = \dfrac{\dfrac{1}{2\alpha_i}}{\displaystyle\sum_{i=1}^{N}\dfrac{1}{2\alpha_i}} = k_i\quad$ for $\ i = 1,2,\ldots, N\ $ (46)

The ratio $\dfrac{\Delta P_i}{\Delta P_D}$ is known as the PARTICIPATION FACTOR of generator I,

represented as k_i. Once all the k_i s, are calculated from eq.(46), the change

in generations are given by $\quad \Delta P_i = k_i\ \Delta P_D\quad$ for $\ i = 1,2,\ldots, N \qquad (47)$

EXAMPLE 5

Incremental cost of three units in a plant are:

$IC_1 = 0.8 P_1 + 160$ Rs / MWh; $IC_2 = 0.9 P_2 + 120$ Rs / MWh;   and

$IC_3 = 1.25 P_3 + 110$ Rs / MWh

where P_1, P_2 and P_3 are power output in MW. Find the optimum load allocation when the total load is 242.5 MW.

Using Participating Factors, determine the optimum scheduling when the load increases to 260 MW.

Solution

Using the equal incremental cost rule

$0.8\ P_1 + 160 = \lambda$; $0.9\ P_2 + 120 = \lambda$; $1.25\ P_3 + 110 = \lambda$

Since $P_1 + P_2 + P_3 = 242.5$ we get $\dfrac{\lambda - 160}{0.8} + \dfrac{\lambda - 120}{0.9} + \dfrac{\lambda - 110}{1.25} = 242.5$

i.e $\lambda\ [\dfrac{1}{0.8} + \dfrac{1}{0.9} + \dfrac{1}{1.25}] = \dfrac{160}{0.8} + \dfrac{120}{0.9} + \dfrac{110}{1.25} + 242.5$ i.e. 3.1611 $\lambda\ = 663.8333$

This gives $\lambda\ = 210$ Rs / MWh

Optimum load allocation is

$P_1 = \dfrac{210 - 160}{0.8} = 62.5\,\text{MW}$; $P_2 = \dfrac{210 - 120}{0.9} = 100\ \text{MW}$; $P_{31} = \dfrac{210 - 110}{1.25} = 80\ \text{MW}$

Participation Factors are:

$$k_1 = \cfrac{\cfrac{1}{0.8}}{\cfrac{1}{0.8} + \cfrac{1}{0.9} + \cfrac{1}{1.25}} = 0.3954$$

$$k_2 = \cfrac{\cfrac{1}{0.9}}{\cfrac{1}{0.8} + \cfrac{1}{0.9} + \cfrac{1}{1.25}} = 0.3515$$

$$k_3 = \cfrac{\cfrac{1}{1.25}}{\cfrac{1}{0.8} + \cfrac{1}{0.9} + \cfrac{1}{1.25}} = 0.2531$$

Change in load $\Delta P_D = 260 - 242.5 = 17.5$ MW

Change in generations are:

$\Delta P_1 = 0.3954 \times 17.5 = 6.9195$ MW

$\Delta P_2 = 0.3515 \times 17.5 = 6.15125$ MW

$\Delta P_3 = 0.2531 \times 17.5 = 4.42925$ MW

Thus optimum schedule is:

$P_1 = 69.4195$ MW; $P_2 = 106.15125$ MW; $P_3 = 84.42925$ MW

Example 6

A power plant has two units with the following cost characteristics:

$$C_1 = 0.6 \, P_1^2 + 200 \, P_1 + 2000 \;\; \text{Rs / hour}$$

$$C_2 = 1.2 \, P_2^2 + 150 \, P_2 + 2500 \;\; \text{Rs / hour}$$

where P_1 and P_2 are the generating powers in MW. The daily load cycle is as follows:

6:00 A.M. to 6:00 P.M.   150 MW

6:00 P.M. to 6:00 A.M.   50 MW

The cost of taking either unit off the line and returning to service after 12 hours is Rs 5000. Considering 24 hour period from 6:00 A.M. one morning to 6:00 A.M. the next morning

a. Would it be economical to keep both units in service for this 24 hour period or remove one unit from service for 12 hour period from 6:00 P.M. one evening to 6:00 A.M. the next morning ?

b. Compute the economic schedule for the peak load and off peak load conditions.

c. Calculate the optimum operating cost per day.

d. If operating one unit during off peak load is decided, up to what cost of taking one unit off and returning to service after 12 hours, this decision is acceptable ?

