UNIT-V

NUMERICAL PROTECTION

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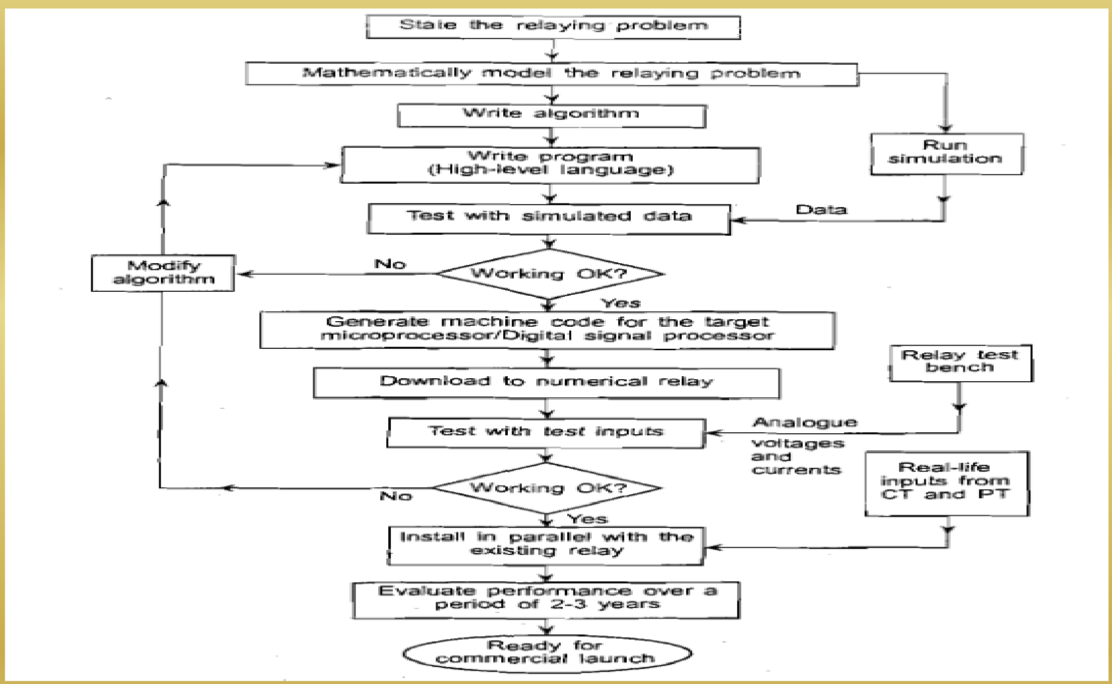
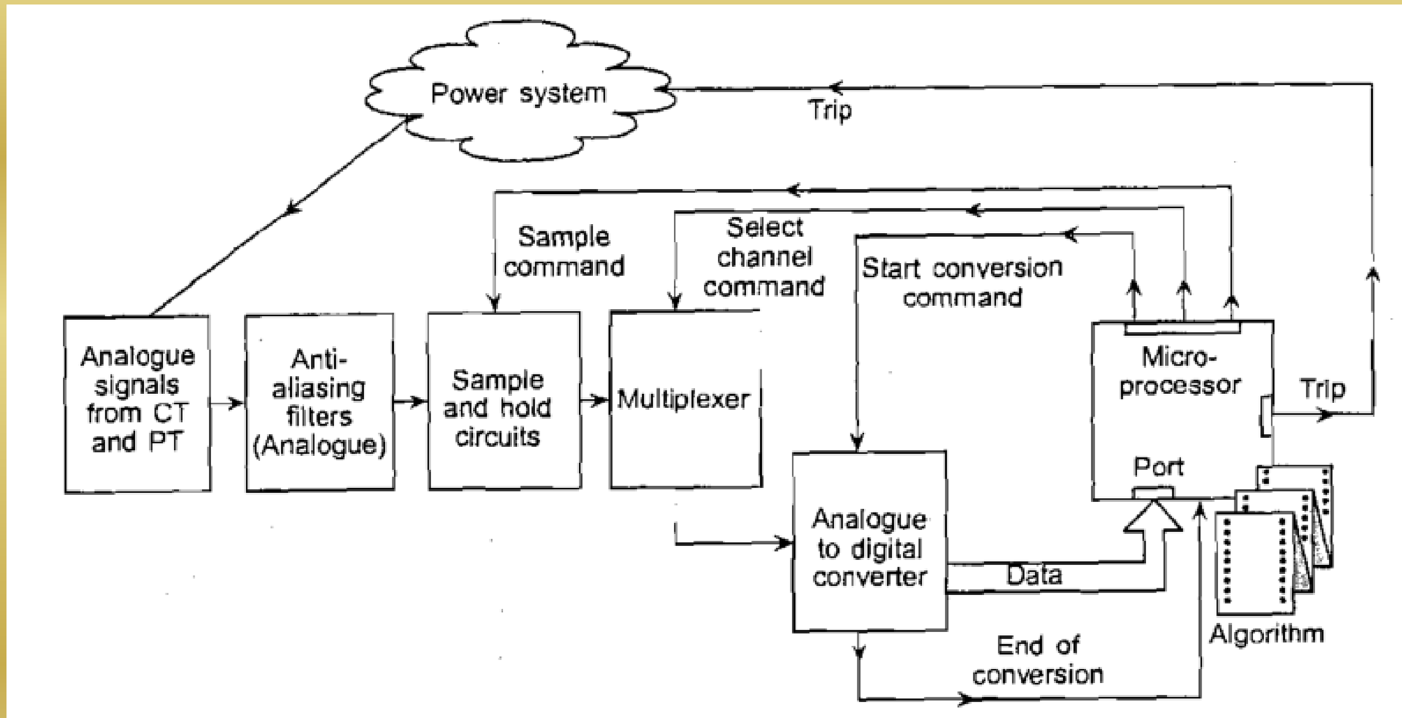
Development cycle of a new numerical relay



Development cycle of a new numerical relay

Block Diagram of Numerical Relay



Block diagram of numerical relay.

Comparison of maximum allowable frequency with and without S/H

Without S/H circuit	*With S/H circuit*
1. $\frac{dv}{dt_{\max}} = \frac{V_{\text{full scale}}}{2^n T_{\text{ADConv}}}$ where T_{ADConv} is the conversion time of the ADC.	1. $\frac{dv}{dt_{\max}} = \frac{V_{\text{full scale}}}{2^n T_{\text{S/H aperture}}}$ where $T_{\text{S/H}}$ aperture is the acquisition time of the S/H circuit.
2. $f_{\max} = \frac{1}{2\pi 2^n T_{\text{ADConv}}}$ n = ADC word length = 16 bits T_{ADConv} = 10 μs (Typical) $V_m = V_{\text{full scale}}$ Gives: $f_{\max} = 0.24$ Hz Thus, without S/H, the ADC can handle only extremely low frequencies.	2. $f_{\max} = \frac{1}{2\pi 2^n T_{\text{S/H aperture}}}$ n = ADC word length = 16 bits $T_{\text{S/H aperture}}$ = 250 ps (Typical) $V_m = V_{\text{full scale}}$ Gives: $f_{\max} = 9.7$ kHz With S/H, the same ADC can now handle much higher frequencies.

Sampling Theorem

$$\omega_{\text{sampling, min}} \geq 2\omega_{\text{signal}}$$

$$\omega_{\text{sampling, min}} \geq 2\omega_{\text{signal, max}}$$

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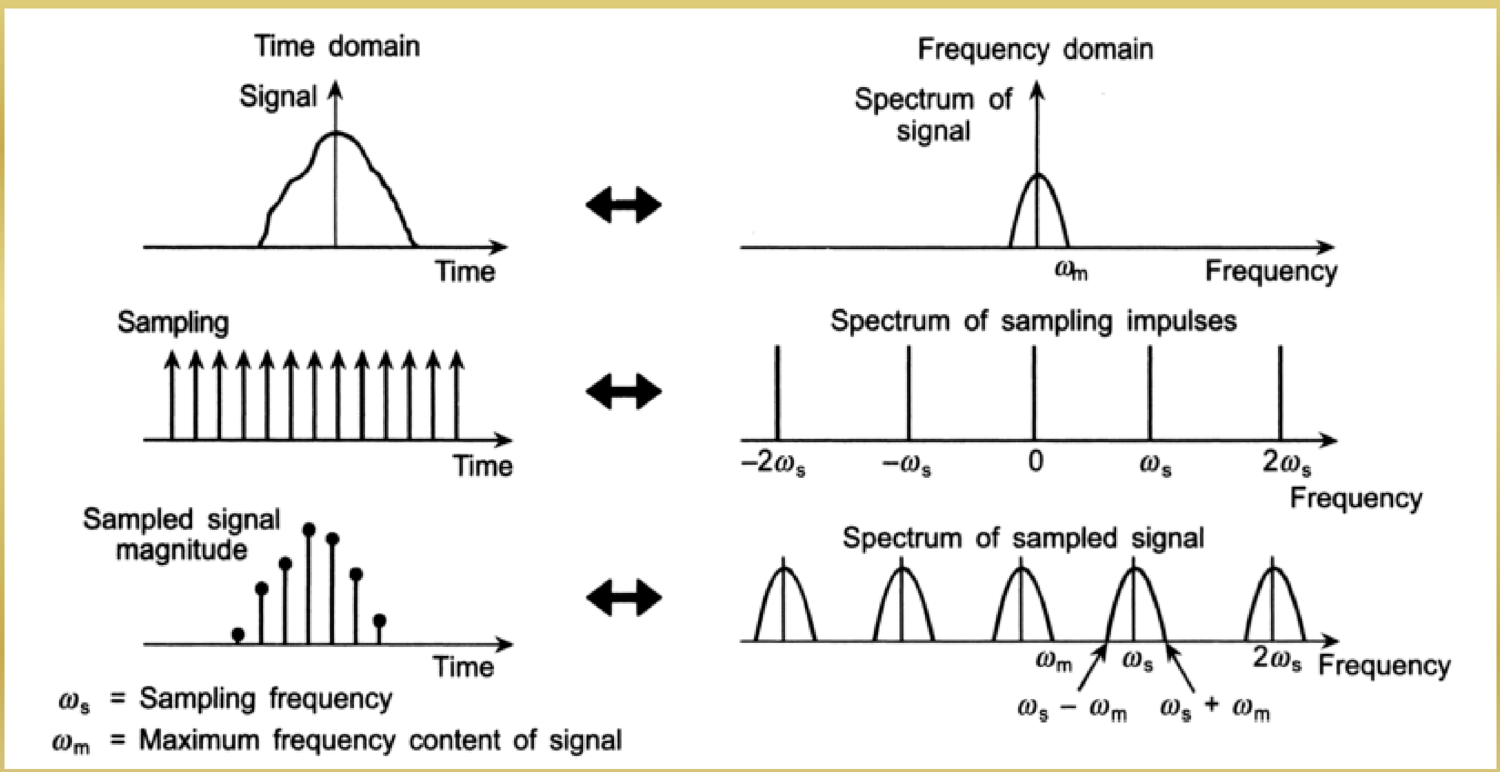