Solution

To meet the peak load of 150 MW, both the units are to be operated.

However, during 6:00 pm to 6:00 am, load is 50 MW and there is a choice

i) both the units are operating

ii) one unit (either 1 or 2 – to be decided) is operating

i) When both the units are operating

$IC_1 = 1.2\ P_1 + 200$ Rs / MWh

$IC_2 = 2.4\ P_2 + 150$ Rs / MWh   Using equal IC rule

$$\frac{\lambda - 200}{1.2} + \frac{\lambda - 150}{2.4} = 50; \quad 1.25\ \lambda = 279.1667 \quad \text{and} \quad \lambda = 223.3333$$

Therefore $P_1 = 19.4444$ MW; $P_2 = 30.5555$ MW

Then $C_T = 14319.42$ Rs / h

For 12 hour period, cost of operation = Rs 171833.04

ii) If unit 1 is operating, $C_1 |_{P1 = 50}$ = 13500 Rs / h

If unit 2 is operating, $C_2 |_{P2 = 50}$ = 13000 Rs / h

Between units 1 and 2, it is economical to operate unit 2. If only one unit is operating during off-peak period, cost towards taking out and connecting it back also must be taken.

Therefore, for off-peak period (with unit 2 alone operating) cost of operation = (13000 x 12) + 5000 = Rs. 161000

Between the two choices (i) and (ii), choice (ii) is cheaper. Therefore, <u>during 6:00 pm to 6:00 am, it is better to operate unit 2 alone.</u>

b. During the peak period, P_D = 150 MW. With equal IC rule

$$\frac{\lambda - 200}{1.2} + \frac{\lambda - 150}{2.4} = 150; \quad 1.25\,\lambda = 379.1667 \quad \text{and} \quad \lambda = 303.3333$$

Therefore P_1 = 86.1111 MW; P_2 = 63.8889 MW

Thus, economic schedule is

During 6:00 am to 6:00 pm P_1 = 86.1111 MW; P_2 = 63.8889 MW

During 6:00 pm to 6:00 am P_1 = 0; P_2 = 50 MW

c. Cost of operation
for peak period

$$= 12\,[\,C_1|_{P1\,=\,86.1111} + C_2|_{P2\,=\,63.8889}\,]$$

$$= 12 \times 40652.78 = \text{Rs } 487833$$

Cost of operation for off-peak period = Rs 161000

Therefore, optimal operating cost per day = Rs 648833

d. If both the units are operating during off-peak period, cost of operation = Rs 171833

If unit 2 alone is operating during off-peak period, cost of operation $\left.\right\}$ = Rs (13000 x 12) + x

= Rs 156000 + x

For Critical value of x: 156000 + x = 171833

x = Rs 15833

Therefore, until the cost of taking one unit off and returning it to service after 12 hours, is less than Rs 15833, operating unit 2 alone during the off-peak period <u>is acceptable.</u>

11 UNIT COMMITMENT

Economic dispatch gives the optimum schedule corresponding to one particular load on the system. The total load in the power system varies throughout the day and reaches different peak value from one day to another. Different combination of generators, are to be connected in the system to meet the varying load.

When the load increases, the utility has to decide in advance the sequence in which the generator units are to be brought in. Similarly, when the load decrease, the operating engineer need to know in advance the sequence in which the generating units are to be shut down.

The problem of finding the order in which the units are to be brought in and the order in which the units are to be shut down over a period of time, say one day, so the total operating cost involved on that day is minimum. This is known as Unit Commitment (UC) problem. Thus UC problem is economic dispatch over a day. The period considered may a week, month or a year.

But why is the problem in the operation of electric power system? Why not just simply commit enough units to cover the maximum system load and leave them running? Note that to "commit" means a generating unit is to be "turned on"; that is, bring the unit up to speed, synchronize it to the system and make it to deliver power to the network. "Commit enough units and leave them on line" is one solution. However, it is quite expensive to run too many generating units. A great deal of money can be saved by turning units off (decommiting them) when they are not needed.

Example 7

The following are data pertaining to three units in a plant.