Time domain
Signal
Time
Frequency domain
Spectrum of signal
ωm
Frequency
Sampling
Time
Spectrum of sampling impulses
−2ωs
−ωs
0
ωs
2ωs
Frequency
Sampled signal magnitude
Time
Spectrum of sampled signal
ωm
ωs
2ωs
Frequency
ωs − ωm
ωs + ωm
ωs = Sampling frequency
ωm = Maximum frequency content of signal

Continued…

$$\omega_s - \omega_m > \omega_m$$

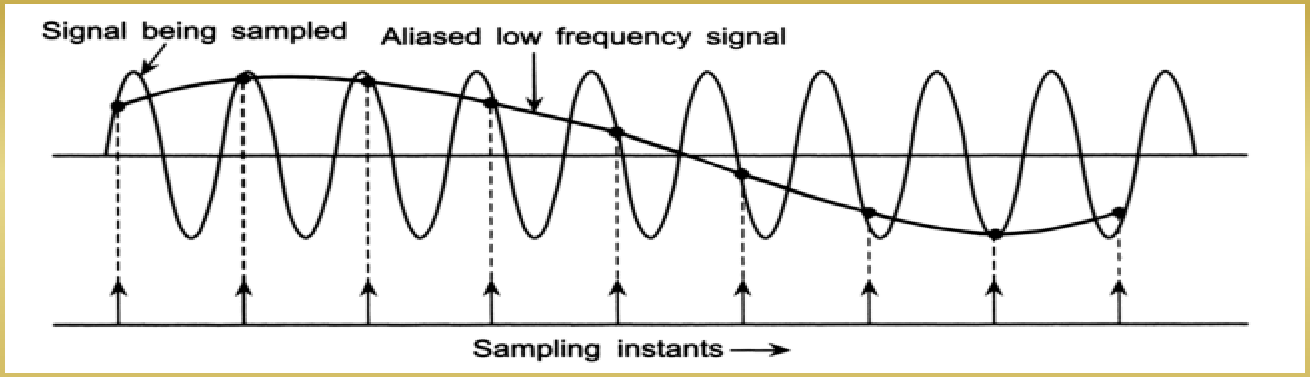
or

$$\omega_s > 2\omega_m$$

Therefore, we have

$$\omega_{sampling,\ min} > 2\omega_{signal,\ max}$$

Continued…



Phenomenon of aliasing.

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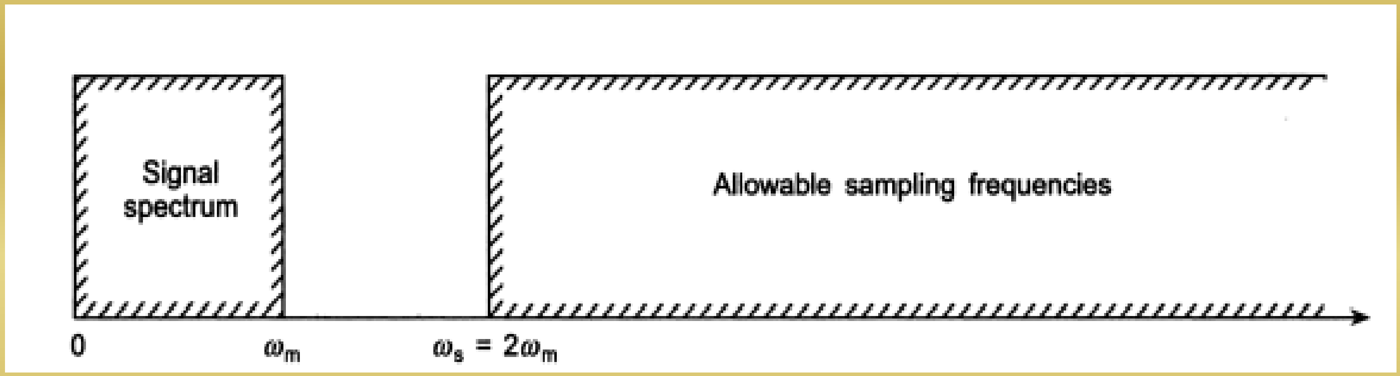

Minimum sampling frequency

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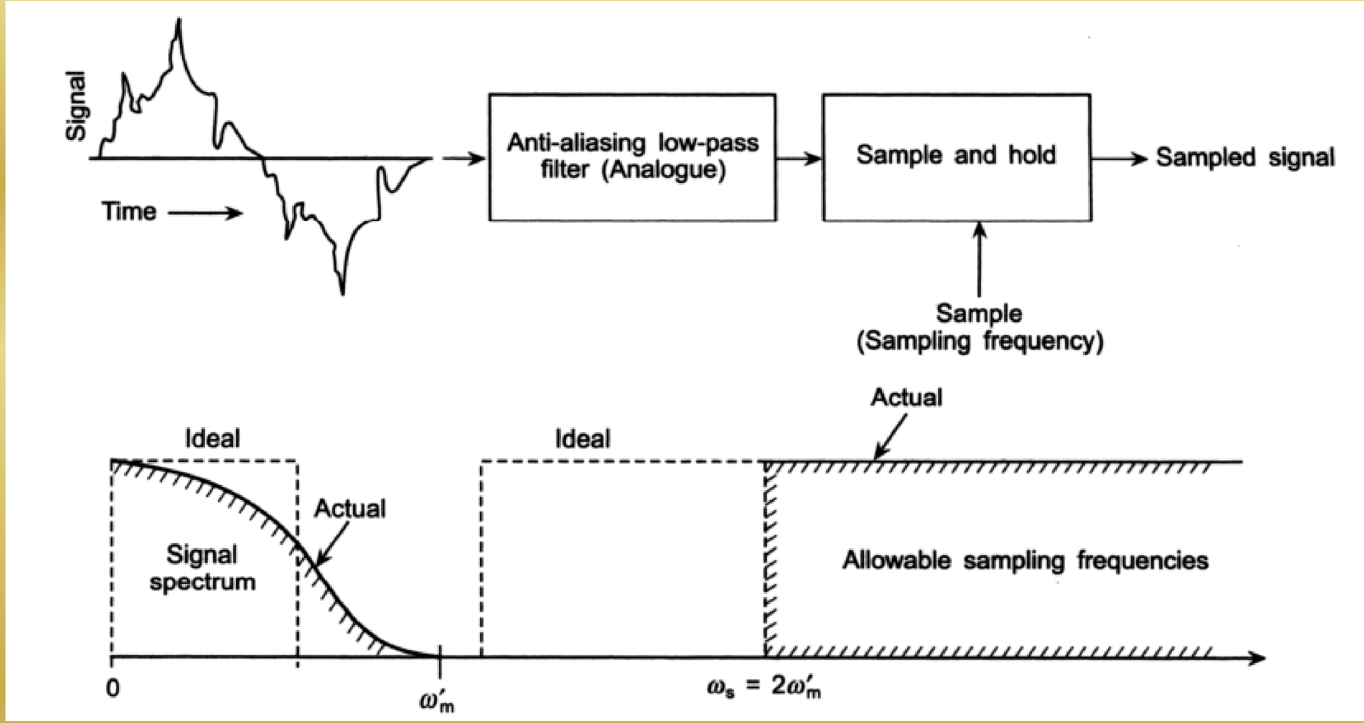


Practical limit on minimum sampling frequency.

Correlation with a Reference Wave

$$c_n = \frac{1}{K}\int_a^b f(t)\phi_n(t)\,dt$$

$$M = \int_a^b \left[f(t) - \sum_{n=0}^{N} a_n \phi_n(t) \right]^2 dt$$

Least Error Squared (LES) Technique

The LES technique helps us in estimating the values of these components.
The assumed signal is:

$$i(t) = K_1 e^{-t/\tau} + \sum_{n=1}^{N} K_{2n} \sin(n\omega_1 t + \theta_n)$$

For the sake of illustration, assuming that the current consists of a dc offset, the fundamental and a third harmonic component, we can write

$$i(t) = K_1 e^{-t/\tau} + K_{21} \sin(\omega_1 t + \theta_1) + K_{23} \sin(3\omega_1 t + \theta_3)$$

We can represent $e^{-t/\tau}$ as a sum of an infinite series, i.e.

$$e^{-t/\tau} = 1 - \frac{t}{\tau} + \frac{t^2}{2!\tau^2} - \frac{t^3}{3!\tau^3} + \cdots$$

Assuming that truncating the series for $e^{-t/\tau}$, to the first three terms, gives adequate accuracy, we get

$$i(t) = K_1 - \frac{K_1}{\tau} t_1 + \frac{K_1 t^2}{2!\,\tau^2} + K_{21} \cos\theta_1 \sin\omega_1 t + K_{21} \sin\theta_1 \cos\omega_1 t$$
$$+ K_{23} \cos\theta_3 \sin 3\omega_1 t + K_{23} \sin\theta_3 \cos 3\omega_1 t$$

Continued…

Unknowns	*Knowns* (These can be precalculated)
$x_1 = K_1$	$a_{11} = 1$
$x_2 = K_{21} \cos \theta_1$	$a_{12} = \sin \omega_1 t$
$x_3 = K_{21} \sin \theta_1$	$a_{13} = \cos \omega_1 t$
$x_4 = K_{23} \cos \theta_3$	$a_{14} = \sin 3\omega_1 t$
$x_5 = K_{23} \sin \theta_3$	$a_{15} = \cos 3\omega_1 t$
$x_6 = -K_1/\tau$	$a_{16} = t$
$x_7 = K_1/2\tau^2$	$a_{17} = t^2$

Thus, we can write

$$
\begin{aligned}
i(t_1) &= a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 + a_{16}x_6 + a_{17}x_7 \\
i(t_2) &= a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 + a_{25}x_5 + a_{26}x_6 + a_{27}x_7 \\
&\vdots \\
i(t_7) &= a_{71}x_1 + a_{72}x_2 + a_{73}x_3 + a_{74}x_4 + a_{75}x_5 + a_{76}x_6 + a_{77}x_7
\end{aligned}
$$