Unit 1:   Min. = 150 MW;   Max. = 600 MW

$$C_1 = 5610 + 79.2\ P_1 + 0.01562\ P_1^2\ Rs\ /\ h$$

Unit 2:   Min. = 100 MW;   Max. = 400 MW

$$C_2 = 3100 + 78.5\ P_2 + 0.0194\ P_2^2\ Rs\ /\ h$$

Unit 3:   Min. = 50 MW;   Max. = 200 MW

$$C_3 = 936 + 95.64\ P_3 + 0.05784\ P_3^2\ Rs\ /\ h$$

What unit or combination of units should be used to supply a load of 550 MW most economically?

Solution

To solve this problem, simply try all combination of three units. Some combinations will be infeasible if the sum of all maximum MW for the units committed is less than the load or if the sum of all minimum MW for the units committed is greater than the load. For each feasible combination, units will be dispatched using equal incremental cost rule studied earlier. The results are presented in the Table below.

Unit	Min	Max
1	150	600
2	100	400
3	50	200

Unit 1	Unit 2	Unit 3	Min. Gen	Max. Gen	P_1	P_2	P_3	Total cost
Off	Off	Off	0	0	Infeasible			
On	Off	Off	150	600	550	0	0	53895
Off	On	Off	100	400	Infeasible			
Off	Off	On	50	200	Infeasible			
On	On	Off	250	1000	295	255	0	54712
Off	On	On	150	600	0	400	150	54188
On	Off	On	200	800	500	0	50	54978
On	On	On	300	1200	267	233	50	56176

Note that the least expensive way of meeting the load is not with all the three units running, or any combination involving two units. Rather it is economical to run unit one alone.

Example 8

Daily load curve to be met by a plant having three units is shown below.
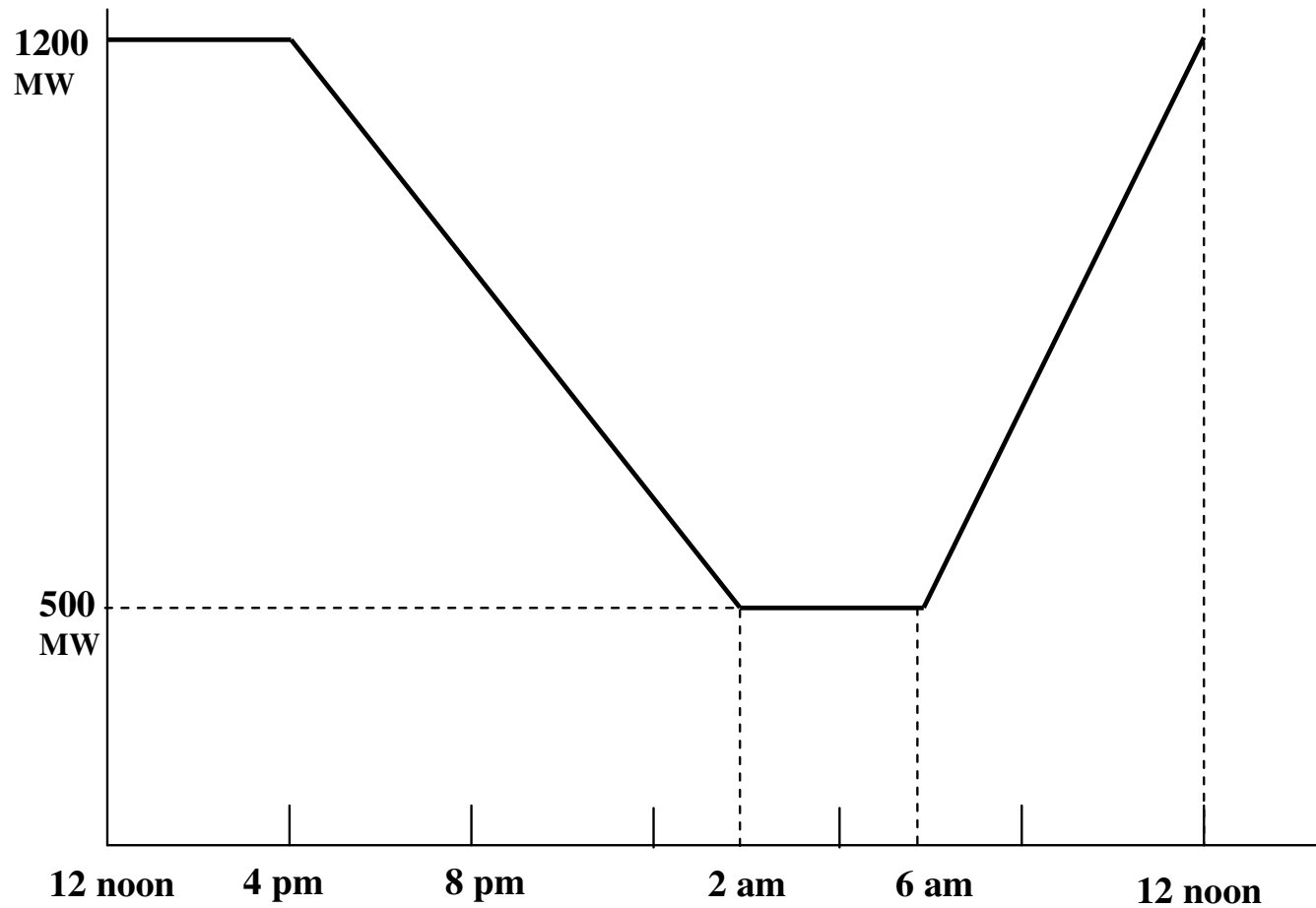
Data pertaining to the three units are the same in previous example.