Continued…

$$\begin{bmatrix} i(t_1) \\ i(t_2) \\ i(t_3) \\ i(t_4) \\ i(t_5) \\ i(t_6) \\ i(t_7) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & a_{17} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} & a_{27} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} & a_{37} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} & a_{47} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} & a_{57} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} & a_{67} \\ a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & a_{77} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix}$$

which in shorthand notation can be written as

$$[i]_{7\times 1} = [A]_{7\times 7}\,[x]_{7\times 1}$$

Thus, we can find the unknowns $[x]_{7\times 1}$ as

$$[x]_{7\times 1} = [A]^{-1}_{7\times 7}\,[i]_{7\times 1}$$

If we are interested in the fundamental, then since

$$x_2 = K_{21}\cos\theta_1$$

$$x_3 = K_{21}\sin\theta_1$$

Continued…

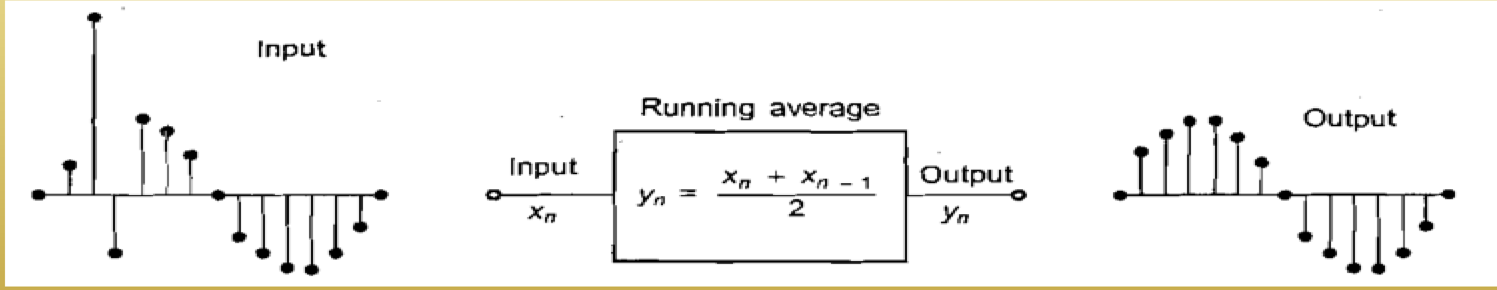
Amplitude of fundamental, $F_1 = \sqrt{x_2^2 + x_3^2}$

Phase angle of fundamental, $\theta_1 = \tan^{-1}\left(\frac{x_3}{x_2}\right)$

Digital Filtering

- Simple Low-pass Filter

$$y_n = \frac{x_n + x_{n-1}}{2}$$



A simple running average filter works as a low-pass filter

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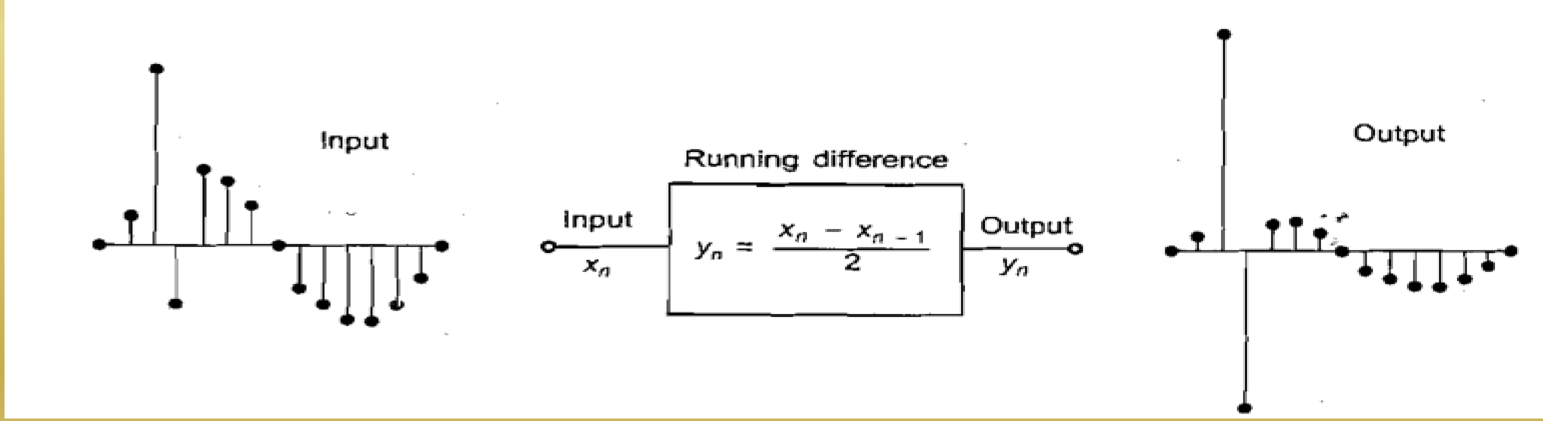
- Simple High-pass Filter

$$y_n = \frac{x_n - x_{n-1}}{2}$$



A simple running difference filter works as a high-pass filter

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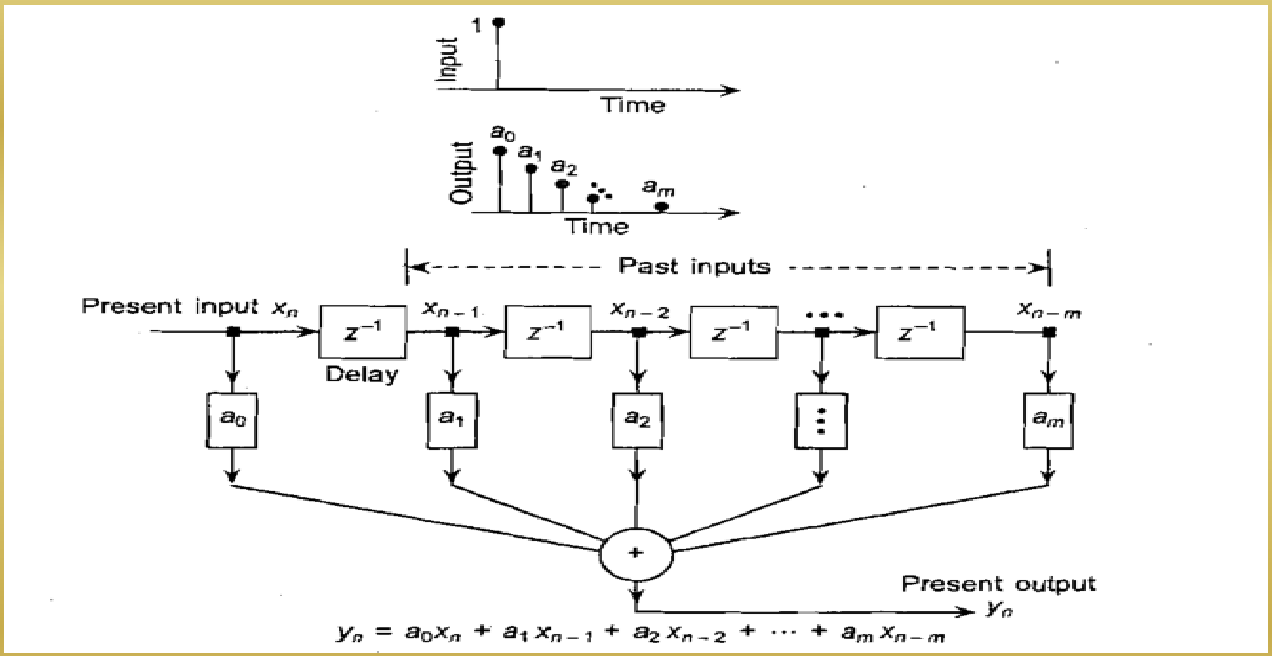
- Finite Impulse Response (FIR) Filters



Block diagram of FIR digital filter (canonical

Continued…

$$y(n) = a_0 x_n + a_1 x_{n-1} + \cdots + a_m x_{n-m}$$

$$f(j\omega) = \sum_{n=0}^{m} e^{(-j\omega n \Delta t)} a_m$$

$$\frac{Y(z)}{X(z)} = \frac{a_0 z^m + a_1 z^{m-1} + a_2 z^{m-2} + \cdots + a_m}{z^m}$$

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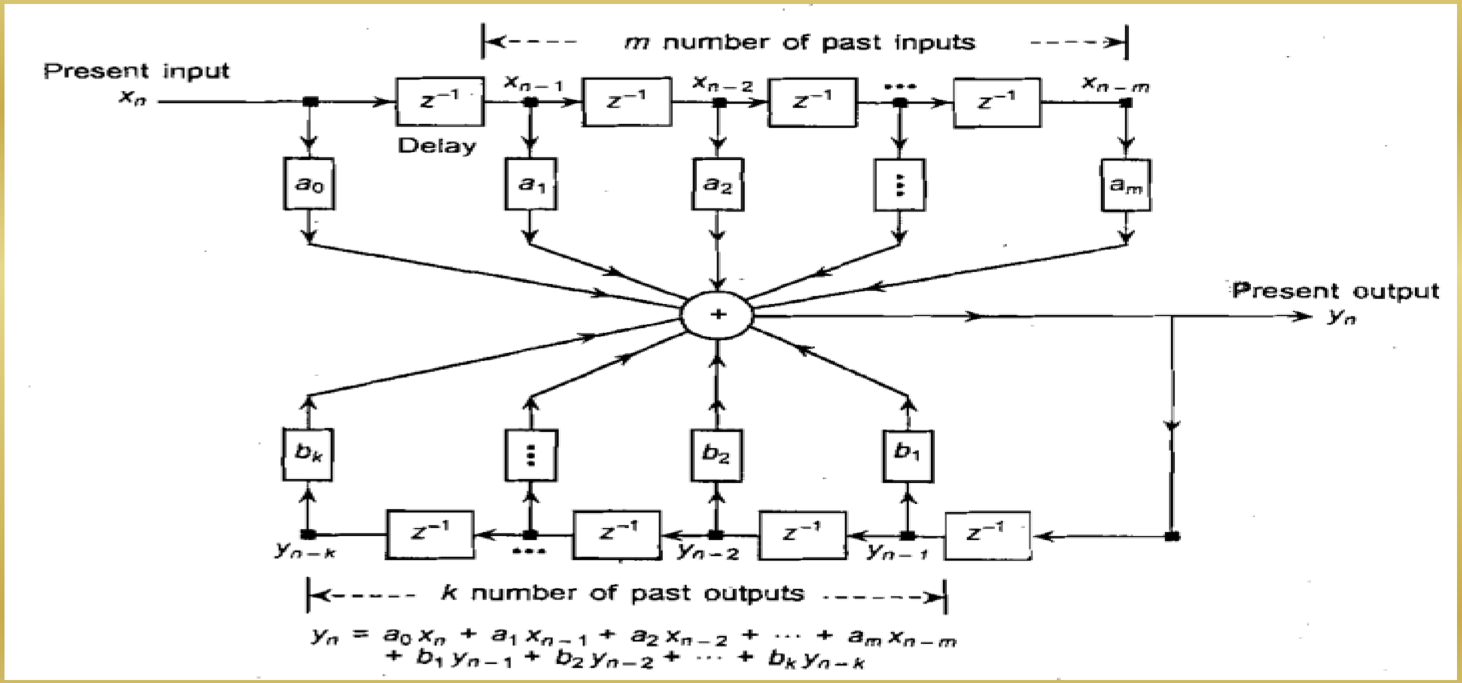
- Infinite Impulse Response (IIR) Filter