Starting from the load of 1200 MW, taking steps of 50 MW find the shut-down rule.

Solution

For each load starting from 1200 MW to 500 MW in steps of 50 MW, we simply use a brute-force technique wherein all combinations of units will be tried as in previous example. The results obtained are shown below.

Load	Optimum combination		
	Unit 1	Unit 2	Unit 3
1200	On	On	On
1150	On	On	On
1100	On	On	On
1050	On	On	On
1000	On	On	Off
950	On	On	Off
900	On	On	Off
850	On	On	Off
800	On	On	Off
750	On	On	Off
700	On	On	Off
650	On	On	Off
600	On	Off	Off
550	On	Off	Off
500	On	Off	Off

The shut-down rule is quite simple.

When load is above 1000 MW, run all three units; between 1000 MW and 600 MW, run units 1 and 2; below 600 MW, run only unit 1.

It is to be noted that, in this brute-force technique, neglecting the number of infeasible solutions, we need to solve the economic dispatch problem $(2^N - 1)^M$ where N is the number of units and M is the load steps. This number will be too large for practical case. UC problem become more complicated when we need to consider power system having several plants each plant having several generating units.

So far, we have only obeyed one simple constraint: *Enough units will be connected to supply the load.* There are several other constraints to be satisfied in practical UC problem.

11 CONSTRAINTS ON UC PROBLEM

Some of the constraints that are to be met with while solving UC problem are listed below.

1. **Spinning reserve:** There may be sudden increase in load, more than what was predicted. Further there may be a situation that the generating unit may have to be shut down because of fault in generator or any of its auxiliaries.

   Some system capacity has to be kept as spinning reserve

   i) to meet an unexpected increase in demand and

   ii) to ensure power supply in the event of any generating unit
       suffering a forced outage.

2. **Minimum up time:** When a thermal unit is brought in, it cannot be turned off immediately. Once it is committed, it has to be in the system for a specified minimum up time.

3. **Minimum down time:** When a thermal unit is decommitted, it cannot be turned on immediately. It has to remain decommitted for a specified minimum down time.

4.   **Crew constraint: A plant always has two or more generating units. It may not be possible to turn on more than one generating unit at the same time due to non-availability of operating personnel.**

5.   **Transition cost: Whenever the status of one unit is changed some transition cost is involved and this has to be taken into account.**

6.   **Hydro constraints: Most of the systems have hydroelectric units also. The operation of hydro units, depend on the availability of water. Moreover, hydro-projects are multipurpose projects. Irrigation requirements also determine the operation of hydro plants.**

7. **Nuclear constraint: If a nuclear plant is part of the system, another constraint is added. A nuclear plant has to be operated as a base load plant only.**

8. **Must run unit: Sometime it is a must to run one or two units from the consideration of voltage support and system stability.**

9. **Fuel supply constraint: Some plants cannot be operated due to deficient fuel supply.**

10. **Transmission line limitation: Reserve must be spread around the power system to avoid transmission system limitation, often called "bottling" of reserves.**

12 PRIORITY- LIST METHOD

In this method the full load average production cost of each unit is calculated first. Using this, priority list is prepared.

$$\left.\begin{array}{l}\text{Full load average} \\ \text{production of a unit}\end{array}\right\} = \frac{\text{Production cost corresponding to full load}}{\text{Full load}}$$

Example 9

The following are data pertaining to three units in a plant.