Block diagram of IIR digital filter (canonical

Continued…

The output at the nth sampling instant is given by

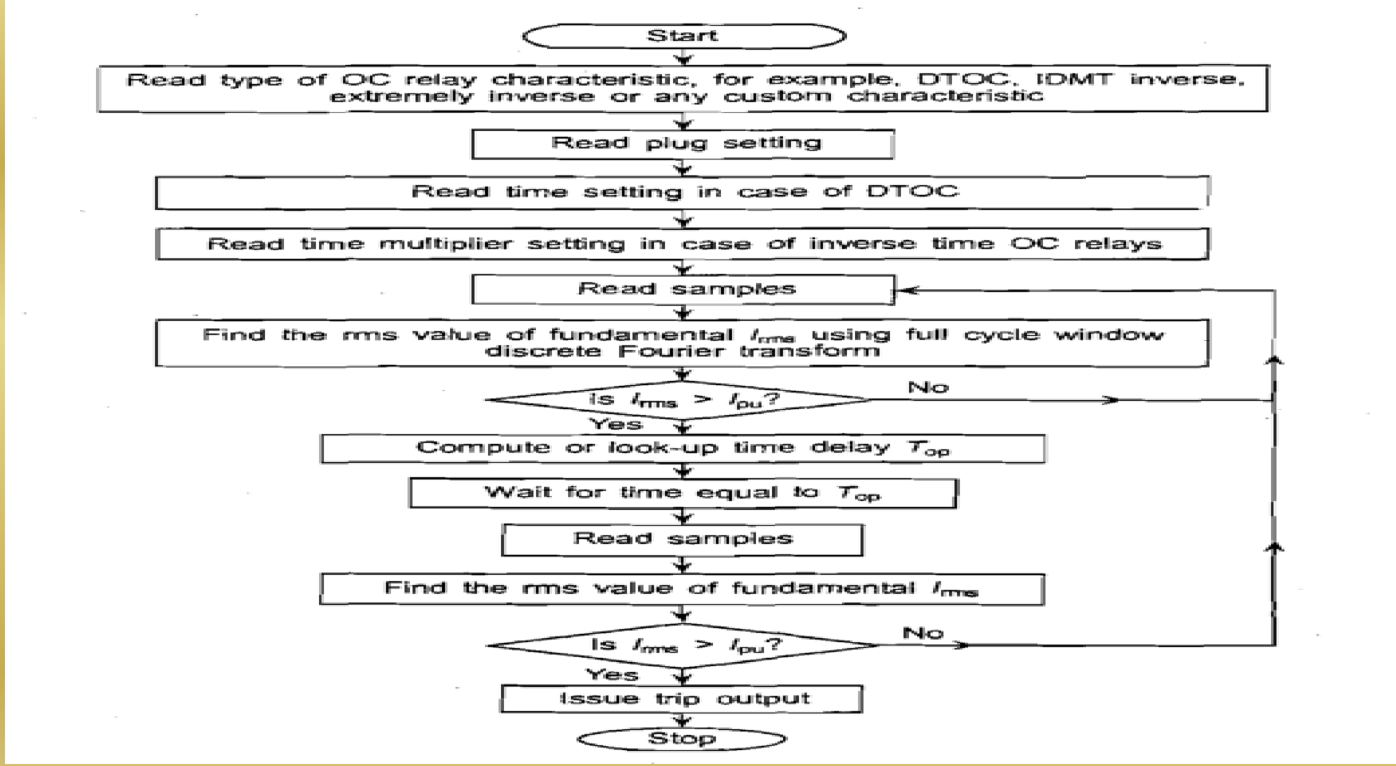
$$y_n = a_0 x_n + a_1 x_{n-1} + \cdots + a_m x_{n-m} + b_1 y_{n-1} + b_2 y_{n-2} + \cdots + b_k y_{n-k}$$

$$\frac{Y(z)}{X(z)} = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_m z^{-m}}{1 - b_1 z^{-1} - \cdots - b_k z^{-k}}$$

Comparison Between FIR and IIR Filters

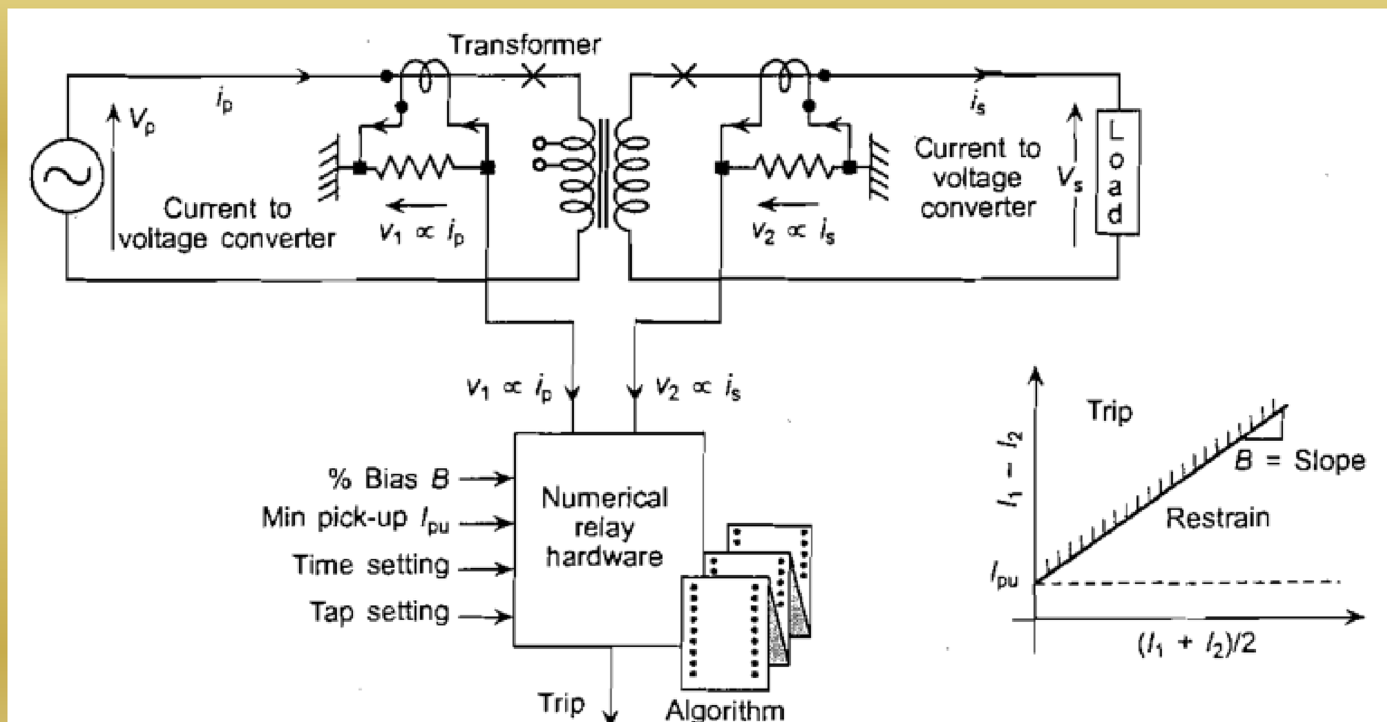
FIR Filter	*IIR Filter*
Output is a function of past m inputs. Therefore, non-recursive.	Output is a function of past m inputs as well as k past outputs. Therefore recursive.
Finite impulse response.	Infinite impulse response.
Always stable since there is no feedback.	Because of feedback, possibility of instability exists.
Has less number of coefficients.	Has more number of coefficients.
Transfer function has only the numerator terms.	Transfer function has both the numerator and denominator terms.
Higher-order filter required for a given frequency response.	Lower-order filter required for a given frequency response.
Has linear phase response.	Has nonlinear phase response.
$y_n = a_0 x_n + a_1 x_{n-1} + \cdots + a_m x_{n-m}$	$y_n = a_0 x_n + a_1 x_{n-1} + \cdots + a_m x_{n-m} + b_1 y_{n-1} + b_2 y_{n-2} + \cdots + b_k y_{n-k}$
$\frac{Y(z)}{X(z)} = \frac{a_0 z^m + a_1 z^{m-1} + a_2 z^{m-2} + \cdots + a_m}{z^m}$	$\frac{Y(z)}{X(z)} = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_m z^{-m}}{1 - b_1 z^{-1} - \cdots - b_k z^{-k}}$
Very simple to implement.	Not as simple as the FIR filter.

Numerical Over-current Protection



Flowchart for a numerical over-current relay algorithm.

Numerical Transformer Differential Protection



Block diagram of numerical protection of transformer

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Algorithm for percentage differential relay will consist of the following steps:

- Read percentage bias B and minimum pick-up I_{pu}.
- Read i_p samples → Estimate phasor $\mathbf{I}_p$ using any technique.
- Read i_s samples → Estimate phasor $\mathbf{I}_s$ using any technique.
- Compute spill current $I_{spill} = \mathbf{I}_p - \mathbf{I}_s$.
- Compute circulating current $I_{circulating} = (\mathbf{I}_p + \mathbf{I}_s)/2$.
- If $I_{spill} > (BI_{circulating} + I_{pu})$ then trip, else restrain.

Review questions

1. Trace the evolution of protective relays.
2. What are the advantages of numerical relays over conventional relays?
3. What paradigm shift can be seen with the development of numerical relays?
4. Draw the block diagram of the numerical relay
5. What do you mean by aliasing?
6. State and explain Shannon's sampling theorem.

Continued…

7. What happens if the sampling frequency is less than the Nyquist limit?
8. What are the drawbacks of a very high sampling frequency?
9. Is sample and hold circuit an absolute must?
10. A 12-bit ADC has conversion time of 10 microseconds. What is the maximum

frequency that can be acquired without using a sample and hold unit?

11. If a sample and hold circuit of 100 picoseconds is available, how will the maximum frequency found out in Question 10 be affected?
12. Explain the statement that all numerical relays have the same hardware but what distinguishes the relay is the underlying software.

Continued…

13. Explain the sample and derivative methods of estimating the rms value and phase angle of a signal. Clearly state the underlying assumptions.
14. What do you mean by Fourier analysis? Explain.
15. How does Fourier transform differ from conventional Fourier analysis?
16. What do you mean by a full cycle window?
17. What are the advantages and disadvantages of a half cycle window?
18. What do you mean by a digital filter? Explain.
19. Draw the block diagram of an FIR and an IIR filter.
20. Compare the FIR and IIR filters.
21. Develop the differential equation algorithm for distance protection of a transmission line.
22. For numerical relaying purpose the differential equation gets converted into a linear algebraic equation. Explain.
23. Discuss the methods to find numerical differentiation and numerical integration.
24. How can certain frequencies be filtered out in solving the differential equation by integration?