Unit 1:   Max. = 600 MW

$C_1 = 5610 + 79.2\, P_1 + 0.01562\, P_1^2$ Rs / h

Unit 2:   Max. = 400 MW

$C_2 = 3100 + 78.5\, P_2 + 0.0194\, P_2^2$ Rs / h

Unit 3:   Max. = 200 MW

$C_3 = 936 + 95.64\, P_3 + 0.05784\, P_3^2$ Rs / h

Obtain the priority list

Solution

Full load average
production of a unit 1 $\Bigg\}$ $= \dfrac{5610 + 79.2 \text{ x } 600 + 0.01562 \text{ x } 600^2}{600} = 97.922$

Full load average
production of a unit 2 $\Bigg\}$ $= \dfrac{3100 + 78.5 \text{ x } 400 + 0.0194 \text{ x } 400^2}{400} = 94.01$

Full load average
production of a unit 3 $\Bigg\}$ $= \dfrac{936 + 95.64 \text{ x } 200 + 0.05784 \text{ x } 200^2}{200} = 111.888$

A strict priority order for these units, based on the average production cost, would order them as follows:

Unit	Rs. / h	Max. MW
2	94.01	400
1	97.922	600
3	111.888	200

The commitment scheme would (ignoring min. up / down time, start – up costs etc.) simply use the following combinations.

Combination	Load P_D
2 + 1 + 3	1000 MW $\leq$ P_D < 1200 MW
2 + 1	400 MW $\leq$ P_D < 1000 MW
2	P_D < 400 MW

Note that such a scheme would not give the same shut – down sequence described in Example 7 wherein unit 2 was shut down at 600 MW leaving unit 1. With the priority – list scheme both units would be held on until load reached 400 MW, then unit 1 would be dropped.

Most priority schemes are built around a simple shut – down algorithm that might operate as follows:

At each hour when the load is dropping, determine whether dropping the next unit on the priority list will leave sufficient generation to supply the load plus spinning reserve requirements. If not, continue operating as is; if yes, go to next step.

Determine the number of hours, H, before the unit will be needed again assuming the load is increasing some hours later. If H is less than the minimum shut – down time for that unit, keep the commitment as it is and go to last step; if not, go to next step.

Calculate the two costs. The first is the sum of the hourly production costs for the next H hours with the unit up. Then recalculate the same sum for the unit down and add the start – up cost. If there is sufficient saving from shutting down the unit, it should be shut down; otherwise keep it on.

Repeat the entire procedure for the next unit on the priority list. If it is also dropped, go to the next unit and so forth.

Questions on " Economic Dispatch and Unit Commitment"

1. What do you understand by Economic Dispatch problem?

2. For a power plant having N generator units, derive the equal incremental cost rule.

3. The cost characteristics of three units in a power plant are given by

$$C_1 = 0.5 P_1^2 + 220 P_1 + 1800 \text{ Rs / hour}$$

$$C_2 = 0.6 P_2^2 + 160 P_2 + 1000 \text{ Rs / hour}$$

$$C_3 = 1.0 P_3^2 + 100 P_3 + 2000 \text{ Rs / hour}$$

where P_1, P_2 and P_3 are generating powers in MW. Maximum and minimum loads on each unit are 125 MW and 20 MW respectively. Obtain the economic dispatch when the total load is 260 MW. What will be the loss per hour if the units are operated with equal loading?

4. The incremental cost of two units in a power stations are:

$$\frac{dC_1}{dP_1} = 0.3\ P_1 + 70\ \text{Rs / hour}$$

$$\frac{dC_2}{dP_2} = 0.4\ P_2 + 50\ \text{Rs / hour}$$

a) Assuming continuous running with a load of 150 MW, calculate the saving per hour obtained by using most economical division of load between the units as compared to loading each equally. The maximum and minimum operational loadings of both the units are 125 and 20 MW respectively.

b. What will be the saving if the operating limits are 80 and 20 MW ?

5.   A power plant has two units with the following cost characteristics:

$C_1 = 0.6\,P_1^2 + 200\,P_1 + 2000$ Rs / hour

$C_2 = 1.2\,P_2^2 + 150\,P_2 + 2500$ Rs / hour

where P_1 and P_2 are the generating powers in MW. The daily load cycle is as follows:

6:00 A.M. to 6:00 P.M.   150 MW

6:00 P.M. to 6:00 A.M.   50 MW

The cost of taking either unit off the line and returning to service after 12 hours is Rs 5000. Considering 24 hour period from 6:00 A.M. one morning to 6:00 A.M. the next morning

a. Would it be economical to keep both units in service for this 24 hour period or remove one unit from service for 12 hour period from 6:00 P.M. one evening to 6:00 A.M. the next morning ?

b. Compute the economic schedule for the peak load and off peak load conditions.

c. Calculate the optimum operating cost per day.

d. If operating one unit during off peak load is decided, up to what cost of taking one unit off and returning to service after 12 hours, this decision is acceptable ?

6. What do you understand by "Loss coefficients"?

7. The transmission loss coefficients B_{mn}, expressed in MW^{-1} of a power system network having three plants are given by

$$B = \begin{bmatrix} 0.0001 & -0.00001 & -0.00002 \\ -0.00001 & 0.0002 & -0.00003 \\ -0.00002 & -0.00003 & 0.0003 \end{bmatrix}$$

Three plants supply powers of 100 MW, 200 MW and 300 MW respectively into the network. Calculate the transmission loss and the incremental transmission losses of the plants.

8. Derive the coordination equation for the power system having N number of power plants.

9. The fuel input data for a three plant system are:

$$f_1 = 0.01 \, P_1^2 + 1.7 \quad P_1 + 300 \quad \text{Millions of BTU / hour}$$

$$f_2 = 0.02 \, P_2^2 + 2.4 \quad P_2 + 400 \quad \text{Millions of BTU / hour}$$

$$f_3 = 0.02 \, P_3^2 + 1.125 \, P_3 + 275 \quad \text{Millions of BTU / hour}$$

where P_i's are the generation powers in MW. The fuel cost of the plants are Rs 50, Rs 30 and Rs 40 per Million of BTU for the plants 1,2 and 3 respectively. The loss coefficient matrix expressed in MW^{-1} is given by

$$B = \begin{bmatrix} 0.005 & -0.0005 & -0.001 \\ -0.0005 & 0.01 & -0.0015 \\ -0.001 & -0.0015 & 0.0125 \end{bmatrix}$$

The load on the system is 60 MW. Compute the power dispatch for $\lambda = 120$ Rs / MWh. Calculate the transmission loss.

Also determine the power dispatch with the revised value of λ taking 10 % change in its value.

Estimate the next new value of λ.

10. **What are Participating Factors? Derive the expression for Participating Factors.**

11. **Incremental cost of three units in a plant are:**

$$IC_1 = 1.2\ P_1 + 21 \quad Rs\ /\ MWh$$
$$IC_2 = 1.5\ P_2 + 15 \quad Rs\ /\ MWh$$
$$IC_3 = 1.6\ P_3 + 24 \quad Rs\ /\ MWh$$

where P_1, P_2 and P_3 are power output in MW. Find the optimum load allocation when the total load is 85 MW.

Using Participating Factors, determine the optimum scheduling when the load decreases to 75 MW.

12. **What is "Unit Commitment" problem? Distinguish between Economic Dispatch and Unit Commitment problems.**

13.   Discuss the constrains on Unit Commitment problem.

14.   Explain what is Priority List method.

ANSWERS

3. 64.29 MW   103.575 MW   92.145 MW   Rs 449.09

4. Rs 111.39   Rs 57.64

5. It is economical to operate unit 2 alone during the off peak period.

     86.1111 MW   63.8889 MW   0   50 MW   Rs 648833   Rs 15833

7. 30.8 MW   0.004   0.06   0.164

9. 18.7923 MW   15.8118 MW   18.5221 MW

     23.6506 MW   18.5550 MW   20.5290 MW

     139.769 Rs / MWh

11. 32.5 MW;   30.0 MW;   22.5 MW

     28.579 MW;   26.863 MW;   19.559 MW